TEACHER NOTES

# Homestead-Miami Chase Activity 

STEM Lesson for TI-Nspire ${ }^{\text {TM }}$ Technology

Objective: Students will create scatter plots given data in a table. Students will then do regressions to model the data.


#### Abstract

About the Lesson: Homestead-Miami Speedway has been rebuilt in different configurations more often than any other track on the NASCAR circuit. Originally built as part of the Hurricane Andrew (1992) recovery efforts, the track opened in 1995 as a $60 \%$ version of the Indy 500, a rectangle with round corners; reflecting the fact that in 1995 the dominant form of motor sports in the US was still Indy Cars. The flat 6 degree banking and short straights made for single groove (boring) racing and treacherous crash angles in the corners. You can still race (digitally) this original layout by going to some on-line racing sites. The track was widened in 1996 to alleviate some of its racing issues, but not very successfully, so the track was rebuilt again as a traditional oval in 1997; however, still with relatively flat low banked turns. Finally in 2003 the track was rebuilt once more but with the current progressive 18 to 20 degree banking allowing for multi-groove racing and more passing. In this activity you will investigate turn radii and banking as part of the drive path around this track. Drivers have to decide if it is better to go faster or take the shortest path. Can you decide?


## Materials: Student Worksheet

Prerequisite skills: The students need to be familiar with all family of graphs including exponentials. Students also need basic trigonometry skills.

## Analysis:

## Using TI-Nspire Technology

1. Open the file named Homestead_Chase_Activity.tns.
2. Move to page 1.2.

Driving on a larger radius path around a curve allows a car to go faster, but the path is longer. The force required to impart the centripetal acceleration to turn the car onto a radius of $r$ is given by the formula first developed by Christian Huygens, circa 1657.

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3. The data in the spreadsheet shows the maximum speed in $\mathrm{ft} / \mathrm{sec}$ at different radii in ft .

| 4.1 | 1.2 | 1.3 *Homestead_nJ $\nabla$ |  |  | x |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{\text {A }}$ radius |  | ${ }^{\text {B }}$ speed | C D |  | 10 |
| - |  |  |  |  |  |
| 1 | 300 | 87.909 |  |  |  |
| 2 | 350 | 94.9526 |  |  |  |
| 3 | 400 | 101.509 |  |  |  |
| 4 | 450 | 107.666 |  |  |  |
| 5 | 500 | 113.49 |  |  | $\checkmark$ |
| A1 | 300 |  |  | 4 | - |

4. Move to page $\mathbf{1 . 3}$ to create a scatter plot of speed vs. radius.
5. What is the independent variable?

## Answer: Radius

6. What is the dependent variable?

## Answer: Speed

Navigator Tip: Quick Poll the students for their answers to \#5 and \#6 and show the results to make sure the students know which variable goes on which axis.
7. Press (1ar) and choose your independent variable. Press - then (ara) and select your dependent variable. Press enter.
8. Press (menu) then Window/Zoom > Zoom Data.

9. The data is not linear. What type of function do you think this is?

## Answer: Answers may vary, but it is actually power function.

10. Find a model for the data by doing a regression. Move back to page 1.2 and move to column C .
11. Press (menu) then choose Statistics > Stat Calc and choose the model you think best represents the data.
12. Fill in the form with the appropriate independent and dependent variables and leave the other boxes as the default settings.
13. Move to page 1.3.
14. Press menu $>$ Graph Type $>$ Function. The function bar will appear at the bottom of the screen. Press $\boldsymbol{\Delta}$ to see $f 1(x)$ and press enter. Your curve for single power should appear.

15. Press tab then $\Delta$ to see $f 1(x)$ and press enter to see your curve.
16. Did your model fit the data? If not repeat steps 11-15 for a different type of function. What is your equation?

Answer: The equation should be approximately $y=5.08 x^{0.5}$.
Navigator Tip: Quick Poll the students for their answers to \#16 and show the results to start a good discussion about models.

The issue is made more complex with track banking. Up to a banking angle of 45 degrees there is a maximum speed that a car can travel around a curve without sliding up the track and into the fence. On a banked track a portion of the force producing centripetal acceleration also adds to the down force on the car, thus increasing its traction on the track.


The extra variable $\theta$ changes the equation to $v=\left(\left(\left(32.17^{*} r\left(0.8^{*} \cos \theta+\sin \theta\right)\right) /(\cos \theta-\right.\right.$ $\left.\left.\left.0.8^{*} \sin \theta\right)\right)\right)^{\wedge}(0.5)$ where 32.17 is the gravitational constant and 0.8 is the coefficient of friction.
17. Move to page 1.4.

The Homestead-Miami track features progressive banking in the corners going from 18 to 20 degrees, so moving higher on the track allows a car to go faster due to both banking and an increase in curve radius. Assume for each half degree in banking the car is running on a path that is 15 ' larger in radius starting at $r=500$ for the 18 degree banking.
18. Compare the speed for the 18 degree banking and the speed for no banking from your model at a radius of $500^{\prime}$.

Answer: Answers will vary but $131 \mathrm{ft} / \mathrm{sec}$ compared to $114 \mathrm{ft} / \mathrm{sec}$ is a $17 \mathrm{ft} / \mathrm{sec}$ increase in speed for the banking.
19. Now compare two drivers going around the track on the same turn. One car decided to take the 18 degree banking part of the turn, but the other car decides to take the 19 degree part of the turn. Which one has the higher speed?

Answer: The car on the 19 degree banking.
20. Which car has to drive the shortest distance?

Answer: The car on 18 degree banking.

It is a trade off between speed and distance. The shorter path is only faster if the tires can hold the car around the tighter radius at a higher speed. Also the car on the shorter path must also brake earlier than the car on the outside. The key is to have the quickest time around the turn. That is what makes a NASCAR driver a professional.

## Using spreadsheet software

Driving on a larger radius path around a curve allows a car to go faster, but the path is longer. The force required to impart the centripetal acceleration to turn the car onto a radius of $r$ is given by the formula first developed by Christian Huygens, circa 1657.


| Radius (ft) | Speed (ft/sec) |
| :---: | :---: |
| 300 | 87.90904 |
| 350 | 94.95262 |
| 400 | 101.5086 |
| 450 | 107.6662 |
| 500 | 113.4901 |
| 550 | 119.0294 |
| 600 | 124.3222 |
| 700 | 134.2833 |

1. Enter the data from the table into spreadsheet software.
2. Use the chart wizard to create a scatter plot of speed vs. radius led for each driver. Chart type will be XY(Scatter). Click Next.
3. What is the independent variable?

## Answer: Radius

4. What is the dependent variable?

Answer: Speed
5. You will have to click on the Series tab to make sure the correct data is on the correct axis. If the wrong data is there, click in the box for the $x$ values then highlight the data for the independent variable.
6. Repeat step 5 for the $y$ values. Click Next.
7. Create titles for the chart and each axis. You can also click the other tabs to change the appearance of your graph. Click Next.
8. Decide if you want the graph to appear in the same window as your table or in a new window. Click Next and your graph should appear.
9. The data is not linear. What type of function do you think this is?

## Answer: Answers may vary, but it is actually power function.

10. Create a model of the data next. Right click on the data and select Add Trendline. Choose the type of function you chose from question 9 . Click the Options tab and click the box beside Display equation on chart. Click OK.
11. Did your model fit the data? If not right click on the trend line and change the type of function. Repeat this step until your model fits the data. What is your equation?

Answer: The equation should be approximately $y=5.08 x^{0.5}$.
The issue is made more complex with track banking. Up to a banking angle of 45 degrees there is a maximum speed that a car can travel around a curve without sliding up the track and into the fence. On a banked track a portion of the force producing centripetal acceleration also adds to the down force on the car, thus increasing its traction on the track.

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12. Use the following data to create a model of speed vs. banking angle.

| Banking Angle (degrees) | Speed (ft/sec) |
| :---: | :---: |
| 18 | 131.2432 |
| 18.5 | 133.5766 |
| 19 | 135.8864 |
| 19.5 | 138.1735 |
| 20 | 140.4384 |

13. Use the chart wizard to create a scatter plot of speed vs. radius led for each driver. Chart type will be XY(Scatter) with smoothed lines. Click Next.
14. What is the independent variable?

## Answer: Banking Angle

15. What is the dependent variable?

Answer: Speed

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16. You will have to click on the Series tab to make sure the correct data is on the correct axis. If the wrong data is there, click in the box for the $x$ values then highlight the data for the independent variable.
17. Repeat step 16 for the $y$ values. Click Next.
18. Create titles for the chart and each axis. You can also click the other tabs to change the appearance of your graph. Click Next.
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