Rational Functions

ID: 8967

Time required
75 minutes

Activity Overview

Students investigate the graphs of functions of the form $y = \frac{1}{x-a}$. They will discover that the graph of such a function has a vertical asymptote at x = a, and a horizontal asymptote at y = 0. They will investigate the graphic and numeric consequences of such asymptotic behavior by observing a trace point on the graph move in response to various inputs for the independent variable.

Topic: Rational & Radical Functions

- *Evaluate a rational function for any real value of the variable*
- *Graph any rational function to verify its domain, range, singularities and asymptotes.*

Teacher Preparation and Notes

- *This investigation could be using as a first introduction to graphs of rational functions. The language of vertical and horizontal asymptote is not introduced in the activity; teachers may wish to summarize the results of the activity using that language. The emphasis here is on how the connections among graphic, numeric, and symbolic representations of rational functions.*
- *This activity could be used in Algebra 2 or Precalculus as an introduction to rational functions.*
- ***To download the student worksheet and the program RATIONAL, go to education.ti.com/exchange and enter "8967" in the keyword search box.***

Associated Materials

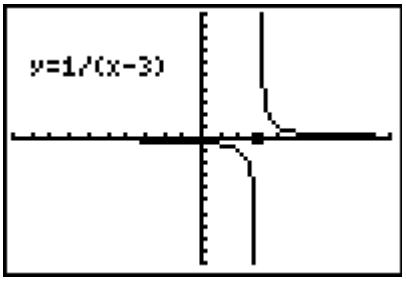
- *RationalFunctions_Student.doc*
- *RATIONAL.8xp*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Radical Functions (TI-84 Plus family) — 8977*
- *Exploring the Exponential Functions (TI-84 Plus family) — 11978*

Problem 1 – Graphing $y = \frac{1}{x-a}$ for various values of *a*.



Students use the calculator program RATIONAL to explore rational functions. When students run the program, it shows the graph of $y = \frac{1}{x-a}$. The value of *a* is shown by a dot on the *x*-axis at (*a*, 0). The starting value of *a* is 3. Students are to press the left and right arrow keys (◄ and ►) to change the value of *a* to help them answer the questions on the student worksheet. The equation and graph update automatically. Students can exit the program by pressing 9.

Student Solutions

1. $x = 2$

2. $x = -1$

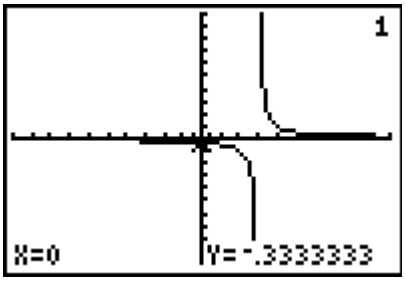
3. $x = a$

4. *y* becomes undefined when $x = a$ because of the 0 denominator. It is at this value that the break in the graph occurs.

5. at $x = a$

Problem 2 – Behavior Near the Vertical Asymptote

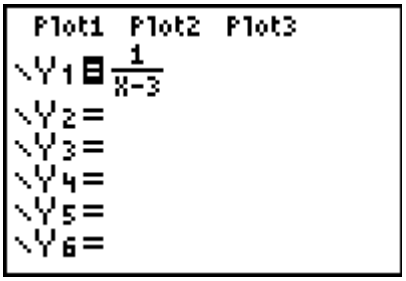
Students will now use the Y= screen to graph a rational function. They can press TRACE to place a point *P* on the graph and use the Y= values to complete the table on the worksheet.



If using Mathprint OS:

Students can display the function as a fraction in the Y= screen. To do this, press Y= and to the right of Y1= press ALPHA [F1] and select **n/d**. Then enter the value of the numerator, press ▼ and enter the expression for the denominator and press ENTER.

Note: Parentheses are not needed in the denominator.



Student Solutions

6.

4	1
3.5	2
3.2	5
2.8	–5
2.5	–2
2	–1

7. sample: *x* goes from –5 to 5, *y* goes from 75 to 125

8. –100

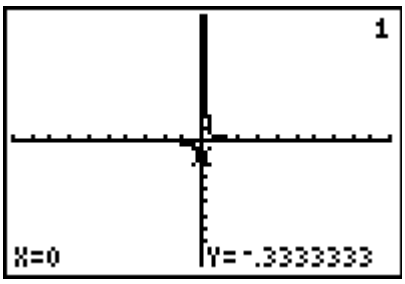
9. yes, make *x* = 3.001

10. yes, make *x* = –2.999

11. yes, by making *x* close to, but slightly greater than 3

Problem 3 – Horizontal Asymptote

In this problem, students continue exploring the function from Problem 2, but zoom the graphing window out so that they can see the horizontal asymptote. Once again, students are to press TRACE to place a point *P* on the graph and use it to answer the question on the worksheet.



Student Solutions

12.

103	0.01
13	0.1
5	0.5
1	–0.5
–7	–0.1
–97	–0.01

13. sample: *x* goes from 450 to 550, *y* goes from –2.5 to 2.5

14. –0.002

15. yes, make *x* = 1,003

16. yes, make *x* = –997

17. yes, by making *x* sufficiently large

18. at *y* = 0