Ain't No River Wide Enough

ID: 9886

Time required
40 minutes

Activity Overview

Students simulate the process a surveyor would use to measure the width of a river by measuring length on one side of the river and angles formed by various reference points. They then prove the Law of Sines and apply it to calculate the river's width, proving that no river is too wide to be measured with trigonometry.

Topic: Trigonometric Identities

- *Prove and apply the law of sines and law of cosines to find the unknown sides or angles of a triangle.*

Teacher Preparation and Notes

- *This activity is appropriate for an Algebra 2 or Precalculus classroom. This activity is intended to be* ***student-centered*** *with brief periods of group instructions. You should seat your students in pairs so they can work cooperatively on their handhelds and circulate among them, offering assistance as needed.*
- *Students should have experience calculating basic trigonometric functions and solving simple trigonometric equations.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9886" in the keyword search box.***

Associated Materials

- *AintNoRiver_Student.doc*
- *AintNoRiver.tns*
- *AintNoRiver_Soln.tns*

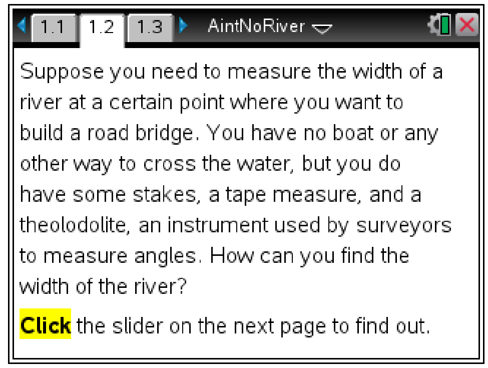
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

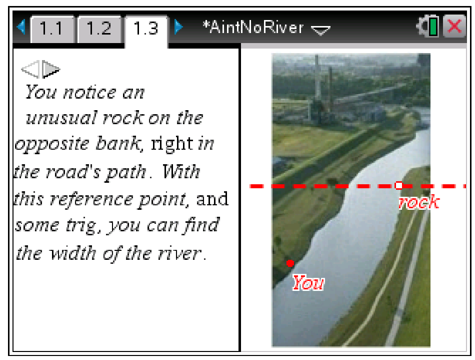
- *Law of Sines (TI-Nspire technology) — 17194*
- *Relatives of the Sine Law (TI-Nspire technology) — 17197*
- *Law of Sines and Cosines (TI-Nspire technology) — 9849*

Problem – Measuring a River

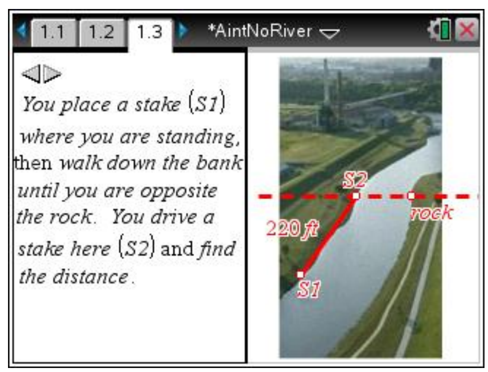
In this activity, the students are part of a team building a new road. To prepare for building a bridge, they need to measure the distance across a river. The river is treacherous and impossible to cross without the right equipment, so students must measure its width *indirectly*, using some stakes to mark points on the bank, a tape measure, and a theolodolite. A theolodolite is a special instrument used by surveyors to measure angles. To measure, the theolodolite must be at the vertex of the angle.



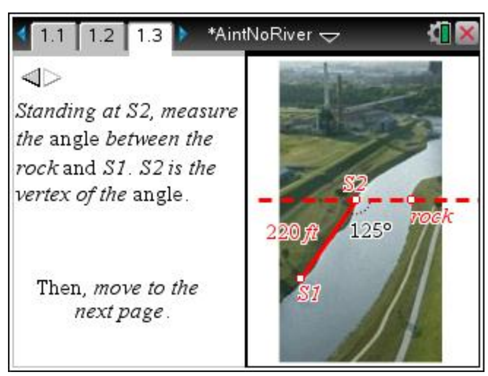
Use the diagram on page 1.3 to lead a class discussion of possible methods to measure the width of the river at the point shown by the dotted line. Students should clearly understand what the problem is asking before working independently than for them to come up with the measuring method shown in this activity.



Clicking on the slider gives directions to guide students through the first few steps of the measurement process. If students have difficulty understanding the process, mark off a “river” in the classroom and act it out.

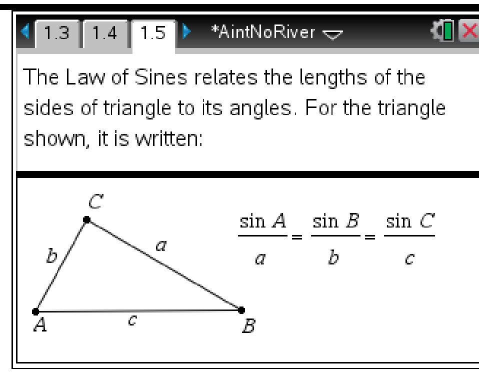


One of the directions given after clicking on the slider directs students to measure the angle between the rock and S1, with a vertex at S2. They should select **MENU > Measurement > Angle**, and then click on the rock, S2, and S1. (Clicking S1, S2, and then the rock works as well.)



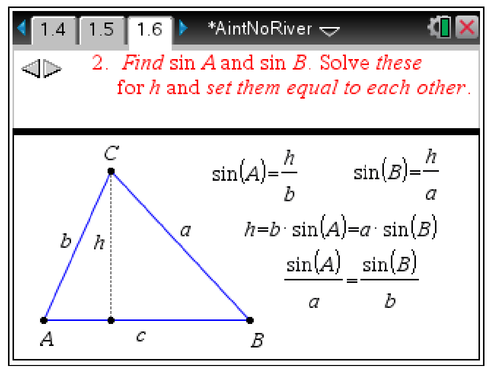
TI-Nspire™ Navigator™ Opportunity: *Live Presenter*

See Note 1 at the end of this lesson.



Page 1.5 introduces the Law of Sines.

Discuss the meaning of the law as a class before having students complete the proof in small groups. Stress that unlike other trigonometry formulas, the Law of Sines applies to *any* triangle, not just to right triangles. The Law of Sines formalizes the relationship between the measures of an angle and the length of the side opposite it. This relationship is intuitive and familiar, as students have already seen that the largest angle is opposite the longest side of a triangle, etc.



Students use the diagram and steps on page 1.6 to prove the Law of Sines. Tips for each step of the proof are given.

1. **Draw altitude *h* from point *C* to the opposite side.**
   It may be necessary to remind students of the meaning of the word *altitude*. An altitude is a straight line through a vertex and perpendicular to the opposite side.

   To draw the altitude, students should use the **Perpendicular** tool. If they wish, they can draw a shorter segment on top of the perpendicular line and hide the line. In the screenshots shown, the line segment is dotted to make the diagram easier to read. To do this, press ctrl + menu while the cursor is on the segment, go to **Attributes**, and change the line style to dotted.

2. **Find sin *A* and sin *B*. Solve these for *h* and set them equal to each other.**
   Students should write expressions using the definition of sine as opposite/hypotenuse, then multiply by *a* and *b* respectively to solve these expressions for *h*. By setting these two equations equal to each other and dividing by *ab*, they obtain the first part of the Law of Sines.

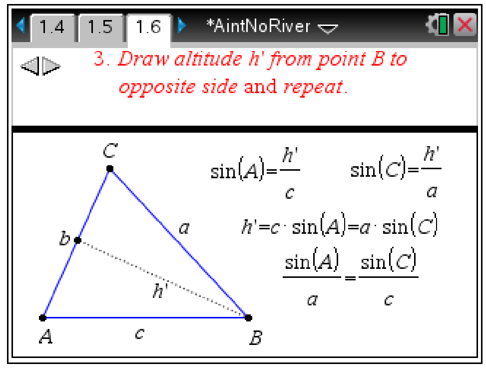
TI-Nspire™ Navigator™ Opportunity: *Live Presenter*

See Note 2 at the end of this lesson.

3. Draw altitude *h*' from point *b* to opposite side and repeat.

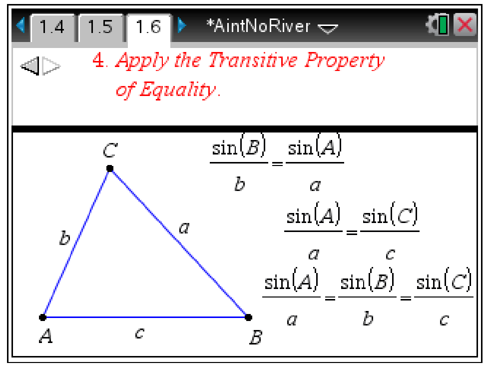
To make the diagram easier to read, students may choose to **Hide** altitude *h* before drawing *h*'.

This time students write and solve expressions for sin *A* and sin *C*.

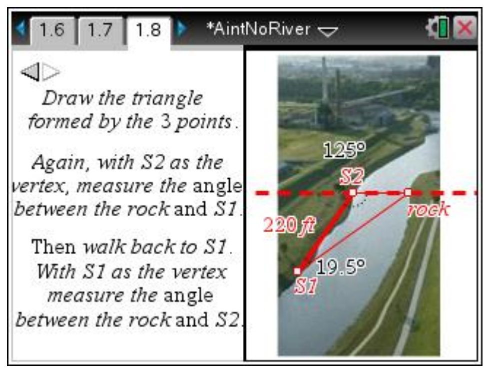


4. Apply the Transitive Property of Equality.

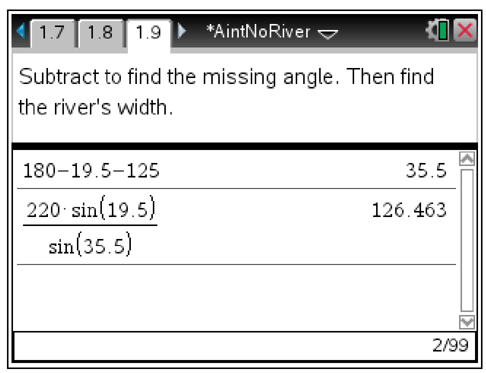
The Transitive Property of Equality states that if *A* = *B* and *B* = *C*, then *A* = *C*. In this case, $\frac{\sin A}{a}=\frac{\sin B}{b}$ and $\frac{\sin A}{a}=\frac{\sin C}{c}$, so $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$, completing the proof.



Page 1.7 signals the transition back to the original river problem, and page 1.8 repeats the diagram for convenience. Students are to draw a triangle and measure the angles that they can without crossing the river.



Have students click the slider on page 1.8 and solve the presented problem. First they should subtract the angle measures they found from 180° to find the measure of the remaining angle, 35.5°. Then they use the Law of Sines to write and solve an equation to find the missing side length, the distance between S2 and the rock.



$$\frac{\sin(35.5^\circ)}{220}=\frac{\sin(19.5^\circ)}{x}$$

$$x\sin(35.5^\circ)=220\sin(19.5^\circ)$$

$$x=\frac{220\sin(19.5^\circ)}{\sin(35.5^\circ)}$$

$$x\approx 126.5$$

The width of the river at this point is approximately 126.5 feet.

TI-Nspire^{TM} Navigator^{TM} Opportunity: *Class Capture* and *Quick Poll*

See Note 3 at the end of this lesson.

TI-Nspire^{TM} Navigator^{TM} Opportunities

Note 1
Question 1, *Live Presenter*
Use Live Presenter to facilitate the discussion of finding the distance across the river as well as how to measure the required angle.

Note 2
Question 1, Live Presenter
Use Live Presenter to demonstrate constructing the perpendicular from vertex C to side c. Also use Live Presenter to facilitate the discussion of finding sin(A) and Sin (B).

Note 3
Question 3, *Class Capture and Quick Poll*
Use Class Capture to monitor student progress as they work through measuring the angles on the triangle and calculate the width of the river. Consider sending a quick poll for students to send their answer for the width of the river.