TEXAS
INSTRUMENTS

EXPLORATIONS™

Computing Derivatives

by

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Important notice regarding book materials

Objectives

♦ To use transformations to remove derivative symbols

♦ To simplify expressions

Materials

♦ TI-89 or Voyage™ 200

♦ Symbolic Math Guide (SMG) handheld application

Computing Derivatives

Activity Overview

These example problems provide ideas about using SMG to help students learn to solve derivative problems. The key transformations in SMG are the sum/difference, constant (or scalar) product, product, quotient, and the chain rule. SMG also includes transformations for derivatives of basic functions. Students can use SMG to learn successful strategies for computing derivatives. Necessary background includes the ability to classify functions as power, exponential, product, composition, and so on. Students also need to know how to decompose composite functions.

Setting Up the TI-89 or TI Voyage™ 200

Before starting, make sure that the TI-89 or TI Voyage™ 200 has SMG installed. If the TI-89 or TI Voyage™ 200 does not have SMG version 2.00 or higher installed, go to education.ti.com to download the free app. If you are not familiar with SMG, the SMG User Guide and the SMG Guided Tour are available at the same link.
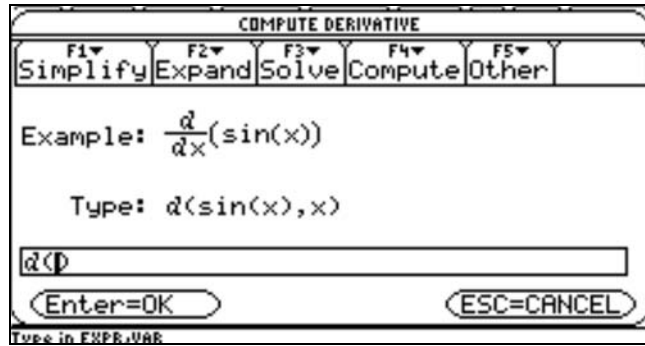
Start SMG by pressing APPS, choosing SMG, and then pressing ENTER.

Entering Derivative Problems

Example

Enter $\frac{d}{dx}(3)$.

1. Press F2 **1** and then press F4 **1**. The display should be similar to the one shown at the right. Press F2 **2** to set the display to enter derivative problems.

```
                    COMPUTE DERIVATIVE
  F1▼     F2▼     F3▼     F4▼     F5▼
Simplify Expand Solve Compute Other

                   d
Example: ───(sin(x))
                  dx

    Type: d(sin(x),x)

d(
 Enter=OK                      ESC=CANCEL
Type in EXPR,VAR
```

2. Type **3,x**. Press ENTER until the window disappears.

The Derivatives of Basic Functions

You begin to learn to compute derivatives by learning to recognize the derivatives of functions such as polynomials, sine, cosine, tangent, ln, log, and so forth. See Student Reference Sheet 1, **Basic Derivatives**, for a table of some of the basic derivatives that SMG knows. In addition, your teacher may choose examples from textbooks or other references for you to solve. An example set of basic derivatives problems is included in Student Work Sheet 1, **Basic Problems**. Master these problems before continuing to the next section.

Example

Compute $\dfrac{d}{dx}(3)$ using SMG. Enter the problem as described in the previous section.

1. The problem appears on the right.

```
 F1▼  F2▼  F3       F4    F5▼   F6▼  F7▼
Set  Prob  ▣+b Trans Tools Ctrl  ?
P20:Compute Derivative

 d
───(3)
dx




main\toures 20/20 Compute Derivative
```
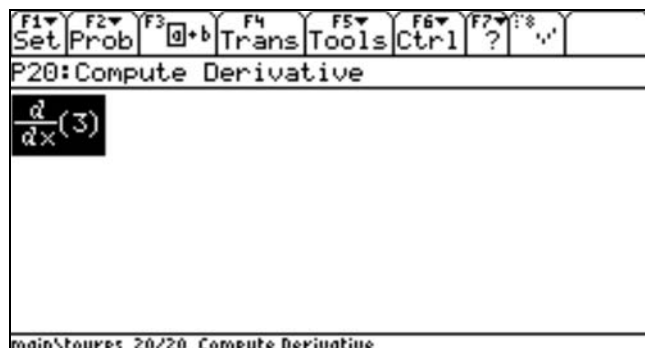
2. Press $\boxed{\text{F4}}$. The display should be similar to the one shown at the right.

3. Press **1** to apply **derivative of constant.**

```
            SELECT TRANSFORMATION
                    d
                   ──(3)
                   dx
┌────────────────────────────────────────┐
│1:│derivative of constant                │
│2:│derivative of polynomial              │
│3:│basic derivatives                     │
│4:│ d                     d              │
│  │──(c·f(x)) → c·──(f(x))              │
│  │dx                    dx              │
│5:│derivative of sum or diff             │
│6:│derivative of product                 │
└────────────────────────────────────────┘
MAIN          RAD AUTO         FUNC 20/20
```
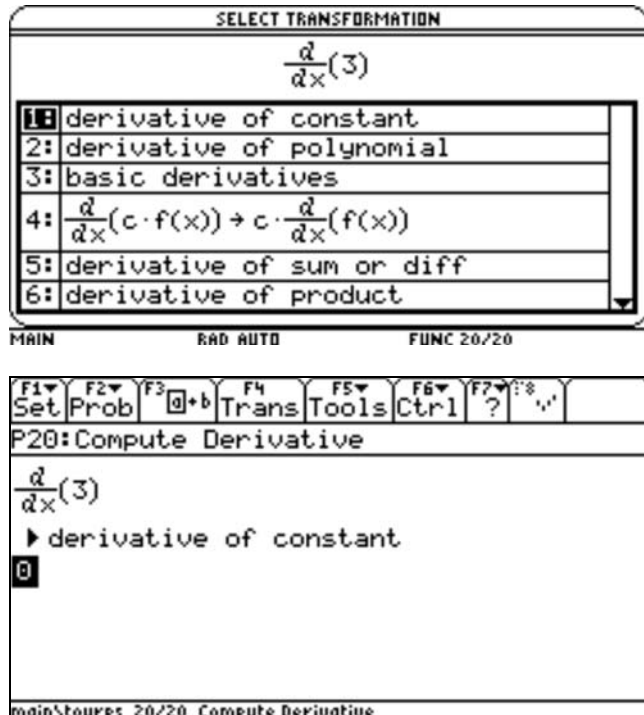
4. The complete solution for this simple problem contains three lines including the name of the transformation you chose.

```
┌F1▾┬F2▾┬F3─┬─F4──┬─F5▾─┬F6▾┬F7▾┐
│Set│Prob│+b│Trans│Tools│Ctrl│ ? │
├───┴────┴───┴─────┴─────┴────┴───┤
│P20:Compute Derivative           │
├─────────────────────────────────┤
│ d                               │
│──(3)                            │
│dx                               │
│                                 │
│ ▶ derivative of constant        │
│                                 │
│ 0                               │
│                                 │
│                                 │
│                                 │
├─────────────────────────────────┤
│main\toures 20/20 Compute Derivative│
└─────────────────────────────────┘
```

Derivatives of Sums and Differences $(f \pm g)' \to f' \pm g'$

The derivative of a sum is the sum of the derivatives and the derivative of a difference is the difference of the derivatives. See Student Work Sheet 2, **Derivatives of Product, Scalar Products, and Quotients**, for practice using these formulas.
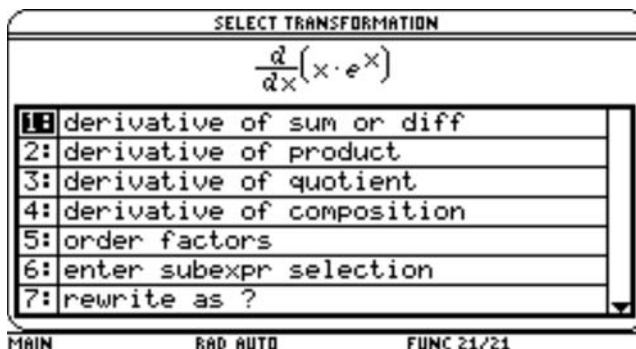
Derivatives of Products $(f \cdot g)' \to f' \cdot g + f \cdot g'$

SMG provides a special window for applying the product formula. This window gives you control over applying the product rule to terms made of three or more factors and may help you not to apply the product rule when you should actually use the chain rule to differentiate a composition.
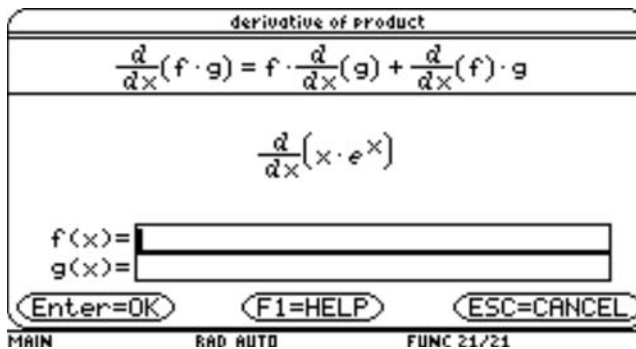
<u>Example</u>

Compute $\dfrac{d}{dx}(x \cdot e^x)$ using SMG. Make sure you use $\boxed{\text{2nd}}$ $[e^x]$ to enter the exponential function. (Note: On the TI-89, press $\boxed{\bullet}$ $\boxed{\text{X}}$.)
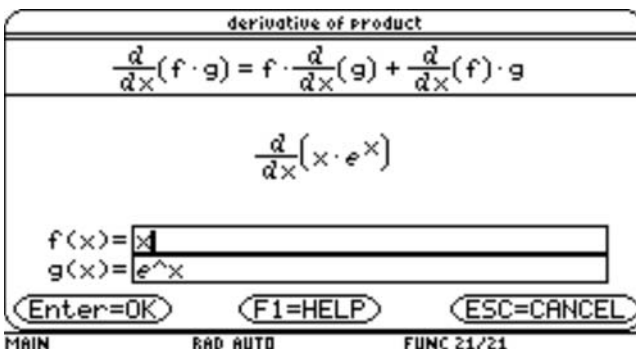
1. Press F4 to list possible transformations for the first step. The function to differentiate is a product of *x* and an exponential function.
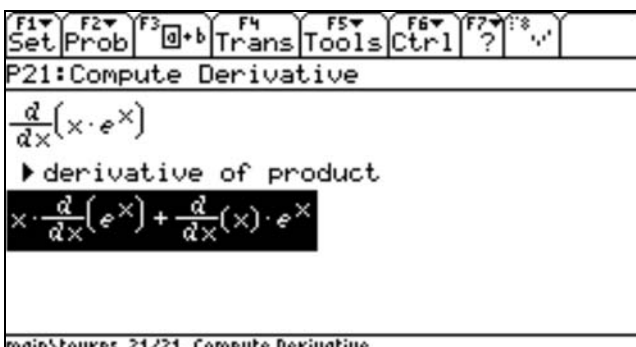
SELECT TRANSFORMATION

$$\frac{d}{dx}\left(x \cdot e^x\right)$$

1:	derivative of sum or diff
2:	derivative of product
3:	derivative of quotient
4:	derivative of composition
5:	order factors
6:	enter subexpr selection
7:	rewrite as ?

MAIN   RAD AUTO   FUNC 21/21

2. Press **2** to select **derivative of product**. SMG displays a window to help you apply the product rule.

derivative of product

$$\frac{d}{dx}(f \cdot g) = f \cdot \frac{d}{dx}(g) + \frac{d}{dx}(f) \cdot g$$

$$\frac{d}{dx}\left(x \cdot e^x\right)$$

f(x)=
g(x)=

(Enter=OK)   (F1=HELP)   (ESC=CANCEL)

MAIN   RAD AUTO   FUNC 21/21

3. Decide how to choose f(*x*) and g(*x*), and then press F1 to have SMG fill in the key information. Make sure that the substitution is the one you want. Change it if it is not.

derivative of product

$$\frac{d}{dx}(f \cdot g) = f \cdot \frac{d}{dx}(g) + \frac{d}{dx}(f) \cdot g$$

$$\frac{d}{dx}\left(x \cdot e^x\right)$$

f(x)=x
g(x)=e^x

(Enter=OK)   (F1=HELP)   (ESC=CANCEL)

MAIN   RAD AUTO   FUNC 21/21

4. Press ENTER until the dialog box disappears.

5. Compare SMG's result with yours. Resolve any differences.

F1▾	F2▾	F3	F4	F5▾	F6▾	F7▾
Set	Prob	⊡+b	Trans	Tools	Ctrl	?

P21:Compute Derivative

$$\frac{d}{dx}\left(x \cdot e^x\right)$$

▶ derivative of product

$$x \cdot \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}(x) \cdot e^x$$

main\toures 21/21 Compute Derivative

Press F4 and choose **basic derivatives** to complete this problem. See Student Work Sheet 2, **Derivatives of Product, Scalar Products, and Quotients,** for practice using the product rule. Remember the goal is for you to be able to work these problems without SMG. Use SMG to make sure you correctly take each step. Your focus should be on choosing a good transformation for the problem you are solving.
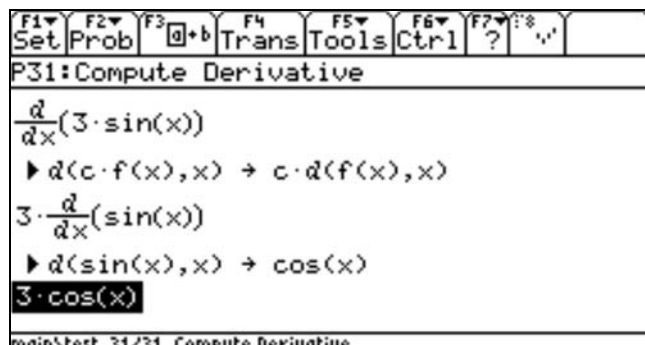
Derivatives of Scalar Products $\frac{d}{dx}(c \cdot f) \rightarrow c \cdot \frac{d}{dx}(f)$

Many students use the product rule when the scalar product rule, which is easier to use, applies. The two screens that follow show the difference between using the product rule and using the scalar product rule when differentiating a constant times function. Choosing the simpler scalar product rule is especially important when differentiating complicated expressions. Begin by entering the problem.

1. Press F4 **2** F1 ENTER.

2. Press F4. Choose **basic derivatives**.

3. Observe that the second term is 0. One term will always be 0 when one of the factors is a constant.
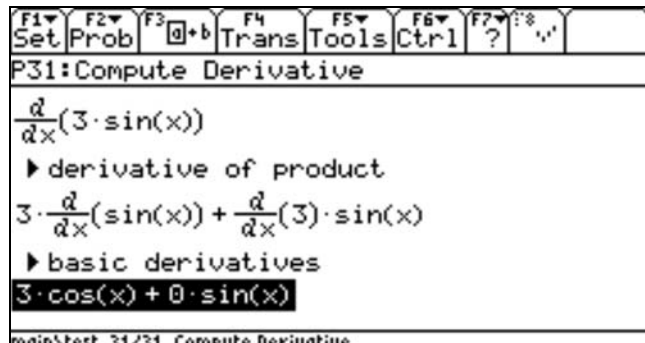
4. Press ⌃ to move to the top of the problem.

5. Press F4 **1**.

6. Press F4 **1**.

7. Observe that the extra term never appears. No extra simplification is necessary.

See Student Work Sheet 2, **Derivatives of Product, Scalar Products, and Quotients,** for additional examples.

Derivatives of Quotients $(f/g)' \rightarrow \dfrac{f' \cdot g - f \cdot g'}{g^2}$

This important formula causes students some difficulty because the derivative of the numerator is in a positive term while the derivative of the denominator is in a negative term. Practice writing the formula from memory until you have mastered it. Note that, once you have learned the chain rule in the next section, you can use the product rule to differentiate quotients. See Student Work Sheet 4, **You Pick the Method(s)**, problem 6. Student Work Sheet 2 contains problems that you can use to practice the quotient rule.
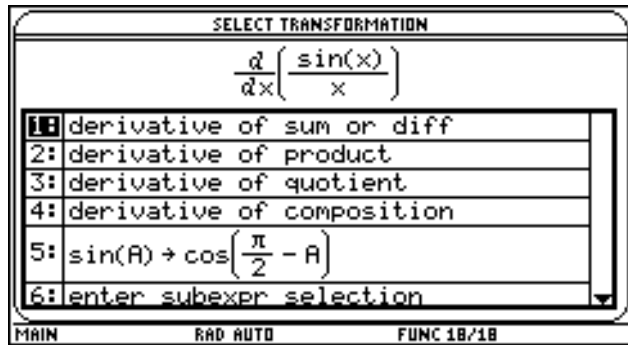
Example

Compute d(sin(x)/x,x) using SMG. Enter the problem and then follow along with pencil and paper anticipating SMG's results.
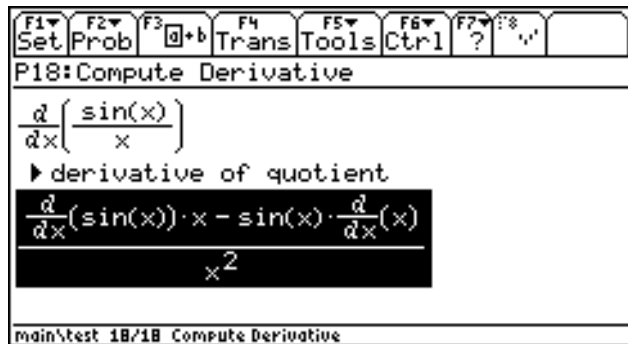
1. The problem appears on the right.
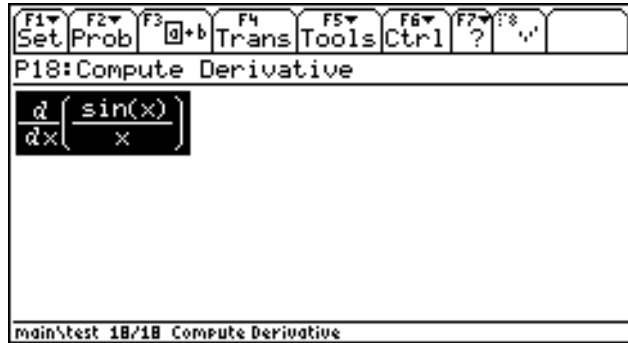
2. Press F4. The display should be similar to the one shown at the right.

3. The problem is to differentiate a quotient, so apply the quotient rule on your paper and then press **3.**

4. Compare your answer with SMG's answer. Make sure you have the minus sign in front of the correct term.

5. You can finish removing the derivative symbols by applying derivative formulas for the basic functions sin(x) and x. Do this on your paper and then press F4 and choose **basic derivatives**. Compare your answer with the result on the right or the one below and on the right.
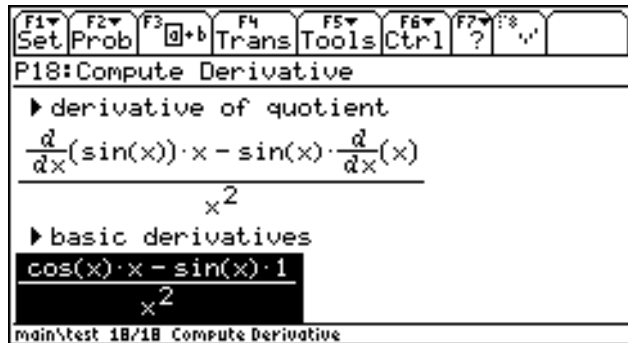
6. Finish by pressing ENTER to simplify the result.
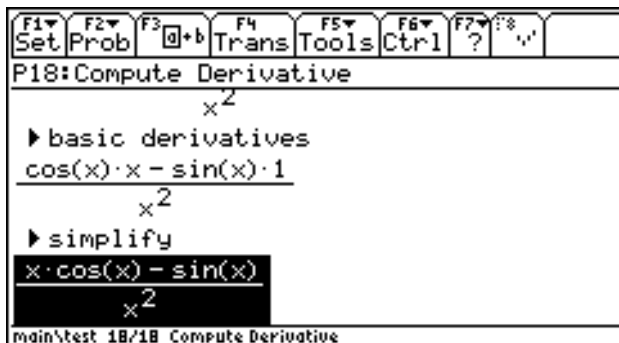
Derivatives of Compositions and the Chain Rule

Some example compositions $(x^3 + 1)^{10}, \sin(x^2), \cos^2(x),$ and $\sqrt{x^2 + 7}$. Note that $\sin(x^2)$ is not a product although it looks like one. In SMG, products are displayed with a dot between factors like $x \cdot (x + 1)$. Look for the dot to make sure the expression contains a product instead of a composition, which will not have the dot.

Example

Compute $\dfrac{d(\sqrt{x^2 + 1})}{dx}$. Enter the problem.

1. To display choices, press F4. The display should appear similar to the one shown at the right.

2. The function to differentiate is a composition, so press **4: derivative of composition**.

3. sqrt(x^2+1) is a composition with outside function f(*u*) = sqrt(*u*) and inside function x^2+1. Press F1 to confirm that this thinking is correct. You may type information yourself. In some cases, SMG will not know how to fill in the information.
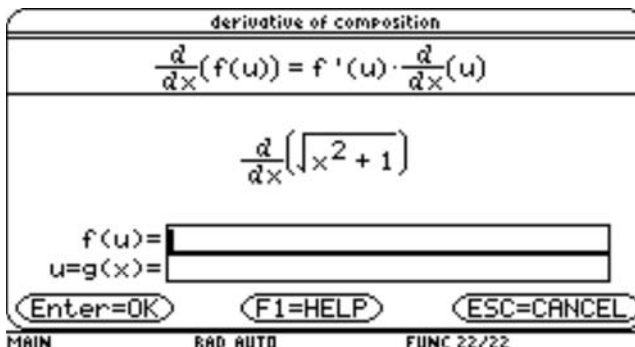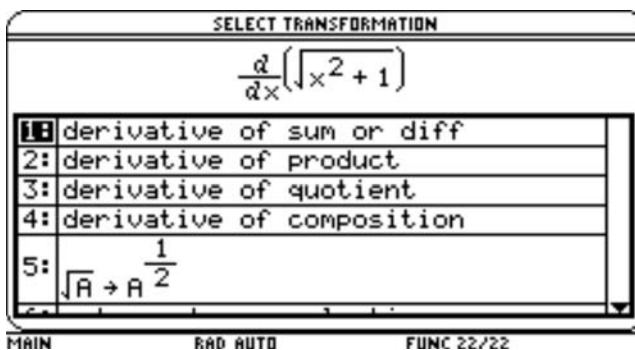
4. Press $\boxed{\text{ENTER}}$ until the window disappears.

To finish the problem, press $\boxed{\text{F4}}$, choose **derivative of polynomial,** and press $\boxed{\text{ENTER}}$ to simplify the result. See Student Work Sheet 3 for practice solving these problems.

Developing Successful Strategies

To learn to compute derivatives:

♦   Learn the basic derivatives and how to choose the proper formula to use them. Students often confuse power and exponential functions. It is helpful to tie formulas to graphs. For example, have student compare the graphs of $y = x^2$ and $y = 2^x$. Then ask them to compare the slopes of tangents at $x = 1$, $x = 10$, and $x = 100$. SMG will place the proper transformation near the top of the list of transformations that result from pressing F4.

♦   Master each major formula. Ask students to apply specific formulas to particular problems.

♦   Learn to recognize sums, products, quotients, and compositions. Students often confuse scalar products and products as well as compositions and products. SMG places the scalar product transformation at the top when it applies this transformation. This serves as a prompt for the students.

♦   Learn to compute derivatives that require the use of more than one major rule. Drawing syntax trees may help. Syntax trees used in computer science serve as an alternate representation of expressions that help students visualize the structure of math expressions. Student Work Sheet 4, **You Pick the Method(s)**, provides students with practice in selecting and applying different rules for a variety of problems.

Teaching Notes

1.   The use of names to identify derivative rules provides an opportunity for students to learn the derivative formulas. Think about the formula for the chain rule before you choose it. When SMG shows you the formula, compare it with your version. Writing your version first may be especially helpful.

2.   Uses of SMG:

♦   To give students an overview of SMG, demonstrate the use of SMG in class.

♦   Give students a goal to work toward. SMG provides a goal for each problem type. Press F7 **1** (2nd [F7] on the TI-89) to find the goal. For derivative problems, SMG says "Apply derivative formulas to remove all indicated derivatives (and then simplify)."

♦   The use of SMG suggests several types of problems that may help students develop successful strategies for attacking "compute the derivative" problems.

♦ Problem Type 1: Give students an outline of a solution and have them fill in a solution. The outline could be a sequence of SMG transformations that solve the problem.

♦ Problem Type 2: Have students develop a plan for computing a derivative. The plan could look like the transformations that SMG reports in the solution display. Some students benefit from structure trees to help them make solution plans.

3. Opportunities to learn with SMG:

♦ Before choosing a named transformation, the student could write (or think) its formula (if it has one).

♦ Before pressing ENTER with TIME TO THINK mode ON, the student could write the result of the transformation.

Student Reference Sheet 1

Basic Derivatives

Name: _____

Date: _____

The following table contains a sample of the formulas available.

Derivative of constant: $\dfrac{d}{dx}(C) \rightarrow 0$
Derivative of identity: $\dfrac{d}{dx}(x) \rightarrow 1$
Power Rule: $\dfrac{d}{dx}(x^r) \rightarrow r \cdot x^{(r-1)}$
$\dfrac{d}{dx}(e^x) \rightarrow e^x$
$\dfrac{d}{dx}(a^x) \rightarrow a^x \cdot \ln(a)$
$\dfrac{d}{dx}(\sin(x)) \rightarrow \cos(x)$
$\dfrac{d}{dx}(\cos(x)) \rightarrow -\sin(x)$
$\dfrac{d}{dx}(\tan(x)) \rightarrow (\sec(x))^2$
$\dfrac{d}{dx}(\cot(x)) \rightarrow -(\csc(x))^2$
$\dfrac{d}{dx}(\csc(x)) \rightarrow -\csc(x) \cdot \cot(x)$
$\dfrac{d}{dx}(\sin^{-1}(x)) \rightarrow \dfrac{1}{\sqrt{1-x^2}}$
$\dfrac{d}{dx}(\cos^{-1}(x)) \rightarrow \dfrac{-1}{\sqrt{1-x^2}}$
$\dfrac{d}{dx}(\tan^{-1}(x)) \rightarrow \dfrac{1}{1+x^2}$

Student Reference Sheet 2

Key Transformations

Key Transformations	Examples/Comments
Derivative of scalar product $$\frac{d}{dx}(c \cdot f) \rightarrow c \cdot \frac{d}{dx}(f)$$	$$\frac{d}{dx}(3 \cdot \ln(x))$$
Derivative of sum or difference $$(f \pm g)' \rightarrow f' \pm g'$$	$$\frac{d}{dx}(x + \ln(x))$$
Derivative of product $$(f \cdot g)' \rightarrow f' \cdot g + f \cdot g'$$	$$\frac{d}{dx}(x \cdot \ln(x))$$
Derivative of quotient $$(f/g)' \rightarrow \frac{f' \cdot g - f \cdot g'}{g^2}$$	$$\frac{d}{dx}\left(\frac{\ln(x)}{x}\right)$$
Derivative of composition [chain rule] $$\frac{d}{dx}(f(u)) \rightarrow f'(u) \cdot \frac{d}{dx}(u)$$	$$\frac{d}{dx}(e^{x^2})$$
$$\frac{d}{dx}(\ln(|f|) \rightarrow \frac{1}{f} \cdot f'$$ $$\frac{d}{dx}(\ln(|x|) \rightarrow \frac{1}{x}$$	$$\frac{d}{dx}(\ln|e^x|)$$
Basic derivatives	Applies transformation for each basic derivative in the expression. $$\frac{d}{dx}(1/x) + \frac{d}{dx}(\sin(x))$$

Student Work Sheet 1

Basic Problems

Name: _____

Date: _____

Use these problems to check your mastery of the basic formuals. Compute each of the derivatives below using SMG unless indicated otherwise.

Basic Derivatives

1. $\dfrac{d}{dx}(3)$

    ◆ deriv of constant

    > 1. Think: "3 is a constant and the derivative of a constant is 0."
    >
    > 2. Press [F4] and choose **derivative of constant.**

2. $\dfrac{d}{dx}\left(\dfrac{1}{x}\right)$

    ◆ d(1/x,x) -> -1/x^2

    > Remember formula or think $1/x = x^{-1}$

3. $\dfrac{d}{dx}(\sqrt{x})$

    > Remember formula or think $\sqrt{x} = x^{1/2}$

4. $\dfrac{d}{dx}(x^{2/3})$

5. $\dfrac{d}{dx}(\ln(x))$

6. $\dfrac{d}{dx}(\log(x))$

7. $\dfrac{d}{dx}(e^x)$

8. $\dfrac{d}{dx}(2^x)$

Student Work Sheet 2

Name: _____

Derivatives of Product, Scalar Products, and Quotients

Date: _____

You should work several problems manually before using SMG to work the problems below.

Derivative of a Product

1. $\dfrac{d}{dx}(x \cdot e^x)$

2. $\dfrac{d}{dt}(\sqrt{t} \cdot (1 - t))$

> Note: You could simplify sqrt(*t*)*(1-*t*) before entering this problem. How would simplification help?

3. $\dfrac{d}{dx}(\sqrt{x} \cdot g(x))$

> Note: Do not enter a formula for *g(x)* when asked to. The solution will contain *d(g(x))/dx*.

    ♦   derivative of product

4. $\dfrac{d}{dt}(t^3 \cdot \cos(t))$

Derivative of a Quotient

5. $\dfrac{d}{dx}\left(\dfrac{x^2 + x - 2}{x^5 + 6}\right)$

6. $\dfrac{d}{dx}\left(\dfrac{e^x}{1 + x^2}\right)$

7. $\dfrac{d}{dx}\left(\dfrac{\tan(x)}{x}\right)$

8. $\dfrac{d}{dx}\left(\dfrac{x}{\sin(x) + \cos(x)}\right)$

> Test your mastery of the quotient rule.
>
> a. The object being differentiated is a quotient. Try the quotient rule.
>
> b. Press F4 and choose derivative of quotient.
>
> c. Write down your result of applying the quotient rule.
>
> d. Press ENTER, fill in the dialog box, press ENTER until the dialog goes away, and compare SMG's result with your result.
>
> e. Resolve any differences.

Derivative of Scalar Product

9. $\dfrac{d}{dx}(3 \cdot x)$

♦ $\dfrac{d}{dx}(c \cdot f(x)) \rightarrow c \cdot \dfrac{d}{dx}(f(x))$

♦ $\dfrac{d}{dx}(x) \rightarrow 1$ [Press ENTER to clean-up.]

10. $\dfrac{d}{dx}(7 \cdot x^2)$

♦ $\dfrac{d}{dx}(c \cdot f(x)) \rightarrow c \cdot \dfrac{d}{dx}(f(x))$

♦ $\dfrac{d}{dx}(x^r) \rightarrow r \cdot x^{r-1}$ [Clean-up]

11. $\dfrac{d}{dx}(-x)$

12. $\dfrac{d}{dx}(y \cdot x)$

13. $\dfrac{d}{dx}(y(x) \cdot x)$

> Note: Do not enter a formula for $y(x)$ when requested; just press ENTER. Is this a scalar product?

Student Work Sheet 3

Derivatives of Compositions

Name: _____

Date: _____

Work several problems by hand before using SMG to help you with the problems below.

1. $\dfrac{d}{dx}(\sqrt{x^2+1})$

2. $\dfrac{d}{dx}(\ln(x^3+1))$

3. $\dfrac{d}{dt}\left(\dfrac{t-2}{2t+1}\right)^9$

4. $\dfrac{d}{dx}\left(\ln(x^2)\right)$

<div style="border:1px solid black">
Note: Log properties might help you simplify this problem.
</div>

5. $\dfrac{d}{dx}\left(\sin(\sin(\sin(x)))\right)$

6. $\dfrac{d}{dx}\left(\ln(\sin(x))\right)$

Student Work Sheet 4
You Pick the Method(s)

Name: _____

Date: _____

Compute each of the derivatives below using SMG unless other instructions are provided.

1. $\dfrac{d}{dx}(x^2 + 3x^3)$

Version A

   ♦ derivative of sum or diff

   ♦ basic derivatives [clean-up]

Version B [This is quicker and can be used once you have mastered differentiating polynomials.]

   ♦ derivative of polynomial [clean-up]

2. $\dfrac{d}{dx}(e^x - 2^x)$

   ♦ derivative of sum or diff

   ♦ basic derivatives

3. $\dfrac{d}{dx}(\dfrac{1}{x} + \sqrt{x})$

4. $\dfrac{d}{dx}(\sqrt{x} - 2e^x)$

5. $\dfrac{d}{dt}(t^2 - \dfrac{1}{\sqrt{t}})$

6. $\dfrac{d}{dx}\left(\dfrac{5e^x + 4}{x^2 + 2}\right)$ [Hint: You can think of this as a product and then use the chain rule.]

7. $\dfrac{d}{dx}(x^{-3/5})$

8. $\dfrac{d}{dx}(x - 3 \cdot \sin(x))$

9. $\dfrac{d}{dt}(\csc(t) + e^{t} \cdot \cot(t))$

- ◆ derivative of sum or difference.

- ◆ Select $\dfrac{d}{dt}(e^{t} \cdot \cot(t))$ then select the derivative of the product.

- ◆ basic derivatives. [Press ENTER to clean-up.]

10. $\dfrac{d}{dx}(\sec(x) \cdot \cot(x))$

- ◆ Select $\sec(x) \cdot \cot(x)$ then rewrite as 1/sin(x)

- ◆ Select 1/sin(x) then select $A/(\sin(B)) \to A \cdot \csc(B)$
  [Clean-up]

- ◆ $\dfrac{d}{dx}(\csc(x)) \to -\csc(x) \cdot \cot(x)$

Note that simplifying the trigonometric expression before differentiating often simplifies the solution.

11. $\dfrac{d}{dx}\left(\dfrac{1}{(x^{2} + x + 1)^{1/3}}\right)$

12. $\dfrac{d}{dx}\left(\ln(|\,x\,|)\right)$