

DIFFERENTIAL EQUATIONS ON THE TI-92 PLUS

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The purpose of this paper is to familiarize the reader with the features and capabilities of the TI-92 Plus calculator which have a direct impact on the teaching of a standard first course in ODE. This paper is based on version 2.05 of the TI-92 Plus operating system.

SECTION ONE: THE DESOLVE COMMAND

The deSolve command ("C" in the Calculus menu) can be used to find symbolic solutions of differential equations. Its use is limited to a single first- or second-order equation. It can be used to find general solutions or to find particular solutions of initial-value and boundary-value problems. Of course, it cannot solve all such problems.

The "prime" operator ([2nd] [B]) is used to enter the differential equation. Use two "prime" symbols to indicate a second derivative. The format for finding a general solution is:

deSolve(D.E. , indep var, dep var).

An example of this format is deSolve($y'' + 2y' - 3y = \sin(x)$, x , y). Note that arbitrary constants are labeled using the @ symbol. The format for finding a particular solution is:

deSolve(first-order DE and condition , indep var, dep var), or

deSolve(second-order DE and condition1 and condition2 , indep var, dep var).

For a 2nd-order equation, the two conditions must be either the values of the function and its derivative at a common point (a "standard" initial value problem) or the value of the function at two different points. Valid examples would include:

deSolve($p' + 2p - q = 0$ and $p(1) = 2$, q , p),

deSolve($3y'' - 2y' - y = e^x$ and $y'(0) = 1$ and $y(0) = 2$, x , y), and

deSolve($3y'' - 2y' - y = 0$ and $y(3) = 1$ and $y(0) = 2$, x , y).

Occasionally, deSolve will return an implicit solution when an explicit solution is possible. In this case, the explicit solution can usually be found using the "solve" command (from the algebra menu), although it may be necessary to make some assumptions as to the range of values for one or both of the variables (e.g. $x > 0$). More frequently, the solution returned by deSolve will be in a more complicated form than necessary and some algebraic manipulation (on the calculator) will be required to simplify the answer.

Basically, the deSolve command will solve almost all of the standard first- and second-order equations students are expected to solve analytically in a first course in ODE (second-order Cauchy-Euler equations seem to be an exception). The reader is encouraged choose a standard text and to use the deSolve

command on randomly selected problems from the sections on separable equations, homogeneous equations, exact equations, first-order linear equations, second order linear equations with constant coefficients, the method of undetermined coefficients, and variation of parameters. Generally speaking, if the students are expected to solve an equation analytically (and it isn't higher than second order), deSolve will solve it.

It's important to understand that the arbitrary constants denoted with the @ symbol can be treated just like other variables in many ways. They can be used as the unknown in solve commands and they can be given temporary values using the "with" operator ([]). They cannot be assigned a permanent value using "define". The @ symbol itself is entered by pressing [2nd] [R].

As an example, suppose the general solution is found for $y'' + y = 0$ with the constants labeled @1 and @2. After defining $y(x)$ to be that solution the following sequence can occur:

entry: solve($y(0) = 5$ and $y(\pi/2) = 7$, { @1, @2 })

answer: @1 = 5 and @2 = 7

entry: $y(x) | @1 = 5$ and $@2 = 7$

answer: $5 \cos(x) + 7 \sin(x)$

which yields the particular solution to the boundary-value problem.

SECTION TWO: THE ODE GRAPHER

The ODE grapher in the TI-92 Plus is designed to handle a system of up to 99 first-order D.E. The independent variable is always t and the dependent variables are called $y1'$, $y2'$, ..., $y99'$. To use the grapher with a higher-order D.E., it must be converted to an equivalent system of first order equations in the usual way (see example 2 below). The grapher can use either of two numerical methods: a modified Runge-Kutta method or Euler's method. In most circumstances, the Runge-Kutta method should be the preferred method. Euler should only be used if accuracy is not as important, and speed is a primary concern.

A graph of the solution may be displayed in a variety of formats. The grapher can generate slopefields for 1-dimensional problems and direction fields for 2-dimensional problems. It can also plot multiple solutions based on interactively chosen initial conditions. Numerical solution data is available through TRACE and TABLE, and it can also be accessed on the home screen or stored in a data variable to generate lists and/or statistics plots.

An ODE Graphing Check-List

To put the calculator in **DIFF EQUATIONS** mode, press [MODE] and choose [6] in the graph options menu.

1. Press [Y=] and enter the differential equation(s) and any initial conditions. Initial conditions are not necessary if you want to graph a slopefield or direction field without a particular solution. On this screen, you may also select which equations will be graphed and what "style" will be used for each graph.
2. While still on the [Y=] screen, press [◆][F] or [F1][9] to access the "graph formats" dialog. Be sure that the "solution method" and "fields" options are set as desired.

- Exit "graph formats" and return to the [Y=] screen. Unless the "SLPFLD" option is selected, press [F7] to access the "axes" dialog and choose which variable(s) will be represented by the horizontal and vertical axes in the graph.
- Press [WINDOW] to select the "window" parameters. These parameters include the boundaries of the xy-screen and the t -interval to be used for the solution. Depending on other settings, there are 2 to 4 additional parameters at the bottom of this screen.
- Select [GRAPH] to graph the solution.

Example 1

Graph the solution of the differential equation $y' = -xy$, with initial condition $y(0)=2$.

In **DIFF EQUATIONS** mode, and go to the [Y=] screen. Remember that the independent variable is t and we can make the dependent variable $y1$. Enter the equation as $y1' = -t*y1$. Enter the initial condition by setting $t0 = 0$ and $y1 = 2$. ($t0$ can also be set on the WINDOW screen.) Make sure the statistics plots (Plot1, etc., located above $t0$) are off, and select the "Line" style for $y1'$ in the Style menu ([F6]). Note the "check-mark" next to $y1'$ means it is "selected". (See Figure 1.)

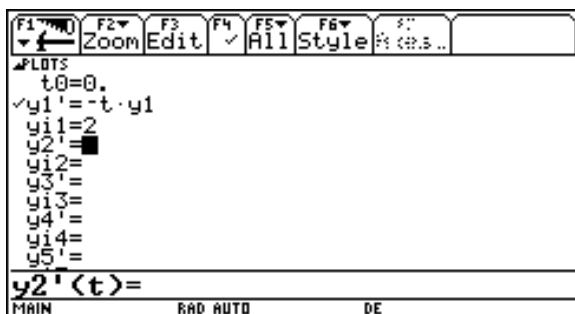


Figure 1



Figure 2

Next press [◆][F] or [F1][9] to access the "graph formats" dialog, and set the solution method to "RK" and the fields option to "FLDOFF" (Figure 2). Return to the [Y=] screen and press [F7] to go to the "axes" dialog and set the axes option to "TIME" (Figure 3). Lastly, set the values on the WINDOW screen to match those shown in figure 4.

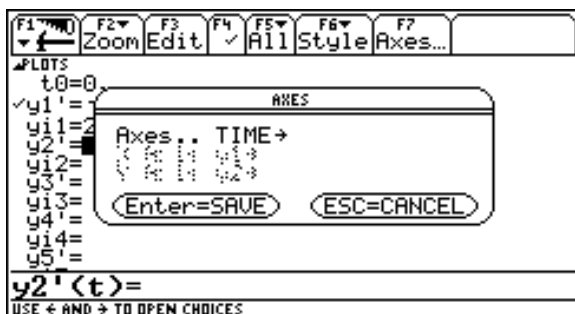


Figure 3

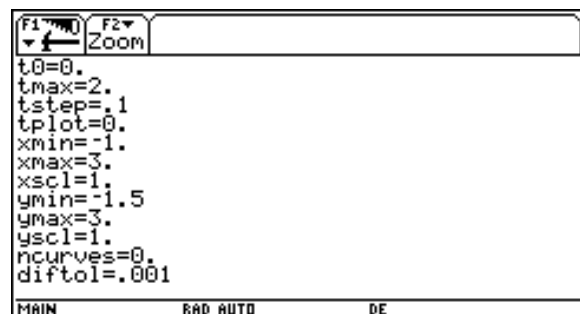


Figure 4

Now press GRAPH to obtain the graph shown in figure 5.

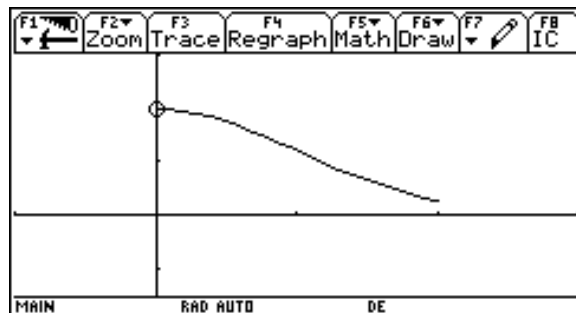


Figure 5

Try the following activities with this differential equation:

- Use TRACE to see numerical values corresponding to the solution. You can move to new t -values using the left/right arrow keys, or by typing in the desired t -value and pressing ENTER. (The value of t must be in the interval specified by t_0 and t_{\max} .)
- Go to the home screen, and type $y_1'(1.75)$ to see the value of the solution at $t=1.75$. Note that this is very unfortunate notation, since this is really $y_1(1.75)$, not $y_1'(1.75)$.
- Go to the TblSet dialog and set $\text{tblStart} = -0.4$ and $\Delta\text{tbl} = 0.4$. Press TABLE to see a table of values for the solution. Note the values outside of the interval $[t_0, t_{\max}]$.
- Go back to the graph and superimpose the analytic solution $y = 2 \exp(-x^2/2)$ using DrawFunc in the DRAW menu. The DrawFunc command will appear on the home screen. Just type the function (without the "y =") following the command and press ENTER.
- Clear the function drawn in (d) using ClrDraw (in the Draw menu), then go to the "graph formats" dialog, and select SLPFLD. If you do this while on the graph screen, it will regraph automatically. Otherwise, press GRAPH again. Notice that the solution is now graphed across the entire slopefield and the values of t_0 and t_{\max} are ignored..
- Try deleting the initial condition on the [Y=] screen and regraphing. The grapher will graph just the slopefield without any solution. (Note: When FLDOFF is selected, initial conditions must be specified.)
- With the slopefield on the screen, choose IC ([F8]). Move the cursor to any point on the screen, and press ENTER to graph the solution which has that point as its initial condition. Now choose IC again and this time, enter the initial conditions by just typing the values for t and y_1 . This process can be repeated indefinitely to add additional solutions to the screen. Curves drawn using IC cannot be traced.

Example 2

Investigate the D.E. $y'' + xy' + 2y = \sin x$, with $y(0)=1$ and $y'(0)=-1$.

First, the equation must be converted to an equivalent system of two first-order equations. This is accomplished by introducing a new variable $w = y'$. We then have:

$$y'' = -xy' - 2y + \sin x \quad \text{or,} \quad w' = -xw - 2y + \sin x, \quad \text{with } y(0)=1 \text{ and } w(0)=-1.$$

Translating (x becomes t , y becomes y_1 , and w becomes y_2), the equations to enter are:

$$\begin{aligned}
 y_1' &= y_2 & (y' &= w) \\
 y_2' &= -t \cdot y_2 - 2y_1 + \sin t & (w' &= -xw - 2y + \sin x)
 \end{aligned}$$

where the initial values are $t_0 = 0$, $y_{i1} = 1$, and $y_{i2} = -1$. Enter these equations and initial conditions on the [Y=] screen as shown in figure 6. Use [F4] to deselect y_2 so it won't graph. Set the style for y_1 equal to "Line". Go to the "graph formats" screen and select FLDOFF. Check that the TIME option is selected in the Axes menu.

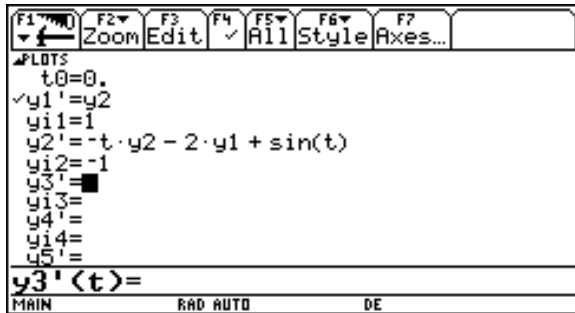


Figure 6

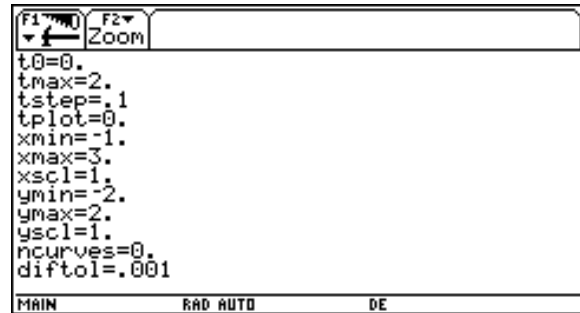


Figure 7

On the WINDOW screen, enter the values shown in figure 7. Now press GRAPH to graph the solution. The resulting graph (figure 8) is the graph of the solution as a function of the independent variable. In the original notation of the problem, it is a graph of the solution $y(x)$ vs. x .

Only y_1 is graphed because only y_1' was selected on the [Y=] screen. Even if both y_1' and y_2' were selected, you can get the same graph by choosing the CUSTOM option in the Axes menu and choosing "t" for the x-axis and "y1" for the y-axis. To investigate further, try the following:

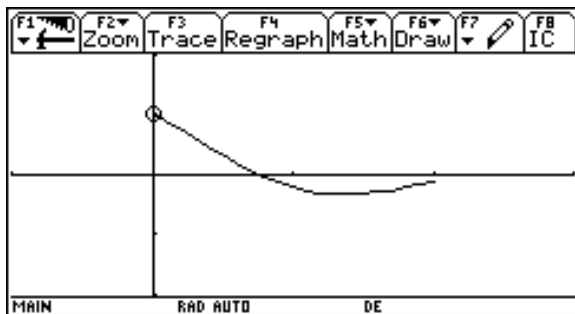


Figure 8

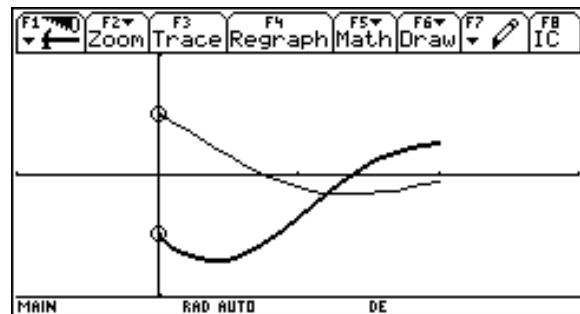


Figure 9

- With the TIME option still selected in the Axes menu, return to the [Y=] screen and select y_2' in addition to y_1' . Check that the style for y_2' is set for "Thick" (the default) and regraph. Both y_1 and y_2 will be graphed as functions of t . In other words, this is a graph of the solution and its derivative. (See figure 9.) Notice the two different graph styles. Both graphs can be traced.
- Note that if you use IC ([F8]) in this setting, you are asked what the axes represent. Normally, one would leave the x-axis set to "t" to avoid changing the meaning of the axes. For the y-axis you can choose either y_1 or y_2 . If the y-axis is set to y_1 , you may then choose new initial conditions for t and for y_1 , and the resulting solution y_1 is plotted. A corresponding graph for y_2 is not plotted. If the y-axis is set to y_2 , then new initial conditions may be set for t and y_2 and the resulting y_2 will be plotted, without a corresponding graph of y_1 .

- (c) Go to the HOME screen and type the command "BldData RKData". This will compute the values for all of the plotted points and place them in a data variable called RKData. (Any variable name may be used in place of RKData.) You can view these values using the Data/Matrix Editor (figure 10). The values may also be used to generate lists and statistics plots.

	F1	F2	F3	F4	F5	F6	F7
	Plot	Setup	Cell	Header	Calc	Util	Stat
DATA	t	y1	y2				
	c1	c2	c3	c4	c5		
1	0.	1.	-1.				
2	.1	.89069	-1.179				
3	.2	.76578	-1.311				
4	.3	.6302	-1.392				
5	.4	.48921	-1.42				
6	.5	.34801	-1.396				
7	.6	.21167	-1.325				

ric1=0.
MAIN RAD AUTO DE

Figure 10

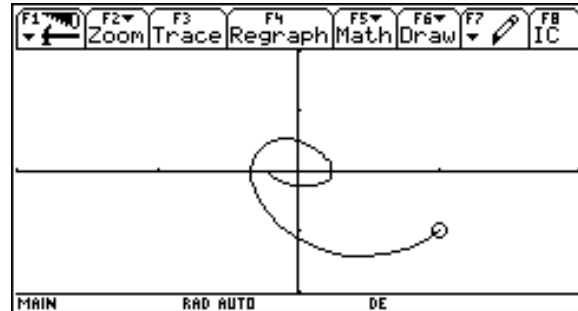


Figure 11

- (d) Change the style for y_2' to "Line" and change the AXES settings to "y1" and "y2" for the x-axis and y-axis respectively. On the WINDOW screen, change the t -interval to $[0,6]$ and the x -interval to $[-2,2]$. Press GRAPH to plot a trajectory in the phase plane. In the notation of the original equation, the resulting graph (figure 11) plots y' vs. y .
- (e) Use TRACE to find numerical values for this phase-plane graph.
- (f) With the phase plane graph still on the screen, use IC to choose other initial conditions and generate trajectories for other solutions. Note that in this context, it's best to accept the settings offered for the axes choices (y_1 and y_2) so that you interactively select the initial values of y_1 and y_2 , but t_0 remains unchanged.
- (g) If DIRFLD is selected in the graph formats menu, a direction field for y_1 vs. y_2 is drawn. Since these equations depend explicitly on t , the shape of the direction field changes as t does. The value of $dtime$ on the WINDOW screen determines which value of t is used to compute the direction field.

Example 3

Explore the rabbit-fox predator-prey model given by $F' = -F + 0.1F \cdot R$ and $R' = 3R - F \cdot R$.

Select FLDOFF in the graph formats menu, TIME for the axes option, and enter the values shown in figures 12-13 on the [Y=] and WINDOW screens. Be sure to "select" both y_1 and y_2 for graphing, and choose different "styles" for the two graphs.

	F1	F2	F3	F4	F5	F6	F7
	Zoom	Edit	✓	All	Style	Axes...	
PLOTS							
	t0=0.						
	✓y1'=-y1 + .1·y1·y2						
	y11=2						
	✓y2'=3·y2 - y1·y2						
	y12=5						
	y3'=■						
	y13=						
	y4'=						
	y14=						
	y5'=						
	y3'(t)=						

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Figure 12

	F1	F2
	Zoom	
t0=0.		
tmax=10.		
tstep=.1		
tplot=0.		
xmin=-1.		
xmax=10.		
xsc1=1.		
ymin=-10.		
ymax=40.		
ysc1=5.		
ncurves=0.		
diftol=.001		

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Figure 13

entry: $x^2 * d(y(x),x,2) + x * d(y(x),x) - y(x) = 0$

answer: $(m^2 - 1) \cdot x^m = 0$

entry: $\text{solve}(\text{ans}(1), m)$

answer: $m > 0 \text{ and } x = 0 \text{ or } m = 1 \text{ or } m = -1$

Of course, one could simplify the last answer by dividing the equation by x^m before solving it or by adding the condition that x is non-zero to the solve command. The solution can also be found in a single step by entering the following command:

$\text{solve}(x^2 * d(x^m,x,2) + x * d(x^m,x) - x^m = 0, m) \mid x \neq 0.$

Obviously, the first approach is much more instructive than the second, but the second is more efficient once the technique is understood.

Example 5

Consider $x^2 y'' - x y' + y = 0$. The technique in the previous example finds that $y = x$ is a solution of this equation, and that $m = 1$ is a repeated root of the auxiliary equation.

Now use reduction of order to find the second independent solution.

entry: Define $y(x) = x * u(x)$

answer: Done

entry: $x^2 * d(y(x),x,2) - x * d(y(x),x) + y(x) = 0$

answer: $(d(u(x),x,2) \cdot x + d(u(x),x)) \cdot x^2 = 0$

This leads to the equation $x \cdot u'' + u' = 0$ which can be solved on the calculator in any number of ways (including simply using the deSolve command). Of course, one could introduce a new variable, $w = u'$ to emphasize that the problem has been reduced to solving a first-order equation.

Example 6

Find a particular solution of $y'' - y' + y = 2 \sin(3x)$ using the method of undetermined coefficients.

entry: Define $y(x) = a * \cos(3x) + b * \sin(3x)$

answer: Done

entry: $d(y(x),x,2) - d(y(x),x) + y(x) = 2 \sin(3x)$

answer: $(-8a - 3b) \cos(3x) + (3a - 8b) \sin(3x) = 2 \sin(3x)$

entry: $\text{solve}(-8a - 3b = 0 \text{ and } 3a - 8b = 2, \{a,b\})$

answer: $a = 6/73 \text{ and } b = -16/73$

Of course, the deSolve command can find the general solution for this equation (although the solution is not expressed in a simple form), but unlike the deSolve command, the technique above can be used with

higher order equations as well.

Example 7

Find a series solution of $y'' + y \cos(x) = 0$. Find the terms up to degree 5.

entry: Define $y(x) = a + b x + c x^2 + d x^3 + f x^4 + g x^5$

answer: Done

entry: $d(y(x),x,2) + y(x) * \text{taylor}(\cos(x),x,5) = 0$

answer:
$$g x^9 / 24 + f x^8 / 24 + (d-12g) x^7 / 24 + (c-12f) x^6 / 24 +$$
$$(b-12(d-2g)) x^5 / 24 + (a-12(c-2f)) x^4 / 24 + (-b/2 + d + 20g) x^3 +$$
$$(-a/2 + c + 12f) x^2 + (b+6d) x + a + 2c = 0$$

Recalling that we expect two arbitrary constants, we begin with the constant term and attempt to write c , d , f , and g in terms of a and b .

entry: Define $c = -a / 2$

answer: Done

entry: (pull down the long answer above)

answer: $\dots\dots + (b + 6d) x = 0$

entry: Define $d = -b / 6$

entry: (pull it down again)

answer: $\dots\dots + (12f - a) x^2 = 0$

entry: Define $f = a / 12$

entry: (pull it down again)

answer: $\dots\dots + (20g - 2b/3) x^3 = 0$

entry: Define $g = 2b / 60$

entry: (pull it down again)

answer: $\dots\dots + 3a x^4 / 8 = 0$

We have written c , d , f , g in terms of a and b , so we are done with the process. To find the first few terms of two independent solutions, do the following:

entry: $y(x) \mid a = 1 \text{ and } b = 0$

answer: $x^4 / 12 - x^2 / 2 + 1$

entry: $y(x) \mid a = 0 \text{ and } b = 1$

answer: $x^5 / 30 - x^3 / 6 + x$

One can now choose initial conditions, use the ODE grapher to plot a numerical solution, and then use DrawFunc to superimpose the graph of the corresponding series solution.

In addition to the examples above, the CAS capabilities of the TI-92 Plus can help with much of the manipulation involved with topics such as the Laplace transform and Fourier series. The calculator can also manipulate symbolic matrices and determinants, and find eigenvalues and eigenvectors. Clearly this is useful in the study of systems of equations.

SECTION FOUR: SAMPLE PROGRAMS AND FUNCTIONS

Programs and user-defined functions can help to extend the capabilities of the TI-92 Plus. Three examples relevant to a first course in ODE are included here.

(a) Program NumDE

This program allows the user to generate numerical solution data for a first-order DE using a choice of Euler's method, Improved Euler Method, or Runge-Kutta 4. The program is run by entering the command NumDE() on the HOME screen. The resulting data is stored in lists for use after the program is run.

(b) Function Wron

This function computes the Wronskian of a list of functions. The single argument is a list of functions, so a typical call might be Wron({ 1, x, x² }).

(c) Function VarPar

This function does part of the work necessary to solve a second-order equation using variation of parameters. The equation is assumed to be in the form

$$y'' + P(x)y' + Q(x)y = f(x).$$

We assume that f_1 and f_2 form a fundamental set of solutions for the corresponding homogenous equation. We are seeking a solution of the form $y = u_1 \cdot f_1 + u_2 \cdot f_2$.

The function should be called with VarPar($f_1(x), f_2(x), f(x)$) and it returns the list { u_1', u_2' }. This requires the user to do the integration to find u_1 and u_2 , and then to write the solution. These last two steps can easily be "automated" as part of the function if desired.