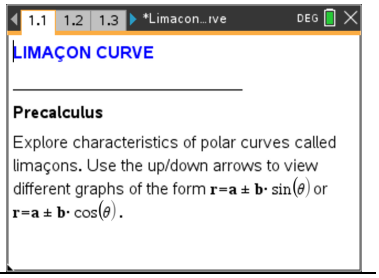
Limaçon Curve

Student Activity

Name _____

Class _____

In this activity, you will investigate the effect of changing the values of *a* and *b* in the polar equations $r = a \pm b * sin(\theta)$ and $r = a \pm b * cos(\theta)$, where $a > 0$ and $b > 0$. You will also explore the relationship between the polar curve $r = a \pm b * sin(\theta)$ (or $r = a \pm b * cos(\theta)$) and the sinusoidal function $f(x) = a + b * sin(x)$ (or $f(x) = a \pm b * cos(x)$).



Polar curves called limaçons have equations of the form $r = a \pm b * sin(\theta)$ or $r = a \pm b * cos(\theta)$. On page 1.2 are special kinds of limaçon graphs called cardioids. Use the up/down arrows to see all the graphs.

Move to page 1.2.

1. Why do you think these graphs are called cardioids?

2. What similarities do you notice about the equations of the six graphs?

3. How do the addition and subtraction signs affect the graphs?

Move to page 1.3.

Use the up/down arrows to see all the graphs.

4. How are the equations different from those on page 1.2? How does this difference affect the graph?

Limaçons have different shapes depending on the ratio $\frac{a}{b}$. We have already seen the cardioid graph that is the result when $a = b$ (or $\frac{a}{b} = 1$).

Move to page 1.5.

Graph the polar functions shown in the table by changing a and b. Complete the table with the values of a, b, and $\frac{a}{b}$ as you observe each graph. Use the up/down arrows to change the values of a and b.

Limaçon	a	b	$\frac{a}{b}$
$i)\ r1 = 1 + 3 * cos(\theta)$			
$ii)\ r1 = 1 + 2 * cos(\theta)$			
$iii)\ r1 = 3 + 2 * cos(\theta)$			
$iv)\ r1 = 2 + 1 * cos(\theta)$			
$v)\ r1 = 3 + 1 * cos(\theta)$			

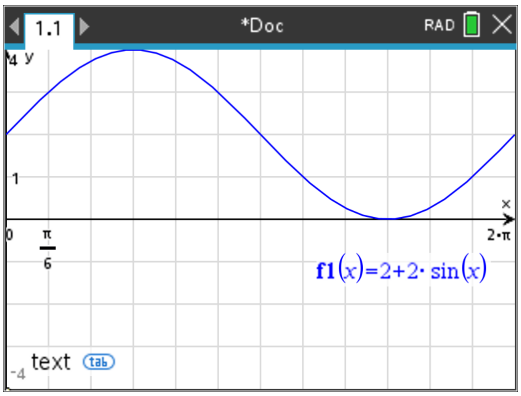
5. Some of the limaçons in the table above have the ratio $\frac{a}{b} < 1$. Write an equation of another polar curve for which $\frac{a}{b} < 1$. Graph your limaçon and describe the shape of the limaçon.

6. One of the polar curves in the table above has a ratio which satisfies $1 < \frac{a}{b} < 2$. Write an equation of another polar curve for which $1 < \frac{a}{b} < 2$. Graph your limaçon and describe the shape of the limaçon.

7. Some of the limaçons in the table above have the ratio $\frac{a}{b} > 2$. Write an equation of another polar curve for which $\frac{a}{b} > 2$. Graph your limaçon and describe the shape of the limaçon.

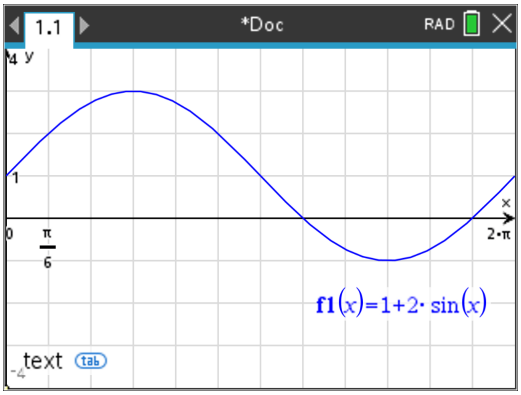
Move to page 2.2.

8. The graph of the sinusoidal function $f(x) = 2 + 2 * sin(x)$ on the interval $0 \leq x \leq 2\pi$ is shown below. The x-scale is $\pi/6$.



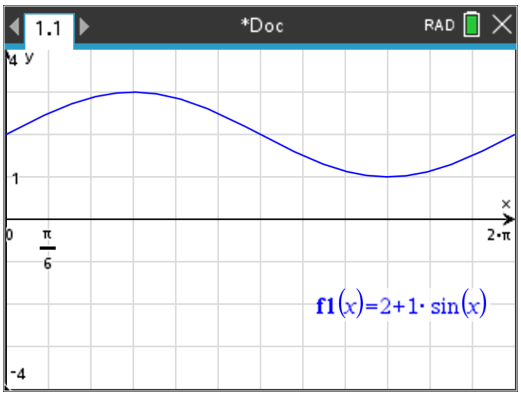
Graph the limaçon given by $r1(\theta) = 2 + 2 * sin(\theta)$. Press menu and select 3 Graph Entry/Edit and then 5 Polar. Now press menu and select 5 Trace and then 1 Graph Trace. Use the right arrow to move your cursor. Observe the change in the r and θ values.
On the interval from x = 0 to x = 2π of the sinusoidal function, the maximum occurs at $x = \frac{\pi}{2}$ ($\approx 1.6\ rad$) and the minimum occurs at $x = \frac{3\pi}{2}$ ($\approx 4.7\ rad$).
How do the y −values at these two points correspond to the r −values on the cardioid?

9. The graph of the sinusoidal function $f(x) = 1 + 2 * sin(x)$ on the interval $0 \leq x \leq 2\pi$ is shown below. The x-scale is $\pi/6$.



Graph the limaçon given by $r1(\theta) = 1 + 2 * sin(\theta)$. Trace along the graph of the limaçon. Observe the change in the r and θ values. Explain why the polar curve $r = 1 + 2sin(\theta)$ has an inner loop in the interval $\frac{7\pi}{6} \leq \theta \leq \frac{11\pi}{6}$ (θ between $\approx 3.7\ rad$ and $\approx 5.7\ rad$).

10. The graph of the sinusoidal function $f(x) = 2 + 1 * sin(x)$ on the interval $0 \leq x \leq 2\pi$ is shown below. The x-scale is $\pi/6$. 2+2*sin



Graph the limaçon given by $r1(\theta) = 2 + 1 * sin(\theta)$. Trace along the graph of the limaçon. Observe the change in the r and θ values. Explain why the polar curve does not contain the point located at the pole.