Comparing Z-Scores

THE NORMAL DISTRIBUTION

Teachers Teaching with Technology™ Professional Development from Texas Instruments

Teacher Notes & Answers

7 8 9 10 **11** **12**

TI-30XPlus MathPrint™ | Worksheet | Student | 50 min

Teacher Notes:

Statistical Analysis – Syllabus Outcome: MS2-12-10, bands 4 – 5.

The sample question is based on Q20 of the NESA example questions for Statistical Analysis. Students can scan the QR code to watch a 3 minute video covering the concept of a z-score, the sample problem and how to solve it on the TI-30XPlus calculator using two different approaches. The remaining questions rehearse and extend content from the sample question. The video is for question two uses a different calculator however the content is still relevant.

Instructions

The first question is provided as a sample. Scan the QR code with your phone to watch the tutorial on YouTube. The tutorial demonstrates how to answer the question, a brief explanation of 'why' and how to use your calculator to help solve the problem.

Sample Question

Two brands of light bulbs are being compared.

The life span of the globes for each brand is normally distributed.

A light bulb is deemed to be defective if it is has a life span of less than 500 hours.

A consumer makes the following claim: "Brand A light bulbs are more likely to be defective than Brand B."

Brand	A	B
Mean Life Span:	600 hours	700 hours
Standard Deviation:	50 hours	100 hours

Question: Determine whether or not the consumer's claim is correct and justify your answer using information provided.

Question: 1.

Basketball scores are normally distributed. Team A has an average score of 80 and a standard deviation of 10. Team B has an average score of 90 and a standard deviation of 5.

a) Which team is most likely to obtain a score that exceeds 100?

Answer: Team A and B are equally likely as $Z_A = \dfrac{100-80}{10} = 2$ & $Z_B = \dfrac{100-90}{5} = 2$ corresponding to 2 standard deviations from the mean.

b) Which team is most likely to obtain a score that exceeds 95?

Answer: Team B is more likely to exceed a score of 95 since: $Z_B = \dfrac{95-90}{5} = 1$ & $Z_A = \dfrac{95-80}{10} = 1.5$ corresponding to 1.5 and 2 standard deviations from the mean.

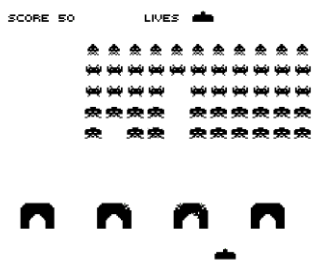
c) Which team is most likely to obtain a score that exceeds 105?

Answer: Team A is more likely to exceed a score of 105 since: $Z_A = \dfrac{105-80}{10} = 2.5$ & $Z_B = \dfrac{105-90}{5} = 3$ corresponding to 2.5 and 3 standard deviations from the mean.

Author: P. Fox

Question: 2.

Space Invaders was introduced in 1978. Players attempted to defeat waves of alien space craft. The console was a commercially produced rigid coffee table sized machine. High scores were recorded on each console and found to be normally distributed. A resurgence of the game's popularity amongst retro-gamers produced a new set of high scores. Modern technology is more responsive so these scores are not directly comparable. The mean and standard deviation for the original and retro gamers are in the table below.



a) Buzz is an original gamer. He had a high score of 6210 points. JD is a Retro-Gamer with a high score of 7280 points. Which gamer might be deemed as the most skilled?

Player	Original - Gamers	Retro-Gamers
Mean High Score:	6000 points	7000 points
Standard Deviation:	200 points	250 points

Answer: Assuming "most skilled" relates to other gamers of their era; Buzz: $Z_B = \dfrac{6210-6000}{200} = 1.05$ and

JD: $Z_J = \dfrac{7280-7000}{250} = 1.12$ suggests JD is further from the mean than Buzz.

Teacher Notes:

This question purposely uses non-integer multiples of the standard deviation. The relative distance from the mean is not as obvious from the figures provided essentially forcing students to 'do the calculation' and interpret the result. Whilst the difference may appear subtle, such questions are often sufficient to challenge some students.

b) Renee is a Retro-Gamer with a high score of 6990. Her mum Skye was an original gamer with a high of 6180. Even though Skye's score is numerically higher, Renee would like to beat her mum's standardised score, a score relative to her mum's era. What score does Renee need to acquire in order to achieve this goal? For additional help for this question, check out the video: http://bit.ly/z-scores

Answer: Skye's (mum) z-score is: $Z_S = \dfrac{6180-6000}{200} = 0.9$. This means that Renee will need to achieve a higher z-score. $Z_R = \dfrac{x-7000}{250} = 0.9$. So Renee needs to score: 7225 or higher.

Teacher Notes:

This question involves the same concepts but requires students to 'solve' a simple linear equation. One way to check student's conceptual understanding here would be provide multiple choice answers such as: (a) 7750, (b) 7300, (c) 7250 (d) 7225 and (e) 7180. Whilst scores above 7225 will beat Skye's score Renee doesn't have to achieve them to beat her mum, Skye. The question implies the 'minimum required score'. Each incorrect response aligns to a potential misconception or simply a guess (7750). Students should be able to 'estimate' the correct answer without guessing. 6180 is less than one standard deviation from the mean. 7225 and 7180 are the only two answers less than one standard deviation from the mean for the retro-gamers. Since 7180 is the same distance from the mean for retro-gamers as 6180 is for the original gamers, (but retro-gamers have a higher standard deviation), this only leaves 7225 as the possible correct answer. If students have this logic and reasoning they have a good conceptual grasp and appropriate numerical skills but may not have the confidence or skills to perform the corresponding algebraic manipulation.

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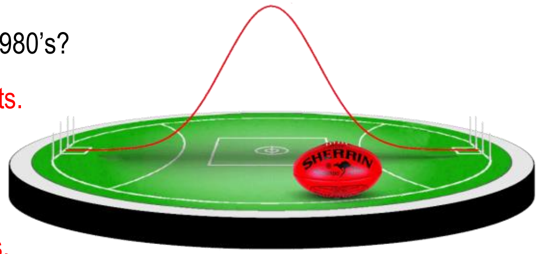
Question: 3.

In any given year Australian Rules Football (AFL) game scores are approximately normally distributed. In the 1980's the average (mean) score was 110 points with a standard deviation of 30. After a series of rule and structure changes, the average score in 2019 was 81 points with a standard deviation of 22.

a) A statistical body declares that a high scoring game occurs when a team scores 2 standard deviations above the mean.

i. What would be classified as a high score in the 1980's?

Answer: High Score = 110 + 2 x 30 = 170 points.



ii. What would be classified as a high score in 2019?

Answer: High Score = 81 + 2 x 22 = 125 points.

iii. Of the games played in the 1980's, what proportion of the scores would have been classified as high scoring in 2019?

Answer: Note that this response requires computation of the probability. From part (ii) we know that a high score in 2019 corresponded to 125 points. So the question is asking for the proportion of scores greater than 125 in the 1980's where μ = 110 and σ = 30.
Using the calculator, approximately 30.9% of scores in the 1980's would have been greater than 125.

iv. Of the games played in 2019, what proportion of them would have been classified as high scoring in the 1980's?

Answer: Note this is the reverse of the previous question. From part (i) we know that a high score in the 1980's corresponded to 170 points. So the question is asking for the proportion of scores greater than 170 in 2019's where μ = 81 and σ = 22.
Using the calculator, approximately 0.003% of scores in the 2019 would have been greater than 170.

Note: Given that 198 games are played in a season it is likely that at most we would see just one game with such a high score. Indeed, there were no games in the 2019 season with such a high score. The highest score was 150 points. A stark contrast to 1980 for example that saw 4 games over 170 points including one with 211 points in a season that contained only 132 games! [There were fewer AFL clubs in 1980.]

v. Identify an appropriate classification for a 'low scoring' game. Of the games played in 2019, what proportion of them would have been classified as low scoring in the 1980's?

Answer: Note this is the reverse of the previous question but also looks at 'low scoring'. Given that a 'high scoring' game is identified as 2 standard deviations above the mean, then classification for a low scoring game could be defined as 2 standard deviations below the mean.
Students could take two approaches to the second part of the question.
Option 1: Students could use theoretical calculations.
Option 2: Students could check actual game results.
In both cases a cut off score needs to be established using the 1980's parameters.
1980's low score = 110 – 2 x 30 = 50 points.
Option 1: Theoretical result using the calculator: 7.9% of games would have been low scoring.
Option 2: Out of a total of 396 scores, 24 were 50 points or lower, approximately 6%.

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b) Each AFL game consists of 4 quarters, each of 20 minute duration. During 2020 the AFL changed the duration to 4 quarters, each of 16 minute duration. [Note: Duration = Actual Game Time]
During 2020 the average score was 61 points with a standard deviation of 20.

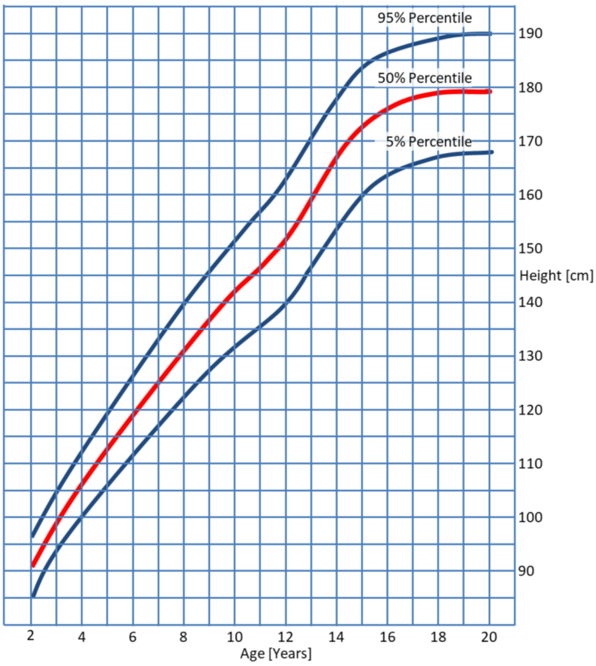
i. What would be classified as a high score in 2020?

Answer: Using the same classification for high scores as before, a high score would be 101 points.

ii. Many people commented that the 2020 scores were much lower than expected. Based on game time, determine whether the 'average' score was 'to be expected'.
Answer: As game time was reduced by 20%, it would be *reasonable* to expect scores to also be reduced by 20%. In 2019 the average score was 81 points, therefore the expected score for 2020 could have been approximately 64 points. So the actual average of 61 points was only slightly lower than the expected result.

Question: 4.

Growth charts provide a range of typical heights for range of age groups. The height of adolescent males is normally distributed. The growth chart (opposite) shows the mean height bounded by the fifth and ninety fifth percentiles.



a) Based on the chart, what is the approximate standard deviation of 20 year old males?

Answer: The 95% percentile (190cm) down to the 5% percentile (168cm) represents 4 standard deviations. Therefore one standard deviation is approximately: 5.5 cm.

b) A high school teacher measured the heights of boys in the class as: 166, 163, 169, 166, 174, 175, 167, 182, 179, 163, 168, 176, 160 and 173. Determine the likely age of the boys. Justify your answer.

Answer: The average height of the boys in the class is 170cm with a standard deviation of 6.5 cm. This aligns quite well with students between 14 and 15 years of age.

Note: The intention here is to introduce the concept of the 'likelihood' that the students fall into the 14 – 15 year age category. They are less likely to have come from a 12 – 13 year or 19 – 20 year age group. This is the idea behind hypothesis testing and confidence intervals. An informal discussion about hypothesis testing is appropriate so that students understand that 'statistics' is a very large area of study and of course relevant to many careers.

c) Nelson Asofa-Solomona is the tallest NRL player at 200cm. Mason Cox plays is the tallest AFL player at 211cm. Assuming males stop growing (height) at the age of 20, what percentiles do Nelson and Mason belong?

Answer: Mean height ≈ 179cm. Standard Deviation ≈ 5.5. Nelson's height is 3.8 standard deviations above the mean. This puts him in the 99.99 percentile. Mason's height is 5.8 standard deviations above the mean. That puts him in the 99.999999 percentile. In other words … Mason is very tall!

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