

Work-Energy Theorem Applied to a Cart Pulled Down a Track

Activity Overview

In this activity, students will analyze the relationship between the maximum speed of a cart pulled by a falling mass and the work done by gravity. Students will measure the speed of the cart, calculate the work in each trial, make a graph of speed vs. work, and determine an equation which will fit the data. This equation will be compared to the Work-Energy theorem.

Materials

- TI-Nspire™ technology
- Dynamics cart
- A known mass
- Meter stick
- Vernier CBR2™
- Track for cart with a pulley attached to one end
- String
- Triple-beam balance

Set-Up

- Measure and record the mass of the cart and the mass of the falling object.
- Place the track on a table and adjust the track so that it is level. The end of the track with the pulley should extend beyond the edge of the table. Place the cart on the track.
- Attach one end of the string to the cart, pass the string over the pulley at the end of the track, and suspend the mass from the free end of the string. The string length should be adjusted so that the cart can coast part of the way down the track after the mass hits the floor.
- Place the Vernier CBR2™ at the end of the track near the starting position of the cart so that it can record the motion of the cart down the track.

Data Collection

- Before beginning data collection, open a Lists & Spreadsheet page on the TI-Nspire™. Label the first column “h”. Use this column to record the height (measured in meters) from which the mass is released. Label the second column “v”, and use this column to record the speed of the cart as it coasts down the track after the mass hits the floor. The mass will be released from several different heights and the speed attained by the cart measured for each trial.
- Connect the CBR2™ to the TI-Nspire™, and open the meter in a Data & Statistics page when prompted. The meter should display the distance to the cart.
- With the mass initially resting on the floor, pull the cart back on the track and hold it in place so that the mass is suspended a short distance (~5 - 10 cm) above the floor. Measure this initial height and record it.
- Press the center button on the NavPad of the TI-Nspire™ to begin data collection. Release the cart when data collection begins (the CBR2™ will begin to tick rapidly), and stop data collection by pressing the button again when the cart reaches the end of the track. The cart should coast for some distance after the mass hits the floor. A graph of distance vs. time will appear on the screen. The graph should have a curved section (as the cart accelerated while being pulled by the falling mass) followed by a linear section (as the cart coasted after the mass hit the floor). The slope of the linear section will be the speed attained by the cart.

- Pull the cart back again so that the mass is raised to a greater height. Measure and record the new height from which the mass will be released. Press the button on the NavPad again. A dialog box will appear prompting you to Store or Discard the previous data. Choose “Store”, and the new data collection will begin.. Release the cart when the CBR2™ begins ticking. Complete several more trials using successively greater release heights. After the last trial, press the Tab key on the TI-Nspire™ until the “X” on the meter window is highlighted. Press the center button on the NavPad to close the meter window.

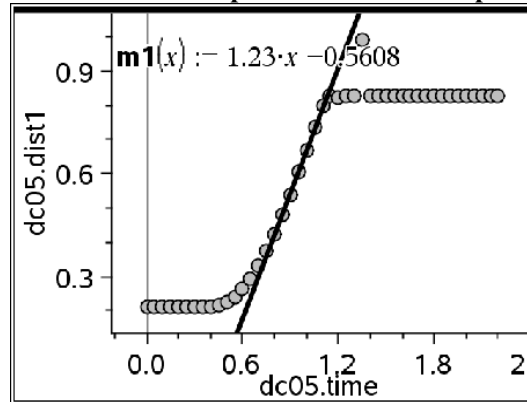
Data Analysis

- Go back to the Lists & Spreadsheet page by holding the CTRL button and pressing the Left Arrow on the NavPad. Enter the heights (measured in meters) in the column labeled “h”.
- Go to the Data & Statistics page by holding the CTRL button and pressing the Right Arrow on the NavPad. The maximum speed of the cart for each trial is the slope of the linear section of the distance vs. time graph. To start with the first data set, move the cursor over each axis label and click the center button on the NavPad. Change the x-axis to dc01.time and change the y-axis to dc01.dist1. Press the Menu button, choose “4: Analyze“, and then choose “2: Add Movable Line”. Using the NavPad, grab the line and align it with the linear section of the data. Record the slope of the line on the Lists & Spreadsheet page in the column labeled “v” next to the corresponding height. Return to the Data & Statistics page and change the axes to dc02.time and dc02.dist1. Continue the process until the speeds for all trials have been determined and recorded.
- The work done by gravity on the system is equal to the weight of the falling mass multiplied by the distance it falls (which is equal to the initial height). Return to the Lists & Spreadsheet page. Label the third column “work”. In the second row of that column, enter the formula “=(mass of the falling object) * 9.8 * ‘h’”. (Note the single quote before the column title h.) This will calculate the work done on the system for each trial.
- To begin analyzing the relationship between the speed of the cart and the work done on the system, make a graph of speed vs. work. On the Data & Statistics page, click on each axis label to change the variable. Set the x-axis as “work”, and set the y-axis as “v”. Consider the shape of the graph. Would a direct proportion fit the data? If not, what function could express the relationship between speed and work?
- According to the work-energy theorem, the work done on a system is equal to the change in kinetic energy of the system. Since the system starts from rest, the work should be equal to the kinetic energy acquired by the system. Since the work done by gravity causes a change in the kinetic energy of the falling mass as well as the cart, the mass of the system (M) is the sum of the mass of the cart and the mass of the falling object. Thus, $work = \frac{1}{2}Mv^2$. Solving for v yields
$$v = \sqrt{\frac{2}{M} * work}$$
- To compare the data with the above equation, we need to linearize the graph. On the Lists & Spreadsheet page, title a new column “sqrtwork”. In the second cell of the column enter the formula “=sqrt(‘work’)”. From the Home menu, add another Data & Statistics page. Set the x-axis as “sqrtwork” and the y-axis as “v”. Press the Menu button, choose 4: Analyze, and then choose 2: Add Movable Line. Press Menu again, choose 4: Analyze, and now choose 3: Lock Intercept at Zero. Grab the line with the cursor and adjust it so that it best fits the data. If the line fits the data well, this demonstrates that the speed of the system is directly proportional to the square root of the work done on the system. Note the slope of the line. Compare the slope to $\sqrt{2/M}$ where M is the mass of the cart plus the mass of the falling object. What might explain any discrepancy?

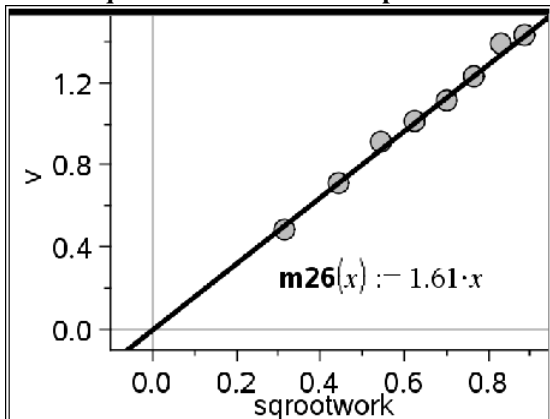
Sample Lists & Spreadsheet Page

	A h	B v	C work	D sqroot...
			=0.2*9.8*h	=√('work)
1	0.05	0.489	0.098	0.31305
2	0.1	0.711	0.196	0.442719
3	0.15	0.912	0.294	0.542218
4	0.2	1.01	0.392	0.626099
5	0.25	1.11	0.49	0.7
6	0.3	1.23	0.588	0.766812
A1	-0.05			

Sample Distance vs. Time Graph Used to Find the Speed of the Cart



Speed vs. Square Root of Work Graph



Speed vs. Work Graph with Function Plotted

