Texas Instruments Activity #11
Title: Taking It to the Limit
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Estimated Time: 40-50 Minutes

NCTM Standards:

Communication Standard – Organize and consolidate their mathematical thinking through communication.

Connections Standard – Recognize and apply mathematics in contexts outside of mathematics.

Algebra Standard – Understand patterns, relations, and functions. Approximate and interpret rates of change from graphical and numerical data. Understand and compare the properties of classes of functions.

Topics in Calculus:

Limits, Properties of Limits

Overview:

In this activity, the students will explore the concept of limits through the use of the TI-89. The students will explore the limit rules for both finite and infinite limits. Each rule has set examples and a section for the students to try other examples to create a basis of understanding of each of the rules.

Supplies: TI-89 Graphing Calculator

Name: ____ Date: _________________________

taking it to the limit

In this activity, you will use the TI-89 graphing calculator to explore limits. Using the steps below, discover each of the Properties of Limits by finding examples to support each of them. Determine whether the rules apply under all circumstances and whether or not the rule applies when the variable approaches infinity. Write the definition for each of the properties of the limits.

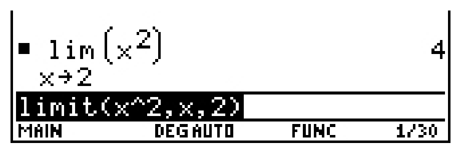
The following steps are used to find a limit of a function using the TI-89.

STEP ONE: Press $\boxed{\text{F3}}$, select 3:limit(and press $\boxed{\text{ENTER}}$.

STEP TWO: Following limit(, enter the function. (In this example, $f(x) = x^2$.)

STEP THREE: Now, following the function, press $\boxed{,}$, then the variable used, press $\boxed{,}$, then enter the parameter (what the variable approaches), finally, close the argument by pressing $\boxed{)}$. (In this case, the variable is x and the parameter is 2.)

STEP FOUR: Press $\boxed{\text{ENTER}}$ to find the limit.



1. Sum Rule: ____

Compute the following limits for the functions listed below:

$f(x) = 4x^2$

$g(x) = -2x^2$

$\lim_{x\to 0}\left(f(x)+g(x)\right) =$ __________ $\quad \lim_{x\to 0}\left(f(x)\right)+\lim_{x\to 0}\left(g(x)\right) =$ ___________

$\lim_{x\to 1/2}\left(f(x)+g(x)\right) =$ _________ $\quad \lim_{x\to 1/2}\left(f(x)\right)+\lim_{x\to 1/2}\left(g(x)\right) =$ _________

$\lim_{x\to 1000}\left(f(x)+g(x)\right) =$ _________ $\quad \lim_{x\to 1000}\left(f(x)\right)+\lim_{x\to 1000}\left(g(x)\right) =$ ________

$\lim_{x\to \infty}\left(f(x)+g(x)\right) =$ __________ $\quad \lim_{x\to \infty}\left(f(x)\right)+\lim_{x\to \infty}\left(g(x)\right) =$ ___________

Based off of the above calculations, does the *Sum Rule* apply to all limits? Try at least three different functions, including exponential and trigonometric functions. List the functions and the results. Now, is the Sum Rule valid for all functions and limits?

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Does the Sum Rule apply to two functions that are approaching infinity? _________Yes _________ No

2. Difference Rule: ____

Repeat the procedures from 1, subtracting instead of adding. Once again, use at least three different functions. NOTE: You do not have to retype all of the functions. You can use the ⏶ key to select the appropriate function. Then press [ENTER]. Finally, use the ◀ ▶ keys to make changes.

List the functions you used and the results. Does the difference rule hold for all of functions and limits?

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Does the Difference Rule apply to two functions that are approaching infinity? _______ Yes _______No

3. Product Rule: ____

Compute each of the limits for the functions listed below.

$f(x) = \sin(x)$

$g(x) = 2\cos(x)$

$$\lim_{x\to 0}\left(f(x)\cdot g(x)\right) = \_\_\_\_\_\_\_\_\_\_ \qquad \lim_{x\to 0}\left(f(x)\right)\cdot\lim_{x\to 0}\left(g(x)\right) = \_\_\_\_\_\_\_\_\_\_$$

$$\lim_{x\to 1/2}\left(f(x)\cdot g(x)\right) = \_\_\_\_\_\_\_\_\_\_ \qquad \lim_{x\to 1/2}\left(f(x)\right)\cdot\lim_{x\to 1/2}\left(g(x)\right) = \_\_\_\_\_\_\_\_\_\_$$

$$\lim_{x\to 1000}\left(f(x)\cdot g(x)\right) = \_\_\_\_\_\_\_\_\_\_ \qquad \lim_{x\to 1000}\left(f(x)\right)\cdot\lim_{x\to 1000}\left(g(x)\right) = \_\_\_\_\_\_\_\_\_\_$$

$$\lim_{x\to \infty}\left(f(x)\cdot g(x)\right) = \_\_\_\_\_\_\_\_\_\_ \qquad \lim_{x\to \infty}\left(f(x)\right)\cdot\lim_{x\to \infty}\left(g(x)\right) = \_\_\_\_\_\_\_\_\_\_$$

From the calculations you performed above, do you think that the product rule can be used for all functions and limits? Calculate the limits of at least three different types of functions. Make several limit calculations for each of the functions. Let each of the limits approach zero, small numbers, large numbers, and infinity.

List the functions you used and the results. Does the multiplication rule for all functions and limits?

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Does the Multiplication Rule apply to two functions that are approaching infinity? _____ Yes _____No

4. Division Rule: ____

Compute each of the limits for the functions listed below.

$f(x) = 2x^2 + 3x - 2$

$g(x) = 4x - 1$

$$\lim_{x\to 0}\left(f(x)/g(x)\right) = \_\_\_\_\_\_\_\_\_\_ \qquad \lim_{x\to 0}\left(g(x)\right)/\lim_{x\to 0}\left(f(x)\right) = \_\_\_\_\_\_\_\_\_\_$$

$$\lim_{x\to 0}\left(f(x)/g(x)\right) = \_\_\_\_\_\_\_\_\_\_ \qquad \lim_{x\to 0}\left(g(x)\right)/\lim_{x\to 0}\left(f(x)\right) = \_\_\_\_\_\_\_\_\_\_$$

$f(x)=2e^{4x}$	$\lim\limits_{x\to\infty}\left(f(x)/g(x)\right)=$ __________	$\lim\limits_{x\to\infty}\left(g(x)\right)/\lim\limits_{x\to\infty}\left(f(x)\right)=$ ___________
$g(x)=e^{2x}-1$	$\lim\limits_{x\to\infty}\left(f(x)/g(x)\right)=$ __________	$\lim\limits_{x\to\infty}\left(g(x)\right)/\lim\limits_{x\to\infty}\left(f(x)\right)=$ ___________

Try several different numbers that x approaches, including small and large numbers, then, calculate the limits of at least three different types of functions. Make several limit calculations for each of the functions.

List the functions you used and the results. Does the division rule for all functions and limits?

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Does the Division Rule apply to two functions that are approaching infinity? ________ Yes _______ No

Are there any restrictions on the value of the limit of the quotient? If yes, what are they? ______________

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5. Power Rule: ____

	$\lim\limits_{x\to 0}\left(f(x)\right)^2=$ __________	$\left(\lim\limits_{x\to 0}\left(f(x)\right)\right)^2=$ ___________
$f(x)=x-1$	$\lim\limits_{x\to 1/4}\left(f(x)\right)^2=$ __________	$\left(\lim\limits_{x\to 1/4}\left(f(x)\right)\right)^2=$ ___________
	$\lim\limits_{x\to 10000}\left(f(x)\right)^2=$ __________	$\left(\lim\limits_{x\to 10000}\left(f(x)\right)\right)^2=$ ___________
	$\lim\limits_{x\to\infty}\left(f(x)\right)^2=$ __________	$\left(\lim\limits_{x\to\infty}\left(f(x)\right)\right)^2=$ ___________

From the calculations you performed above, do you think that the power rule can be used for all functions and limits? Calculate the limits of at least three different types of functions. Make several limit calculations for each of the functions. Let each of the limits approach zero, small numbers, large numbers, and infinity.

List the functions you used and the results. Does the multiplication rule for all functions and limits?

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Does the Power Rule apply when a function approaches infinity? __________ Yes __________No

6. Constant Multiple Rule: ____

Using the TI-89, show that the Constant Multiple Rule is valid using examples. Then, explain why the Constant Multiple works for infinity. (HINT: Use the properties of infinity.)