About the Lesson

In this activity, students are given a manufacturing situation and asked to write a system of linear inequalities to represent it. Once they have written the inequalities, students examine its solution set by testing values of the variable using lists and viewing its graph. In Part 1, a single inequality is presented to represent the constraint. In Part 2, a second constraint and a second variable are added to the situation. In Part 3, a third constraint is introduced, creating a system of inequalities that has no solution, which is explored in a similar fashion.

As a result, students will:

- Graph a linear inequality in two variables and describe the three regions into which it divides the plane.
- Graph a pair of linear inequalities in two variables and describe the region of their intersection.
- Determine whether a given point belongs to the solution set of a pair of linear inequalities in two variables.

Vocabulary

- linear inequality
- constraint
- system of inequalities
- solution set

Teacher Preparation and Notes

- It would be beneficial for students to clear all lists and functions. Press 2nd + and select **ClearAllLists**. Press y=, move to any equation that is defined and press clear.
- This activity can easily be extended to include linear programming by introducing a profit function $P(x,y)$.

Activity Materials

- Compatible TI Technologies:

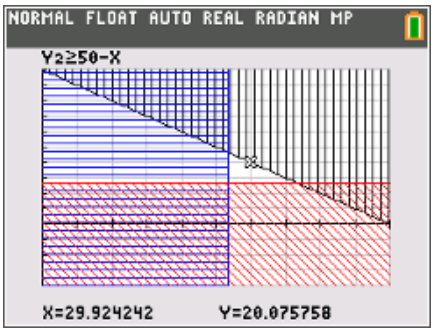
  TI-84 Plus*

  TI-84 Plus Silver Edition*

  TI-84 Plus C Silver Edition

  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

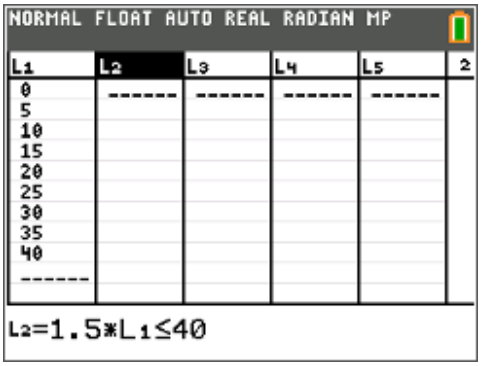
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

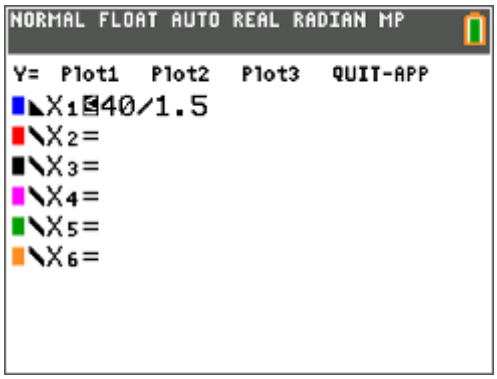
- TheImpossibleTask_Student.pdf
- TheImpossibleTask_Student.doc

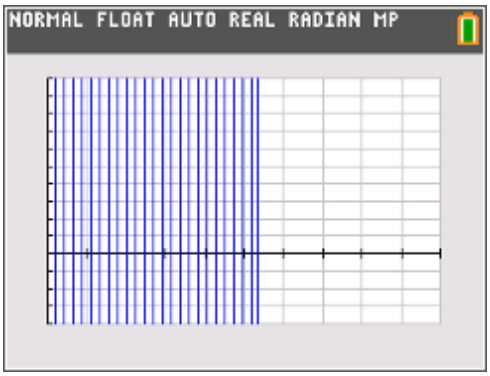
Part 1 – The First Constraint

In this problem, students are given a manufacturing situation and asked to write an inequality to represent it. Caution students to be aware that some of the information in the problem is given in hours and some of the information in the problem is given in minutes — they will need to convert one to the other to write their inequalities.



Once they have written the inequality, students examine its solution setting by testing values of the variable using lists and viewing its graph.





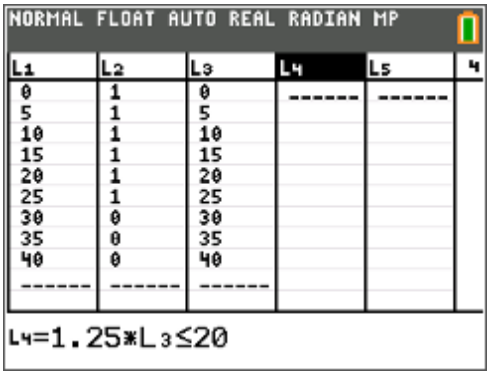
Tech Tip: If students are using the TI-84 Plus or the TI-84 Plus C, pressing (alpha) and the function key corresponding to the correct symbol at the bottom of the screen will change the equals sign to the proper inequality symbol. For example, to make ≤, press (alpha) [F3].

1. Can the owner make 10 birdhouses in one week? 20? 30?

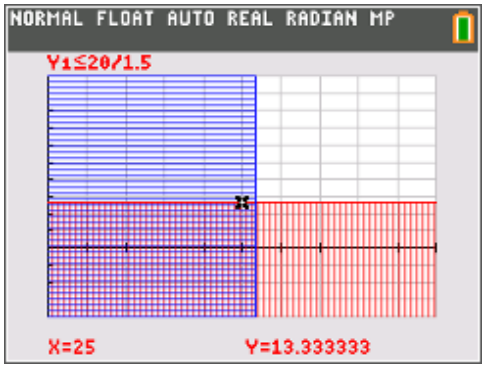
Answer: Yes; yes; no

Part 2 – Another Constraint

A second constraint and a second variable are added to the situation. Again, students should be aware of units as they write their inequalities. The solution set to this inequality is again explored in the list editor. The idea of a system of inequalities, with a solution set equal to the intersection of the two inequalities in the system, is introduced and further explored.



Students then graph the second inequality on top of the first and compare the solution set shown by the graph with that found by testing values. Students must solve the inequality for *y* before they can graph it.



Tech Tip: Students may want to change the default color, **blue**, of the graph of **Y1** to some other color like **red** to contrast it with the default color of the **X1** graph.

2. Can the expert make 10 birdhouses in one week? 20? 30?
Answer: Yes; no; no

3. What does the solution (10, 15) represent in this situation?
Answer: It means that in one week, the owner can make 10 birdhouses and the expert can make 15 birdhouses.

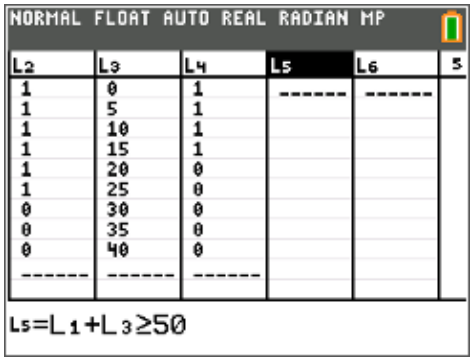
4. List as many solutions to the system as you can.
Answer: Answers will vary. Sample answers: (10, 10); (10, 15); (15, 10); (15, 15); (20, 10); (20, 15); (25, 10); (25, 15).

5. List several points that are within this area.
Answer: Answers will vary. Sample answers: (10, 10); (10, 15); (15, 10); (15, 15); (20, 10); (20, 15); (25, 10); (25, 15).

6. Compare your answer to Question 5 with your answer to Question 4.
Answer: The solutions to the system (the answer to Question 4) are points within the intersection (the answer to Question 5).

Part 3 – A Final Constraint

A third constraint is introduced and explored in a similar fashion. First the solution set of the inequality by itself is discussed, and then students are prompted to search for solutions to the system created by all three inequalities taken together.

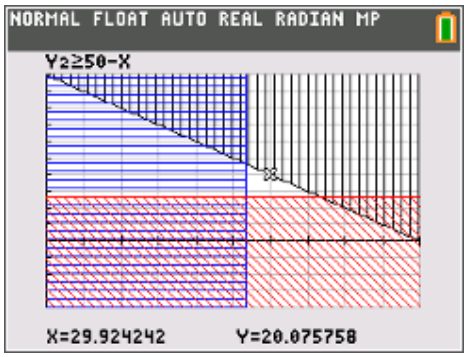


7. List several solutions to this inequality.
Sample Answer: (25, 25); (25, 30); (25, 40); (30, 25); (30, 30); (30, 40); (40, 25); (40, 30); (40, 40).

8. Are there any such rows that are solutions? List as many solutions to the system as you can.
Answer: There are no such rows.

9. What does your answer to Question 8 mean in this situation?
Answer: This system has no solution.

They should find that there are no such (*x*, *y*) pairs and conclude that the task is impossible. This conclusion is verified when they graph the system and see that there is no area where all three solution sets (shaded areas) overlap.



10. Is there an area where all three of these overlap?
Answer: No.

11. What does this mean about the solutions to this system?
Answer: This system has no solution.

Challenge

12. Use the **Points of Interest – Trace** feature of the **Inequality Graphing App** to find the maximum number of birdhouses that the owner and the expert can make in a week.
Answer: At most, they can make 42 birdhouses (the owner can make a maximum of 26 birdhouses a week and the expert can make a maximum of 16 birdhouses a week).