Calculus Mini Labs with the TI-89 and the TI-92 Plus

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On many graphing calculators it is possible to do a calculation and then edit it to do a similar calculation. However, the TI-89 allows us to do more. We can save the home screen in the text editor as a script of executable commands. This script can be accessed and modified as needed. I will illustrate this process with several scripts that I have used in teaching calculus.

I provide the script and a few sample screen dumps for each example. Figure 1 illustrates the TI-89 in split screen mode. The Text Editor is displayed in the upper half with the home screen in the lower half. The bold rectangle under the toolbar enclosing the upper half of the window indicates that the text editor is the active screen. Figure 2 shows the final steps of the solution to the problem on the home screen. A printout of the script follows the figures. The script files were transferred from the TI-89 to a computer with a link cable and Graph Link software. The scripts were then copied and pasted into this document. I use the same format to illustrate all the examples.

Our first example is on arc length in three dimensions. I consider the case of the helix given in parametric form (cos(t), sin(t), t) and compute the length on a closed interval [a,b].
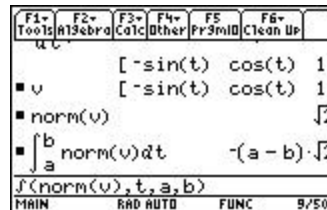

Figure 1

Figure 2

The script as it appears in the text editor follows.

C:"The Helix"
C:NewProb
C:Define x(t)=cos(t)
C:Define y(t)=sin(t)
C:Define z(t)=t
C:[[x(t),y(t),z(t)]]
C:d([[x(t),y(t),z(t)]],t) $\rightarrow$ v
C:v
C:norm(v)
C:$\int$(norm(v), t, a, b)

Once the script has been saved, it can be easily edited. I replaced the coordinate functions and the limits of integration to solve a related problem in James Stewart's text, *Calculus, Concepts and Contexts* (# 45 on pg. 734).

> The DNA molecule has the shape of a double helix. The radius of each helix is about 10 angstroms (1 angstrom = 10^{-8} cm.). Each helix rises about 34 angstroms during each complete turn and there are about 2.9 x 10^8 complete turns. Estimate the length of each helix.
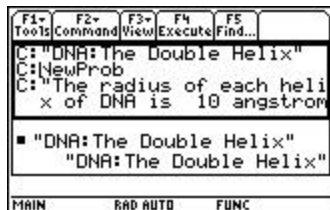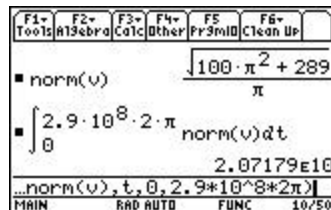
Figure 3

Figure 4

C:"DNA:The Double Helix"
C:NewProb
C:"The radius of each helix of DNA is 10 angstroms (1 angstrom=10^(-8) cms). Each helix rises 34 angstroms for each turn and there are 2.9*10^8 complete turns."
C:Define x(t)=10*cos(t)
C:Define y(t)=10*sin(t)
C:Define z(t)=34t/(2p)
C:[[x(t),y(t),z(t)]]
C: d([[x(t),y(t),z(t)]],t) $\rightarrow$ v
C:v
C:norm(v)
C:$\int$(norm(v), t,0,2.9$*$10^8$*$2 p)

Our next example is on the equality of the mixed partial derivatives. Prior to proving the equality of the mixed partial derivatives (or, in some cases, in lieu of such a proof) we might want to have our students verify the result for specific functions. This script does just that as it utilizes the computer algebra system of the TI-89.
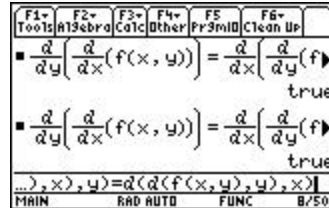
Figure 5

Figure 6

C:"Mixed Partials"
C:NewProb
C:"In this script I illustrate"
C:"equality of the mixed partials."
C:Define f(x,y)=sin(x^2+y^2)
C:$d(d(f(x,y),x),y)$
C:$d(d(f(x,y),y),x)$
C:$d(d(f(x,y),x),y)= d (d (f(x,y),y),x)$

Harmonic functions are the subject of the next script. Recall a function, $f(x,y)$, is called a harmonic function if it satisfies the Laplace equation, $f_{xx} + f_{yy} = 0$.
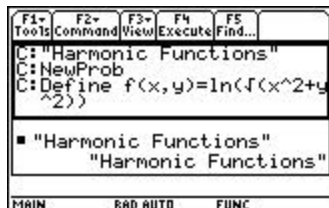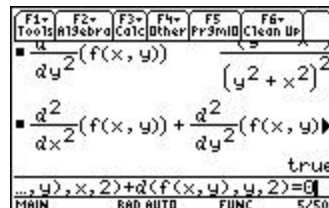
Figure 7

Figure 8

C: "Harmonic Functions"
C:NewProb
C:Define f(x,y)=ln($\sqrt{x^2 + y^2}$)
C: $d(f(x,y),x,2)$
C: $d(f(x,y),y,2)$
C: $d(f(x,y),x,2)+ d(f(x,y),y,2)=0$

In order to find the local extrema of a real valued function of two or three variables we apply the Second Derivatives Test. This necessitates finding the critical points where the first derivatives

simultaneously vanish. The SOLVE command of the TI-89 and a suitable script are just what we need.
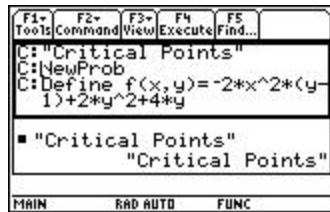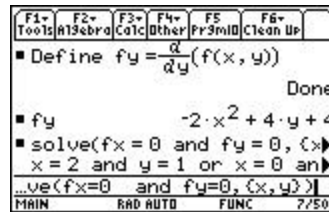

Figure 9

Figure 10

C: "Critical Points"
C:NewProb
C:Define f(x,y)= -2*x^2*(y-1)+2*y^2+4*y
C:Define fx=d(f(x,y),x)
C:fx
C:Define fy=d(f(x,y),y)
C:fy
C:solve(fx=0 and fy=0,{x,y})

Finally, I examine Lagrange Multipliers. I consider the problem of finding the dimensions of a rectangular box with largest volume and total surface area 64 cm^2. The TI-89 solves a system of 4 nonlinear equations. This is not an easy task to perform with pencil and paper!
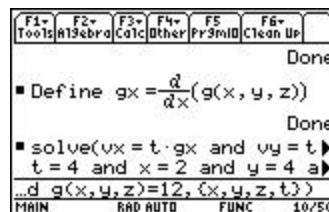

Figure 11

Figure 12

C: "Lagrange Multipliers"
C:NewProb
C:Define v(x,y,z)=x*y*z
C:Define vx=d(v(x,y,z),x)
C:Define vy= d (v(x,y,z),y)
C:Define vz= d (v(x,y,z),z)
C:Define g(x,y,z)=2*x*y+2*y*z+2*x*z
C:Define gz= d (g(x,y,z),z)
C:Define gy= d (g(x,y,z),y)
C:Define gx= d (g(x,y,z),x)
C:solve(vx=t*gx and vy=t*gy and vz=t*gz and g(x,y,z)=64,{x,y,z,t})

I have used all these examples in class. The use of scripts can save class time in checking solutions. I believe that when the students create the necessary steps to make the procedures work correctly, they develop a better understanding of the concepts.

Many different types of commands including graphing, program, and user-defined functions, can be done on the home screen. The following script was used to explore Riemann Sums in introductory integral calculus. Program **rect** and function **rarea** were written by Carl Leinbach (Connors, 1999).
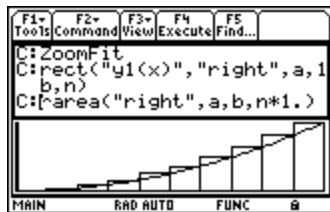

Figure 13

Figure 14

C:NewProb
C:Define y1(x)=x^2
C:0 →a
C:10 →b
C:10 →n
C:a →xmin
C:b →xmax
C:ZoomFit
C:rect("y1(x)","right",a,1b,n)
C:rarea("right",a,b,n*1.)
C:rect("y1(x)","left",a,b,n)
C:rarea("left",a,b,n*1.)
C:rect("y1(x)","mid",a,b,n)
C:rarea("mid",a,b,n*1.)

References

Connors, Mary Ann., Editor and Contributor. (1999). *Calculus Short Course: CAS-CALC I.* Columbus, OH: Technology College Short Course Program.

Connors, Mary Ann. (1999). *The TI-89 in Discrete Dynamical Systems and Calculus*. USMA, West Point, NY: Adam Fellowship Technology Project.

Leinbach, Carl and Thomas, Sally, Editors and Contributors. (1999*). Calculus Short Course: CAS-CALC II*. Columbus, OH: Technology College Short Course Program.

Stewart, James. (1998). *CALCULUS: Concepts and Contexts*. Pacific Grove, CA:
Brooks/Cole Publishing Company.