Math Objectives

- Students will understand the definition of the natural logarithm function in terms of a definite integral.
- Students will be able to use this definition to relate the value of the natural logarithm function to the area under a curve.
- Students will be able to use the derivative $\left(\frac{1}{x}\right)$ of the natural logarithm to determine properties of its graph.

Vocabulary

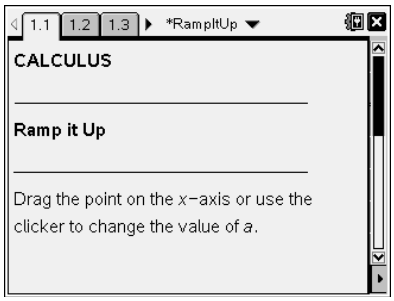
- natural logarithm function
- area under a curve
- increases without bound
- derivative
- Fundamental Theorem of Calculus

About the Lesson

- This lesson is a follow-up lesson to the activity *Natural Logarithm*.
- As a result, students will:
  - Conjecture about the value of the natural logarithm function as *x* increases without bound.
  - Analyze the natural logarithm function to solve a real-world problem.

TI-Nspire Navigator™ System

- Use Screen Capture to demonstrate that students can grab and drag the point *x* properly.
- Use Screen Capture to demonstrate the definition of the natural logarithm function.
- Use Document Collection to assess student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- In the *Graphs & Geometry* application, you can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
RampItUp_Student.pdf
RampItUp_Student.doc

TI-Nspire document
RampItUp.tns

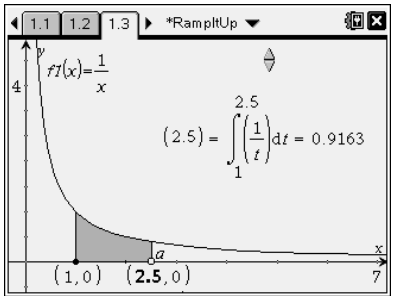
Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☜) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text*. Then press (ctrl) [☜] to grab the point and close the hand (✊). Students may also use the clicker, the up/down arrows, to change the value of *x*.

TI-Nspire Navigator Opportunity: *Screen Capture and/or Live Presenter*
See Note 1 at the end of this lesson.

Move to page 1.2.



The **natural logarithm**, ln(*a*), can be defined as the area under the curve of $\frac{1}{t}$ from 1 to *a*. The graph on page 1.3 represents this area. Grab and drag the point *a*, or use the slider in the top-right portion of the page, to see the definition in action.

Teacher Tip: The shaded region may not be immediately visible. Students may have to click on the graph or move the point to make the shading for the integral appear.

Tech Tip: To find all the values in the table, students will need to click the slider. Dragging the point along the *x*-axis will only move in increments of 1. However, the slider moves in increments of 0.5.

1. The computed area of the shaded region is equivalent to ln(*a*), the value of the natural logarithm function.

   a. Complete the following table.

Answers:

a	0	0.5	1.0	1.5	2.0	2.5
ln(*a*)	**–∞**	**–0.6931**	**0**	**0.4055**	**0.6931**	**0.9163**

b. When is ln(*a*) negative? Why?

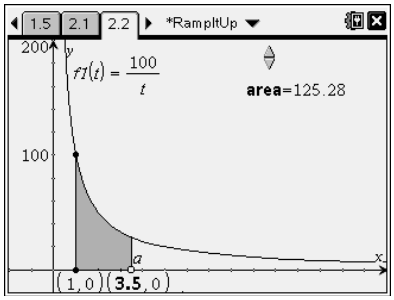
Answer: Values of ln(*a*) are negative between 0 and 1 because the lower bound of the integral is 1.

$$\int_{1}^{0.5} \frac{1}{t} dt = -\int_{0.5}^{1} \frac{1}{t} dt$$

c. When is ln(*a*) equal to zero? Use the graph and the definition of ln(*a*) to explain your answer.

Answer: When *a* = 1: ln(1) = 0. The integral is the area under the curve from 1 to some other number. Because there is no movement from 1 to itself, there is no area under the curve. $\int_{1}^{1} \frac{1}{t} dt = \ln(1) - \ln(1) = 0$

Move to page 2.1.



2. The Calvert Construction Company is designing a ramp for a new project, starting with a model. A sketch of a cross section of part of the ramp is given on page 2.1. The shaded area represents the support system for the ramp. Your job is to help the design team with some measurements for the model.

a. The graph shows $\mathbf{f1}(t) = \int_{1}^{a} \frac{100}{t} dt$. What expression can be used to find the area under the model ramp for any value of *a*?

Answer: The area under the ramp can be found using the expression 100 · ln(*a*).
$\int_{1}^{a} \frac{100}{t} dt = 100 \cdot \int_{1}^{a} \frac{1}{t} dt = 100 \cdot \ln(a)$

b. What is the exact area of the support system under the model ramp when *a* = 8.75? Use the bottom of page 2.3 for calculations.

Answer: 100 · ln(8.75) = 216.905

c. Complete the following table. Round your answers to the nearest hundredth.

Answers:

a	7	7.25	7.75	8.25	8.6	9.3
Area	**194.59**	**198.10**	**204.77**	**211.02**	**215.18**	**223.00**

d. **Extension:** If the model ramp is designed to have a support system 7.25 inches long (a = 7.25) and 15 inches deep, what would be the **volume** of the support system under the ramp?

Answer: 198.10 square inches · 15 inches = 2,971.50 cubic inches

TI-Nspire Navigator Opportunity: *Embedded Assessment*
See Note 2 below.

Wrap Up

Upon completion of this activity, students should understand:

- The relationship between the area under the curve and the definition of the natural logarithm function.
- The properties of the graph of the natural logarithm function.

TI-Nspire Navigator Opportunities

Note 1

Question 1, Name of Feature: Screen Capture and Live Presenter

As students begin this activity, use Screen Capture to be assured that each student is able to grab and drag the appropriate point and to use the slider to get more accurate answers.

Note 2

Question 2, Name of Feature: Embedded Assessment

At the conclusion of the activity, collect student responses to the questions and use the TI-Nspire Navigator™ Class Analysis feature to show the slide show of student responses and discuss the results.