Name _____

Date _____

EXPLORATIONS

ACTIVITY 2

Getting Down to Basics

Linear models are among the most commonly used algebraic models for real-life situations. Situations can be modeled with a linear equation when you can find two pairs of related values that represent points on a line.

The model can be expressed in slope-intercept form $y = mx + b$ if the constant rate of change (the slope) and the initial value (the y-intercept) can be determined from the two points.

Instructions

Part A—Finding a Linear Model When Two Points Are Known

A common application for linear modeling is finding conversion formulas. For example, you need to convert a number of temperature readings taken in degrees Fahrenheit to degrees Celsius but have forgotten the formula. However, you do recall the corresponding values for both freezing and boiling in each scale.

1. Using the values for freezing (32° F, 0° C) and the values for boiling (212° F, 100° C), find two ordered pairs of data points in the form (Fahrenheit, Celsius) and record them below.

   point 1 = _____

   point 2 = _____

2. To find the slope-intercept form of the linear model, you must first find the slope. You can find the constant rate of change (or the slope in our model) by dividing the change in the y-values by the change in the x-values.

   Because the y-values represent Celsius readings and the x-values represent Fahrenheit readings, the slope is actually the change in Celsius divided by the change in Fahrenheit, or degrees Celsius per degrees Fahrenheit.

   The slope formula is commonly written as

   $$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

   Turn on your Voyage™ 200 PLT. To reset to the default settings, press [2nd] [MEM] [F1], select 1:RAM, select 2:Default, and then press [ENTER] [ENTER].

   Press [♦] [CALC HOME] to access the Home screen. Clear all one-character variables by pressing [F6], selecting 1:Clear a–z, and pressing [ENTER].

   Find the slope of the line m through the points in step 1 by entering them in the formula above and storing this value to m as follows:

   Press
   [(] **100** [−] **0** [)] [÷] [(] **212** [−] **32** [)] [STO▸] **M** [ENTER]

   Record the value of m below.

   $m =$ _____

3. Find the value of the y-intercept b by substituting one of the ordered pairs from step 1 into the equation $y = mx + b$ and solving for b as follows:

   Press [F2] and select 1:solve(.

   Press **0** [=] **M** [×] **32** [+] **B** [,] **B** [)].
   Your entry line should look like this:
   solve(0=m*32+b,b)

   Press [ENTER] to solve for b. Record your value for b below.

   $b =$ _____

   Enter your value for b on the entry line and press [STO▸] **B** [ENTER].

4. Using the values from step 2 and step 3, write the equation to convert Fahrenheit to Celsius in the form $y = mx + b$.

   _____

   Verify that this is correct by entering **Y** [=] **M** [×] **X** [+] **B** and pressing [ENTER].

5. To find the traditional form of this formula, factor the equation $y = mx + b$ as follows:

   Press [F2] and select 2:factor(. Next, press [▲] to highlight the equation in the history area and press [ENTER]. This pastes the equation onto the entry line.

   Press [)] [ENTER].

   Replace x with F (for Fahrenheit) and replace y with C (for Celsius). Record the equation on the line below.

   _____

Part B—Finding a Linear Model for Two Points in General

If the equation of a line can be found for two specific data points, it can be found for any two points in general. Once the equation of the line through any two points has been found, a rule for finding the slope and y-intercept for any two points will have been derived.

Use the Voyage™ 200 PLT to find this equation.

1. To begin, press [F6], select 1:Clear a–z, and press [ENTER] to clear all values stored to one-character variables.

2. Suppose you have any two points (e, f) and (g, h). Find the slope m of this line by substituting these variables into the slope formula given in step 2 in *Part A*. Record your answer on the line below.

   $m =$ _____

   Store this value in your Voyage 200 PLT by pressing [(] **H** [−] **F** [)] [÷] [(] **G** [−] **E** [)] [STO▸] **M** [ENTER].

3. To find the y-intercept b, substitute either point into the equation $y = mx + b$ and solve for b as follows.

   Press F2 and select 1:solve(.
   Choose the point (e, f), and press
   F = **M** × **E** + **B** , **B**).
   The line you entered should look like this:
   solve(f=m∗e+b,b)

   Press ENTER to find b. Record your answer below.

   $b =$ _____

   Store the y-intercept you found for b in your Voyage™ 200 PLT by entering the expression found above and pressing STO▸ **B** ENTER.

4. Use your values for m and b from step 2 and step 3 to write the slope-intercept form of the line containing (e, f) and (g, h) on the line below.

   $y =$ _____

   Verify that this is correct by pressing
   Y = **M** × **x** + **B** ENTER. Do they match?

   _____

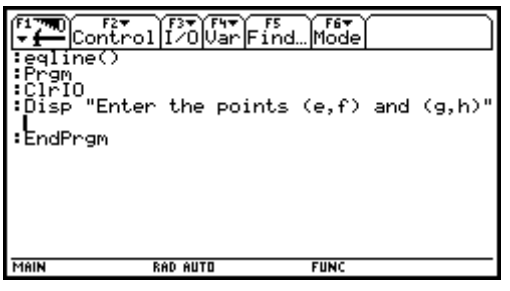
Part C—Writing a Simple Linear Model Program

The expressions for the slope and y-intercept that you found in the model in step 4 of *Part B* will always yield the correct values for any nonvertical line. They are too complicated, though, to be used easily on a regular basis. By using the expressions you already derived, however, you can write a simple program on the Voyage 200 PLT that will make it easy to find a linear model for any two points.

1. To begin a new program in the program editor, press APPS, select Program Editor, and then select 3:New.

   Press ▼ ▼ to move the cursor to the Variable box. Enter the name of your program, such as *eqline*, as the program variable.

   Press ENTER ENTER to display the template for a new program. The program name, Prgm, and EndPrgm are shown automatically.



2. Type the following program lines into the Voyage™ 200 PLT, between the Prgm and EndPrgm lines. Press [ENTER] at the end of each line.

   ***Note**: You can type commands such as **Disp** or **Prompt** directly on a program line, or you can access them by pressing [F3] and selecting the appropriate choice. You also can find them in the catalog by pressing [2nd] [CATALOG] and then pressing the first letter of the command.*

   ClrIO
   Clears the program I/O screen.

   Disp "Enter the points (e,f) and (g,h)"
   Tells the user what is being requested.

   Prompt e,f,g,h
   Asks the user for the x- and y-values of the points.

   (f-h)/(e-g) [STO▸] m
   Stores the result to m.

   (e*h-f*g)/(e-g) [STO▸] b
   Finds the y-intercept using the previously developed rule and stores the result to b.

   Disp "The slope is ", m
   Displays the slope on the Program I/O screen.

   Disp "The y intercept is ", b
   Displays the y-intercept on the Program I/O screen.

   m*x+b [STO▸] y
   Stores the slope-intercept form of the equation as y.

   Disp "The equation is y = ", y
   Displays the equation on the Program I/O screen.

3. When you have entered all of the lines, press [♦] [CALC HOME] to return to the Home screen.

   To start the program *eqline*, press **EQLINE** [(] [)] [ENTER].

   ***Note:** The empty parentheses are necessary for the program to run.*

   Enter the points from step 1 of *Part A* to confirm that your program is running properly.

Questions

Solve the following problems using any method learned in this activity.

1. Sarah and Matt are riding in the car to visit their grandparents in New Orleans. A sign beside the road says NEW ORLEANS: 372 MILES or 600 KILOMETERS. Later they pass a sign that reads NEW ORLEANS: 255 KILOMETERS.

   Use a linear model to convert kilometers to miles based on the information on the signs and the initial conditions. What is your model? How many more miles do they need to go?

   _____

   _____

2. Bill begins weight training and finds that he can lift 80 pounds on day one. On the tenth day of training, he is able to lift 108 pounds. Bill decides that the appropriate model for his strength gain is linear.

   Find a linear model that will predict how much weight Bill will be able to lift on any given day of training. Give an example that shows he was wrong when he chose a linear model.

   _____

   _____

   _____

Hint: *How much weight does his model predict he will lift in a year?*

Getting Down to Basics *(Continued)*

3. Brenda is driving down the highway with her brother Larry who is doing his homework in the back seat. Larry needs a linear model to convert miles to kilometers and asks Brenda for help. She glances at the speedometer and quickly gives him the answer. How did she find two data points to use in finding her model?

   Using only the speedometer of a parked car, find Brenda's model.

4. Maria and Mai are planning a graduation party at a local restaurant. They were given a set fee for their class and told that if they added another 12 guests, the cost would rise to $750. If they added 20 guests, the total cost would be $930. Find a linear model for the total cost of the party based on how many additional guests attend.

   How much will the party cost if no additional guests are invited? How much does the restaurant charge for each additional guest?

