

Activity 1

The First Twelve Days
of School**Concepts/Skills:**

Organize data, look for patterns, problem solving

Calculator:

TI-30Xa SE or TI-34

Objectives:

Students will count the number of "coins" in a variation of *The Twelve Days of Christmas* song. They are expected to generalize the patterns, perhaps through some symbolic expression.

Getting Students Involved

Ask if anyone has heard *The Twelve Days of Christmas*. You might want to play a recording of the original version of the song or invite someone to sing the song.

Making Mathematical Connections

If students know about triangular numbers (that is, 1 , $1+2=3$, $1+2+3=6$, $1+2+3+4=10$, and so forth), this activity may be very easy. If not, students might generalize some ways to compute these values during the activity. Encourage those students who show they are ready to generalize.

You may want to review use of the $\boxed{\text{SUM}}$ or $[\text{SUM}]$ key.

▮ Transparency Master A: Find the Sum of Numbers

Carrying Out the Investigation

First, introduce the scenario. You may want to use the transparency at the end of the activity (page 6) and act out the first two or three days of the poem.

- ◆ How many coins would you receive on the first day? How many coins would you receive on the second day? How many coins for both days together? *1 coin the first day; $1+2=3$ coins the second day; 4 coins for the two days together.*
- ◆ What could we do to organize the data from this poem to find out the total number of coins on all 12 days? *Perhaps make a table.*

Then let students begin to work on completing the chart. Encourage them to look for shortcuts that will help make the computations.

Making Sense of What Happened

Have students present their answers and explain any shortcuts they used to compute the answers. Pay special attention to the generalizations they made through use of algebraic expressions.

If you believe your students are ready to develop formulas, help them see how to find a symbolic expression that can be used to compute solutions to the questions.

Continuing the Investigation

Have students create a different version of the poem.

Solutions

1. Answers will vary.
2. The sum of the first n whole numbers is $n(n+1)/2$. The sum of the first n triangular numbers is $n(n+1)(n+2)/6$.

| Day | Number of Coins on This Day | Total Number of Coins |
|------------|------------------------------------|------------------------------|
| 1 | 1 | 1 |
| 2 | 3 | 4 |
| 3 | 6 | 10 |
| 4 | 10 | 20 |

3. Answers will vary.

4. The number of coins on the 12th day is $1+2+3+\dots+12=12(13)/2=78$. The total number of coins for 12 days is $1+3+6+10+\dots+78=12(13)(14)/6=364$. The **[SUM]** key could be used to keep a running total; **[RCL]** can be used to check the running total at any point.

[1] **[SUM]**

[1] **[+]** **[2]** **[=]** **[SUM]**

[1] **[+]** **[2]** **[+]** **[3]** **[=]** **[SUM]**

and so on

| Day | Number of Coins on This Day | Total Number of Coins |
|-----|-----------------------------|-----------------------|
| 1 | 1 | 1 |
| 2 | 3 | 4 |
| 3 | 6 | 10 |
| 4 | 10 | 20 |
| 5 | 15 | 35 |
| 6 | 21 | 56 |
| 7 | 28 | 84 |
| 8 | 36 | 120 |
| 9 | 45 | 165 |
| 10 | 55 | 220 |
| 11 | 66 | 286 |
| 12 | 78 | 364 |

5. If the poem were extended for a 13th day, there would be 91 coins on the 13th day.
6. The total would be 455 coins for all 13 days.
7. There would be 1,275 coins on the 50th day.
8. The total would be 22,100 coins for all 50 days.
9. There would be 16,290 coins on the 180th day.
10. The total would be 988,260 coins for all 180 days.

(Some students may suggest that a spreadsheet would be a more efficient way to generate the values for different extensions of the poem.)