Transformations – Shift Happens!

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Student Activity

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TI-Nspire™ Investigation Student 50 min

Teacher Notes:

Prior to commencing this activity, students should have completed the activity "Focusing on Dilations". The previous activity is a warm-up (stretches) to the approach used here. This collection of activities around transformations is designed to avoid misconceptions built from *bottom-up processing* where students use sensory input observations and gradually build their own ideas and general rules. While there are many benefits to the constructivist's approach, a growing body of research specifically around transformations of functions highlights the misunderstandings that fester and build.

"The cognitive load of moving graphs, visualising horizontal transformations (including dilations) requires an extra mental step, explaining their persistent difficulty compared to vertical transformations". [Eisenberg & Dreyfus 1994]. "Technology helps students coordinate algebraic and graphical views, especially for vertical stretches. Horizontal dilations still lag in comprehension" [Koyunkaya & Boz-Yaman 2023]. "Inside function changes feel counterintuitive. Students often predict the wrong effect because they expect the parameter to act "directly" on the graph, not on the input. A key framework for understanding horizontal dilation errors. [Zazkis, Liljedahl & Gadowsky 2003]

By focusing on the logical transformation of a single point, some relatively simple linear algebra, this activity aims to build understanding.

Australian Curriculum Standards

AC9M7SP03

Describe transformations of a set of points using coordinates in the Cartesian plane, translations and reflections on an axis, and rotations about a given point

AC9M9SP02

Apply the enlargement transformation to shapes and objects using dynamic geometry software as appropriate; identify and explain aspects that remain the same and those that change

AC9M9A06

Investigate transformations of the parabola $y = x^2$ in the Cartesian plane using digital tools to determine the relationship between graphical and algebraic representations of quadratic functions, including the completed square form, for example: $y = x^2 \rightarrow \ y = \frac{1}{3}x^2$ (vertical compression) …

Lesson Notes

The instructions provided in the first step has students creating a new document. Teachers may choose to have students add to their previous document on transformations, however, as 'dilations' is an area that students experience the most difficulties, it is recommended that students save this activity under translations, even though the visual and algebraic approach is the same.

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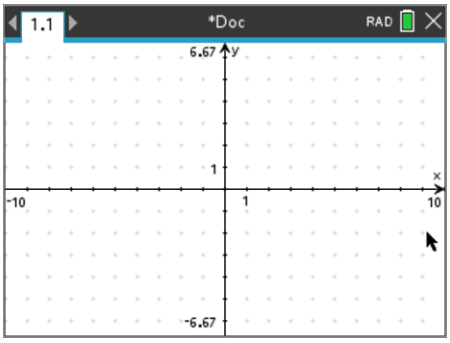
Calculator Instructions:

Create a new TI-Nspire document and insert a Graphs application.

Displaying the grid will make it easy to keep the numbers simple.

menu > **View** > **Grid** > **Dot Grid**

To hide the Graph entry, press: esc

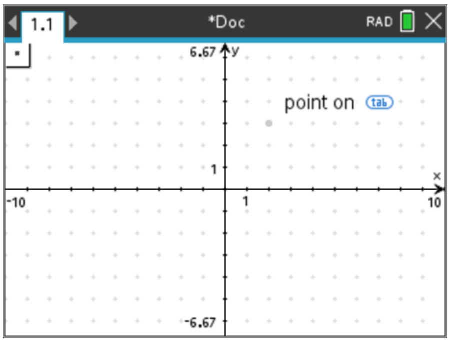


Place a point **on** the grid, "point on" appears as a prompt when the pen is close to the grid.

Once the point has been created, get the coordinates of the point:

ctrl + menu > **Coordinates & Equations**

Press esc to release the Point tool, then grab the point an move it around. The point should maintain integer coordinate values since the axis scale in both directions are provided in integer amounts.



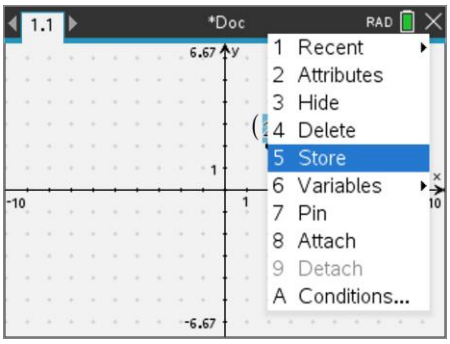
Hover the mouse over the abscissa (x – coordinate) then press:

ctrl + menu > **Store**

Store the abscissa as **xp**.

Repeat this process and store the ordinate (y – coordinate) as **yp**.

The coordinates will now appear bold. These values can be accessed and used in calculations in any other application within this problem.



Create a new point using the keyboard shortcut: P, select:

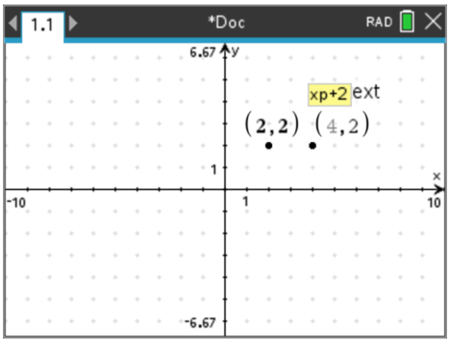
Point by Coordinates

For the abscissa, type: $xp + 2$

For the ordinate, type: yp

Note: To navigate from the abscissa to the ordinate press: enter

This new point is referred to as "an image of point P".



Think about where you would expect the image of point P to appear. Is this translation (shift) logical? Move point P around, does P′ continue to behave the way you expect?

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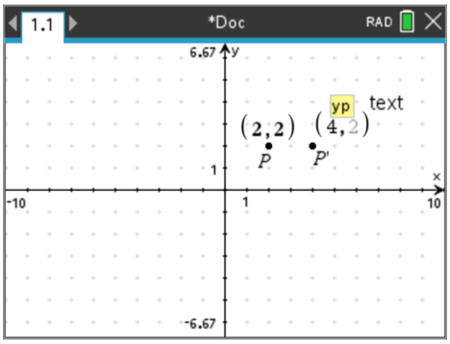
Label the points: P and P′ where P′, pronounced “P prime” is the dilation of the original point P.

Move the mouse over the point then:

ctrl + menu > **Label**

The ‘prime’ notation can be accessed from: ?!▸



The coordinates of points P and P′ automatically move with their respective points. Sometimes this can make reading the coordinates difficult. The coordinates can be moved away from their respective points. Think of them as ‘magnetic’, once moved far enough away they will detach and remain stationary.

Question: 1.

Move point P horizontally and vertically:

a) Describe how point P′ moves.

b) Imagine point P as a point on a line drawing. Moving point P around represents tracing that image. What would the image look like and where would it be?

Answer:

a) Point P′ is always to units to the right (positive x direction) from point P.

b) The image would appear the same, however, it would be 2 units to the right of the existing image.

Question: 2.

Edit the formula used for the abscissa of point P′ so that it is 2 units to the left of xp.

a) Write down the formula you used for the x coordinate (abscissa) of point P′.

b) Imagine point P as a point on a line drawing. Moving point P around represents tracing that image. What would the image look like and where would it be?

Answer:

a) Point P′: (xp – 2, yp).

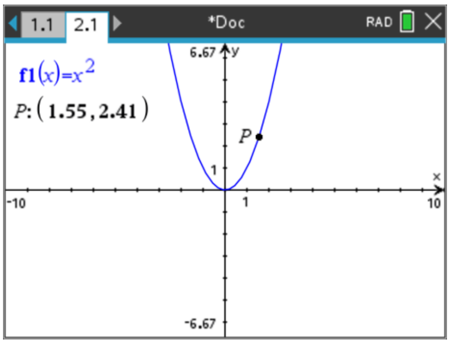
b) The image would appear the same, however, it would be 2 units to the left of the existing image.

Horizontal Transformation Algebra

Insert a new problem into the current TI-Nspire document.

doc▾ > **Insert** > **Problem**

- Insert a Graphs application and graph the function: $y = x^2$.
- Place a point on the graph. Display the coordinates and store them as xp and yp.
- Label the point: P



You may also want to move the coordinates so they don’t move around when point P is being moved.

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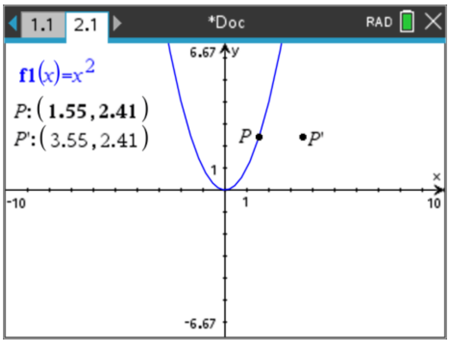
Insert a **Point by Coordinates** and set them as follows:

(xp + 2, yp)

Label the point as: P′

Drag point P along the function and observe the path traced out by: P′.



Point P(xp,yp)[1] is no longer free to move anywhere; it now moves along the function: $y = x^2$. The coordinates of P′ are defined by xp and yp, so the path of P′ can also be defined, the aim here is to determine the equation for that path.

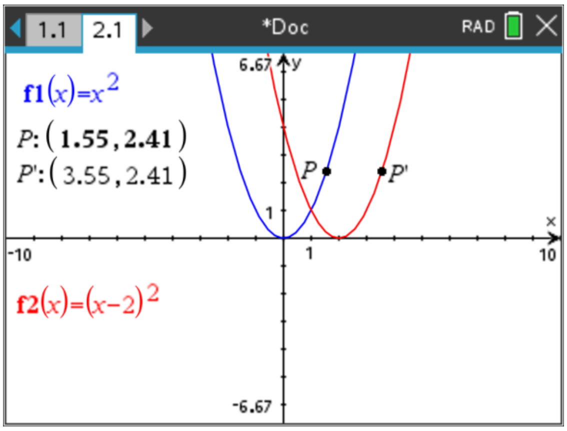
Point P′ will be defined as: P′(x',y') & Point P as P(x, y)

We need to find a rule that relates x' to y'. Our relationships, as defined on the calculator are as follows:

	Equation 1	Equation 2	Equation 3
	$x' = xp + 2$	$y' = yp$	$y = x^2$
Step 1:			
Changing the notation:	$x' = x + 2$	$y' = y$	$y = x^2$
Step 2:			
Express Eqn1 & Eqn2 in terms of x and y respectively:	$x' - 2 = x$	$y' = y$	
Step 3:			
Substitute Eqn 2 into Eqn 3			$y' = x^2$
Substitute Eqn 1 into Eqn 3			$y' = (x' - 2)^2$

With the relationship established, the 'prime' notation can now be removed and the function graphed:

$$\boldsymbol{y} = (x - 2)^2$$



- Point P lies on the graph with equation: $\boldsymbol{y} = x^2$
- Point P′ lies on the graph with equation: $\boldsymbol{y} = (\boldsymbol{x} - \mathbf{2})^{\mathbf{2}}$
- P′ is a horizontal translation of 2 units in the positive x direction.

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[1] Point P is expressed in terms of (xp, yp). This notation reflects the limitation of the digital platform rather than mathematical terminology. Assigning values to variable names such as x and y on the calculator means they will no longer treated as variables, however, it is important to maintain correct mathematical notation, written notes are not bound by such limitations.

Question: 3.

Change the definition for P′: The abscissa changes to xp – 2 while the ordinate remains as yp.

a) Thinking about the translation, what is happening to point P′ ?

b) Determine the relationship between (equation) x' and y'.

c) What is the translation for this graph?

Answer:

a) Point P′ is translated horizontally by 2 units in the negative x direction.

b) Equation: $y = (x + 2)^2$

c) Translation: 2 units in the negative x direction.

- $y = (x - 2)^2$ represents a translation of 2 units in the positive x direction
- $y = (x + 2)^2$ represents a translation of 2 units in the negative x direction

If this seems counterintuitive, think about this:

For the function $y = x^2$, when 2 is subtracted from x, before squaring, the value for x needs to be 2 units bigger to obtain the same result.
Example:
If x = 3, then for $y = x^2$, y = 9, we get the point (3, 9)
To get the same result (9) for $y = (x - 2)^2$ then x = 5, we get the point (5, 9)

Look at the graph, with this thinking lens on, move point P along the function and watch point P′. Don't panic if you find this conceptually challenging, research over more than 20 years confirms that this notion is difficult. Take the time to understand, review it often, and refer to the individual point created before determining the relationship. The individual point is an input. The graph is the output. One of the reasons why the translation and equation seem counterintuitive is that we are working on 'inputs', the graphical representation is an 'output', so there is a lot happening in between!

Question: 4.

Change the definition for P′: The abscissa changes to xp + 3 while the ordinate remains as yp.

a) Thinking about the translation, what is happening to point P′ ?

b) Determine the relationship between (equation) x' and y' and therefore the equation.

Answer:

a) Point P′ is translated horizontally by 3 units in the positive x direction.

b) Equation: $y' = y \text{ and } x' = x + 3.$ Using $y = x^2$ then $y' = (x' - 3)^2$ which can then be written as: $y = (x - 3)^2$

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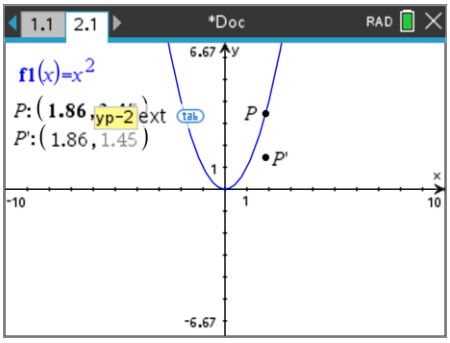
Vertical Transformation Algebra



Edit the relationship for point P′ as follows:

P′(xp, yp – 2)

Move point P along the graph of $y = x^2$ and observe the movements of the point P′.

Question: 5.

An equation needs to be determined for the path of P′ for the transformation provided in the previous instructions.

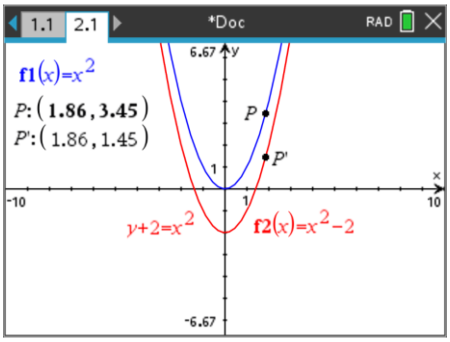
a) Write an expression for x'.

b) Write an expression for y'.

c) Transpose the equations as necessary and use the known relationship: $y = x^2$ to obtain a relationship between x' and y'.

d) Determine the rule for the movement of point P′ and use the calculator to check the graph.

Answer:



a) Expression for x' = x

b) Expression for y' = $y - 2$

c) x' = x and y' + 2 = y $\therefore$ y' + 2 = $(x')^2$

Note: This can be graphed using the direct result thanks to the relational graphing tool, however it is typically expressed in the form y = …

The advantage of the direct result is that it more closely resembles what is happening with the horizontal translation.

d) Equation: $y = x^2 - 2$

Question: 6.

Point P on the graph $y = x^2$ is translated parallel to the y axis by +3 units. The translated point is P′(x', y')

a) Write an expression for x'.

b) Write an expression for y'.

c) Determine the rule for the movement of P′.

d) Check your answer using the calculator.

Answer:

a) Expression for x' = x

b) Expression for y' = $y + 3$

c) x' = x and y' - 3 = y $\therefore$ y' - 3 = $(x')^2$

d) Equation: $y = x^2 + 3$

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