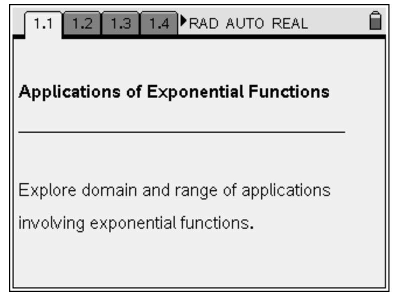
Applications of Exponential Functions

TIMATH: EXPONENTIAL FUNCTIONS

Math Objectives

- Students will identify the domain and range of exponential functions in the form of $y = a^x$, where $a > 0$ and $a \neq 1$.
- Students will identify reasonable domain and range values in the context of the application.
- Students will use graphs of the exponential functions to find solutions to questions regarding applications.



Vocabulary

- domain
- range
- exponential function

About the Lesson

- This lesson explores applications involving bacteria growth and decay. Exponential functions are used to represent the data.
- Using graphs of the exponential functions, students will explore the domain and range in the context of the applications.
- Students will also use the graphs of the exponential functions to find solutions to questions regarding the applications.
- Students will compare and contrast the characteristics of the growth function and the decay function.

TI-Nspire™ Navigator™ System

- Use Screen Capture to examine patterns that emerge.
- Use Teacher Edition computer software to review student documents.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Applications_of_Exponential_Functions_Student.pdf

Applications_of_Exponential_Functions_Student.doc

TI-Nspire document

Applications_of_Exponential_Functions.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (hand icon) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text*. Then press ctrl (click) to grab the point and close the hand (closed hand icon).

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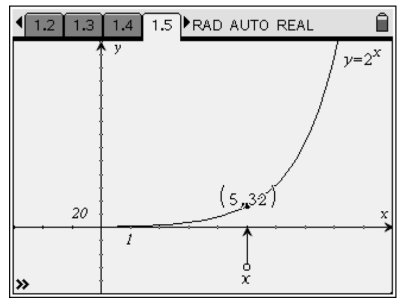

1. If *x* represents time, what does *x* = 6 mean? Would this be a domain or range value of the function? Explain.

   Answer: *x* = 6 represents the 6th time the bacteria in the petri dish was counted. Because the bacteria are checked and counted every 15 minutes, this would have occurred 90 minutes after starting the experiment. Because *x* represents an input value of the function, it would be part of the domain.

   Teacher Tip: A common error students will make is interpreting the *x*-values as the number of minutes. It might be helpful to remind students that the petri dish is checked in 15-minute intervals.

2. What does *y* represent in the context of the application? Would this be the domain or the range of the function? Explain.

   Answer: The *y* variable represents the number of bacteria present in the petri dish. Since *y* represents the output of the function, this would be part of the range.

3. Grab and drag the point labeled *x* on the graph. What domain values make sense in the context of the problem? How would this affect the range values?

   Answer: Because *x* represents time, negative numbers would not make sense in the context of the application. Zero would represent the number of bacteria present at the start of the study. The bacteria present in the petri dish are checked and counted every 15 minutes, so each integer *x*-value represents a 15-minute interval. Therefore, only positive integer values and zero make sense in the context of the application. The range values would then be integers greater than or equal to 1.

4. Find the number of bacteria present in the petri dish at one hour by grabbing and dragging the point labeled *x*.

Answer: Because there are four 15-minute intervals in an hour, the *x* point should be moved until the ordered pair reads (4, 16). Therefore, there are 16 bacteria present in the petri dish at one hour.

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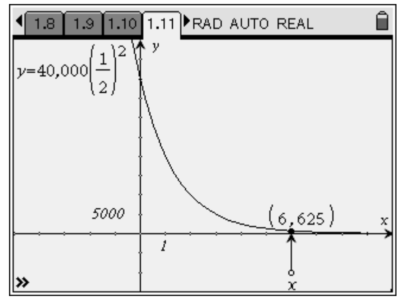

5. Grab and drag the point labeled *x* until the ordered pair is (0, 4**E** + 4). Interpret the ordered pair in the context of the application.

Answer: Zero represents the start of the study. 4**E** + 4 represents the 40,000 bacteria present in the petri dish before the antibiotic is added.

Teacher Tip: If students have trouble interpreting the *y*-value notation in the ordered pair, a quick review of scientific notation may help.

6. What domain values make sense in the context of the application? Explain.

Answer: The *x*-values represent time at 4-hour intervals. Zero represents the start of the study. Therefore, the domain values that make sense in the context of the application are positive integers including zero.

7. What range values make sense in the context of the application? Explain.

Answer: Given that the domain values are integers greater than or equal to zero, the range values are integers less than or equal to 40,000, but greater than zero.

8. According to the exponential function graphed, will the number of bacteria ever be zero? Explain.

Answer: The number of bacteria will not be zero given the function $y = 40{,}000\left(\frac{1}{2}\right)^x$. In order for the graph to have an *x*-intercept, *y* must be zero, and there is no solution to the equation when the function is set equal to zero. Therefore, even though when the *x*-values get larger, the *y*-values become smaller, *y* will not be zero.

9. Approximately how long will it take for the number of bacteria in the petri dish to be fewer than 1,000?

   Answer: While moving the *x* point, at $x = 5$, the *y*-value is 1,250, and at $x = 6$, the *y*-value is 625. Therefore, sometime between 20 and 24 hours, the bacteria present in the petri dish will be fewer than 1,000.

10. Compare and contrast the two graphs. What is similar about the two graphs? What is different?

    Answer: The two graphs are similar in the fact that they both represent exponential functions. The difference is in the graph of $y = 2^x$, when the *x*-values increase, so do the *y*-values. But in the graph of $y = 40{,}000\left(\frac{1}{2}\right)^x$, when the *x*-values increase, the *y*-values decrease. Also, the domain values are the same. Answers may vary.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to identify the domain and range of an exponential function in the form of $y = a^x$, where $a > 0$ and $a \neq 1$.
- How to interpret domain and range in the context of an application.
- How to use graphs of exponential functions to find solutions in regards to an application.

Assessment

Have students solve similar problems like the ones below. Have students identify how these changes to the applications would affect the exponential function, domain, range, and graph.

1) Suppose you started with 100 bacteria in the petri dish in order to study their mitosis and counted the bacteria every half hour.
2) In order to study how the bacteria are affected by an antibiotic, suppose you started with 10,000 bacteria in the petri dish and noticed the number of bacteria present was reduced by a third every two hours.