



# Exploring Space Through MATH

*Applications in Algebra 1*



**EDUCATOR  
EDITION**

## Weightless Wonder

### Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, Evaluate) will be used to accomplish the following objectives.

Students will

- solve quadratic equations and evaluate and graph quadratic functions;
- find the maximum, the  $y$ -intercept, the  $x$ -intercepts, and interpret their significance; and
- determine the effects of parameter changes on the graph of a quadratic equation.

### Prerequisites

Students should have prior experience working with quadratic equations and the properties of a parabola.

### Background

*This problem is part of a series that applies algebraic principles to the U.S. Space Exploration Policy.*

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

The U.S. Space Exploration Policy includes returning the space shuttle safely to flight, completing the International Space Station, developing a new exploration vehicle and all the systems needed for embarking on extended missions to the Moon, Mars, and beyond.

In our quest to explore, humans will have to adapt to functioning in a variety of gravitational environments. Earth, Moon, Mars and space all have different gravitational characteristics. Earth's gravitational force is referred to as one Earth gravity, or  $1 g$ . Since the Moon has less mass than the Earth, its gravitational force is only one sixth that of Earth, or  $0.17 g$ . The gravitational force on Mars is equivalent to about 38% of Earth's gravity, or  $0.38 g$ . The gravitational force in space is called

### Key Concept

Quadratic Functions

### Problem Duration

150-180 minutes

### Technology

Graphing Calculator

### Materials

- Student Edition
- Graph Paper
- Colored Pencils

### Degree of Difficulty

Moderate to Difficult

### Skills

Analyze data to solve a real-world problem; connect quadratic equations and their parabolic graphs to a real-world situation

### NCTM Standards

- Algebra
- Problem Solving
- Communication
- Connections
- Representation



microgravity and is very close to zero- $g$ .

When astronauts are in orbit, either in the space shuttle or on the International Space Station, they are still affected by Earth's gravitational force. However, astronauts maintain a feeling of weightlessness, since both the vehicle and crew members are in a constant state of free-fall. Even though they are falling towards the Earth, they are traveling fast enough around the Earth to stay in orbit. During orbit, the gravitational force on the astronauts relative to the vehicle is close to zero- $g$ .



Figure 1: C-9 jet going into a parabolic maneuver.



Figure 2: Astronaut crew training onboard the C-9 aircraft in preparation for the Microgravity Science Laboratory missions flown on the Space Shuttle Columbia in April and July of 1997.

The C-9 jet is one of the tools utilized by NASA to simulate the gravity, or reduced gravity, astronauts feel once they leave Earth (Figure 1). The C-9 jet flies a special parabolic pattern that creates several brief periods of reduced gravity. A typical NASA C-9 flight goes out over the Gulf of Mexico, lasts about two hours, and completes between 40 and 60 parabolas. These reduced gravity flights are performed so astronauts, as well as researchers and their experiments, can experience the gravitational forces of the Moon and Mars and the microgravity of space.

By using the C-9 jet as a reduced gravity research laboratory, astronauts can simulate different stages of spaceflight. This can allow crew members to practice what might occur during a real mission. These reduced gravity flights provide the capability for the development and verification of space hardware, scientific experiments, and other types of research (Figure 2). NASA scientists can also use these flights for crew training, including exercising in reduced gravity, administering medical care, performing experiments, and many other aspects of spaceflight that will be necessary for an exploration mission. A flight on the C-9 jet is the next best thing to blasting into orbit!

For more information about the U.S. Space Exploration Policy, NASA's Weightless Wonder, and reduced gravity research, visit [www.nasa.gov](http://www.nasa.gov).

To view a video on NASA's Weightless Wonder and reduced gravity research, go to [http://microgravityuniversity.isc.nasa.gov/video/RGSFOP\\_video.mpg](http://microgravityuniversity.isc.nasa.gov/video/RGSFOP_video.mpg).



## NCTM Principles and Standards

### Algebra

- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil in simple cases and using technology in all cases.
- Draw reasonable conclusions about a situation being modeled.

### Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

### Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.

### Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

### Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.



## Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

### 1 – Engage (35 minutes)

- Have students read the Background section aloud to the class.
- Play the video (13 min.) at [http://microgravityuniversity.jsc.nasa.gov/video/RGSFOP\\_video.mpg](http://microgravityuniversity.jsc.nasa.gov/video/RGSFOP_video.mpg).
- Distribute the worksheet, **1 – Weightless Wonder Video**.
- Instruct students to write their answers to the questions on the worksheet.
- Arrange students in groups of 3-4 and ask them to compare and discuss their answers to the questions.

### 2 – Explore (20 minutes)

- Distribute the worksheet, **2 - Interpreting Graphs of Quadratic Functions**.
- Ask students to discuss the questions and answers as they work as a team.
- Call on students to give their answers and discuss as a class.

### 3 – Explain (45 minutes)

- Take out or distribute graphing calculators.
- Distribute the worksheet, **3 – Weightless Wonder Problem**.
- Have a student read each of the paragraphs aloud to the class or students may read to themselves silently.
- Discuss the variables and expressions of the given function. Note: Using a graphing calculator  $y$  represents the altitude ( $h$ ), and  $x$  represents the time ( $t$ ).
- Have a student read the first question aloud to the class. Explain that the start of the parabola is at  $t = 0$ .
- Have the groups work together to answer the questions.
- Call on students to give their answers and discuss. If available students can use the presentation technology to demonstrate the graphing calculator aspects of the questions.

### 4 – Extend (45 minutes)

- Distribute the worksheet, **4 – Changing the Parameters of Quadratic Functions**.
- Have a student read the paragraph aloud to the class.
- Discuss the variables and expressions of the given function.
- Have each group work together on the *Changes in Initial Altitude* section, recording their solutions with colored pencils on graph paper and posting them around the classroom. Discuss the results with the class.
- Have each group work together on the *Changes in Initial Velocity* section, recording their solutions on graph paper and posting them around the classroom. Discuss the results with the class.
- If available students can use the presentation technology to demonstrate the graphing calculator aspects of the questions.
- Have students complete the *Summary* section and discuss.

### 5 – Evaluate (35 minutes)

- Distribute the worksheet, **5 – Weightless Wonder: Wrap Up**.
- Have students complete it individually.

**ENGAGE****1 – Weightless Wonder – Video**

## Solution Key

Please answer the following questions about the Weightless Wonder video.

1. In the flight of the C-9 what part of one maneuver is a true parabola and why?  
From the point on ascent where the pilot cuts thrust and the plane continues to rise, then noses over into descent (about 20 seconds), during which time the plane is in free fall simulating microgravity.
2. What else can this type of flight simulate besides zero- $g$ ?  
Gravity on the Moon. A lunar parabola is about one-sixth  $g \approx 0.17 g$ .  
Gravity on Mars. A martian parabola is about one-third  $g \approx 0.38 g$ .
3. What changes might occur to the body during this type of flight?  
Motion sickness, vomiting, hypoxia, lips and nails turn blue, feeling happy, dull or lightheaded.
4. What types of experiments do you think might be performed in the reduced gravity environment of a parabolic flight?  
Answers will vary.  
The effects of microgravity on fluid physics, combustion, material science, life sciences, new technologies, and the human body, i.e. breathing, blood pressure, heart rate, bone loss, buoyancy.
5. What do the students do to prepare themselves for the reduced gravity flight?  
Answers will vary.  
Plan the experiment, prepare the experiment hardware, test the experiment, participate in the orientation and safety review at Ellington Field. Pass the flight physical, the NASA Physiological Training course, and the Hyperbaric Chamber training (simulates air at 25,000 ft.). Put on their Air Force flight suits and receive their motion sickness meds and bags.
6. What are other instances where one might feel reduced gravity on Earth?  
Answers will vary.  
Roller coaster, bungee jumping, elevator ride, speeding over a hill in a car, bull riding.



## EXPLORE

## 2 – Interpreting Graphs of Quadratic Functions

## Solution Key

The graph below shows the altitude of a C-9 jet during one parabolic maneuver. Use this graph to answer the questions below:

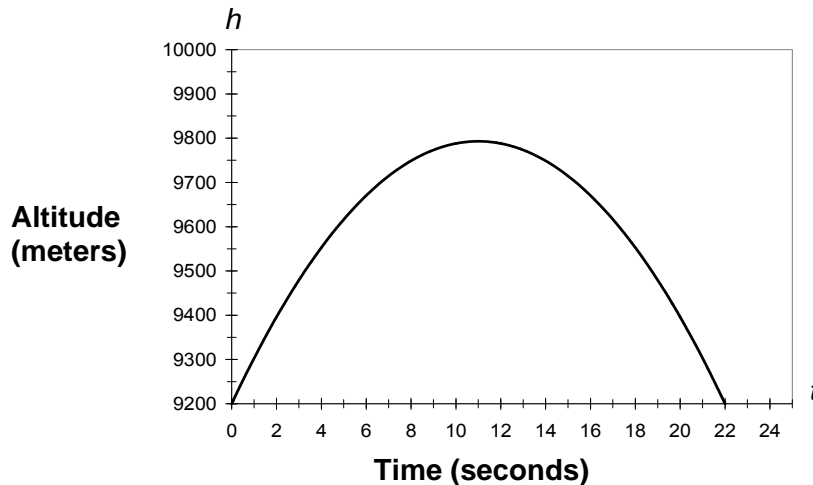


Figure 3: Altitude of C-9 During one parabolic maneuver

1. What does 9200 meters represent in this situation?  
The altitude where the plane started the parabolic maneuver.
2. When does the C-9 first reach an altitude of 9400 meters? How long does the plane remain above 9400 meters? Justify your answer.  
The plane first reaches an altitude of 9400 meters at 2 seconds. It continues to rise and then starts to fall. The altitude decreases to 9400 meters at 20 seconds. The C-9 remains above 9400 meters for the difference between 2 and 20 seconds, or 18 seconds.
3. Between what two whole number seconds was the plane at 9600 meters?  
Between 4 and 5 seconds and again between 17 and 18 seconds.
4. What is the approximate vertex of the parabola? What does this vertex tell you about this part of the flight?  
The vertex is at approximately (11, 9800). This tells you that the maximum altitude the plane reached during this parabolic maneuver was about 9800 meters and it occurred at 11 seconds.
5. What is a reasonable domain for this part of the flight? What does the domain tell you about the flight?  
The domain is  $0 \leq t \leq 22$ . This domain tells you that the parabolic maneuver lasted 22 seconds.
6. What is a reasonable range for this part of the flight? What does the range tell you about the flight?  
The range appears to be  $9200 \leq h \leq 9800$ . This tells you that the plane flew between 9200 meters to 9800 meters during this part of the flight.





## EXPLAIN

### 3 – Weightless Wonder Problem

#### Solution Key

To prepare for an upcoming mission, an astronaut participated in a C-9 flight simulating microgravity, or close to zero- $g$ . The pilot flew out over the Gulf of Mexico, dove down to increase to a maximum speed then climbed up until the nose was at a  $45^\circ$  angle with the ground. To go into a parabolic maneuver, the pilot then cut the thrust of the engine letting the nose of the plane continue to rise then come back down at a  $-45^\circ$  angle with the ground. Ending the maneuver, the pilot throttled the engine back up and began another dive to prepare for the next parabola. The pilot completed 50 parabolas during the 2 hour flight.

The figure below shows the movement of the plane during a typical flight. The parabolic maneuver, where microgravity is felt, is highlighted. This is the part of the flight that you will focus on for the following questions.

The function  $h = -4.9t^2 + 87.21t + 9144$  describes the altitude ( $h$ ) in meters (m) of the plane in relation to the time ( $t$ ) in seconds (s) after it started the parabolic maneuver. You will use this function to analyze the parabolic flight of the C-9. Round all answers to the nearest tenth.

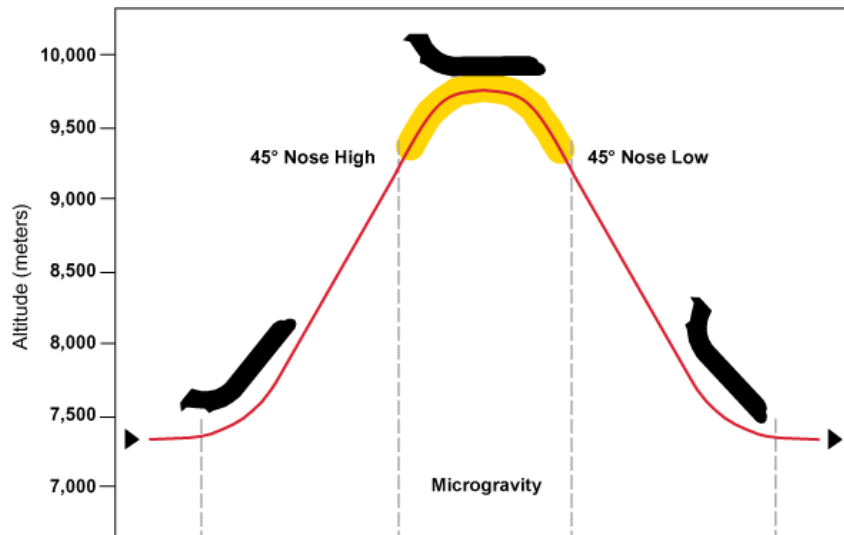


Figure 4: A typical microgravity maneuver.

- Using the defined function, at what altitude did the astronaut first start to feel microgravity?

Let  $t = 0$

$$h = -4.9t^2 + 87.21t + 9144$$

$$h = 9144$$

The altitude would be 9144 meters.



2. Consider the first parabolic maneuver to examine its beginning and end.
  - a. Use algebra to find the times when microgravity began and ended during this one maneuver.

$$h = -4.9 t^2 + 87.21 t + 9144$$

$$9144 = -4.9 t^2 + 87.21 t + 9144$$

$$0 = -4.9 t^2 + 87.21 t$$

$$0 = t(-4.9 t + 87.21)$$

$$t = 0 \text{ s}, 17.8 \text{ s}$$

- b. What was the length of time the astronaut experienced microgravity during this one maneuver? Explain your answer.

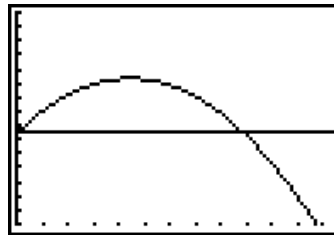
The astronaut experienced 17.8 seconds of microgravity, since the plane began the parabolic maneuver at 0 seconds and ended at 17.8 seconds.



- c. Use the graphing calculator to graph  $y_1 = -4.9 x^2 + 87.21 x + 9144$  and  $y_2 = 9144$ .  
 Note: Using a graphing calculator  $y$  represents the altitude ( $h$ ), and  $x$  represents the time ( $t$ ). Set your **WINDOW** to show only the first parabolic maneuver and write these values on the **WINDOW** screen below. Then sketch the two graphs on the blank screen.

```

WINDOW
Xmin=0
Xmax=25
Xscl=2
Ymin=8500
Ymax=10000
Yscl=100
Xres=1
    
```

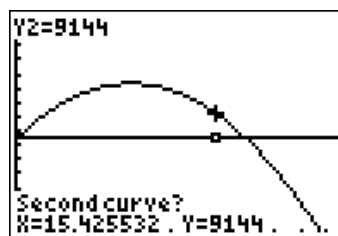
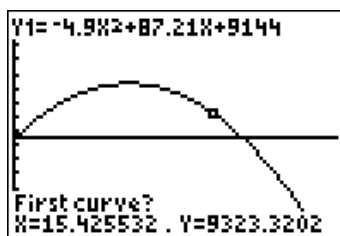


- d. How many times do the graphs intersect? Find the  $x$  and  $y$  values of the point(s) of intersection. Explain what these  $x$  and  $y$  values represent.

The graphs intersect twice.

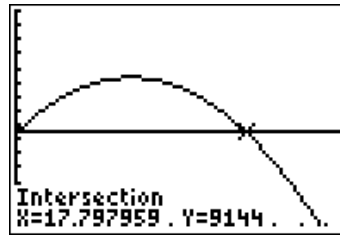
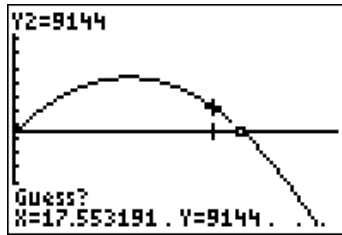
The first time they intersect is at (0, 9144). To find the second time they intersect, use the **CALC** functions, found by pressing **2ND** and **TRACE**.

Select **#5: intersect** and press **ENTER**.



When the screen displays **First curve?** a cursor should be blinking on the parabola. Press **ENTER**. When the screen displays **Second curve?**, the cursor should be blinking on the horizontal line. Press **ENTER**.





When the screen displays **Guess?** move the cursor close to the intersection point that you are trying to find. After moving the cursor, press **ENTER** and your intersection point will be shown. Therefore, when  $y$  (altitude) is again 9144 meters,  $x$  (time) is 17.8 seconds.

3. Consider the first parabolic maneuver to examine its maximum altitude.
- Use algebra to find the maximum altitude of the plane during this one parabolic maneuver. Explain your procedure.

The maximum would occur halfway between the two points of intersection. The average of the two  $x$  values, or  $x = 8.9$  seconds, would be the time at which the plane reaches its maximum altitude. Substituting this value in the equation gives:

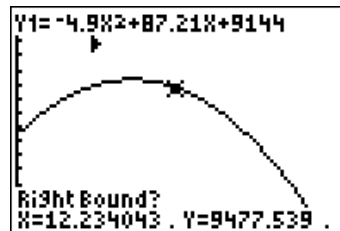
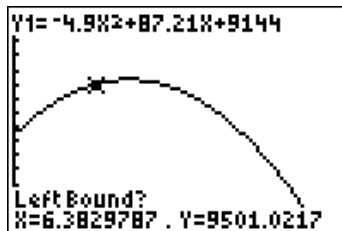
$$h = -4.9(8.9)^2 + 87.21(8.9) + 9144$$

$$h = 9532.0 \text{ m}$$

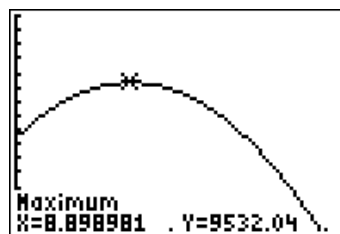
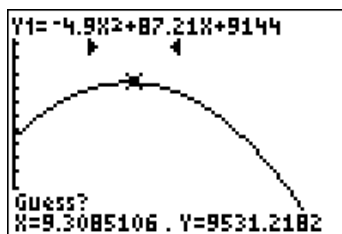


- Use the graphing calculator to find the maximum altitude and when it occurs.

Delete  $y_2 = 9144$  from the **Y=** screen in your calculator. Select **#4: maximum** from the **CALC** menu and press **ENTER**.



The screen will display **Left Bound?**. This must be a point to the left of your maximum point. Move the cursor so that it is to the left of the maximum and press **ENTER**. You must now similarly move the cursor to the right of the maximum point when it displays the **Right Bound?**.



After you enter the left and right bounds, the calculator now displays **Guess?**. Move your cursor close to the vertex point and press **ENTER**. The calculator will display the maximum point. The maximum altitude is 9532.0 meters and it occurs at 8.9 seconds.



4. What percent of the astronaut's total flight was spent in microgravity?

The trip lasted for 2 hours which is 7200 seconds. Each parabola lasted for 17.8 seconds and there were 50 parabolas flown.

$$\frac{17.8 \times 50}{7200} \times 100 = 12.4\%$$

## EXTEND

### 4 – Changing the Parameters of Quadratic Functions

#### Solution Key

The function  $f(t) = \frac{1}{2} a t^2 + v_0 t + h_0$  describes the altitude of the C-9 plane during one of its parabolic maneuvers with respect to the time ( $t$ ) in seconds. This is an equation often used in physics and can be applied to any object in free fall. The variable  $a$  is the acceleration due to gravity. The variable  $v_0$  is the vertical velocity of the airplane when it starts the parabolic maneuver. The variable  $h_0$  is the altitude of the airplane when it starts the maneuver.

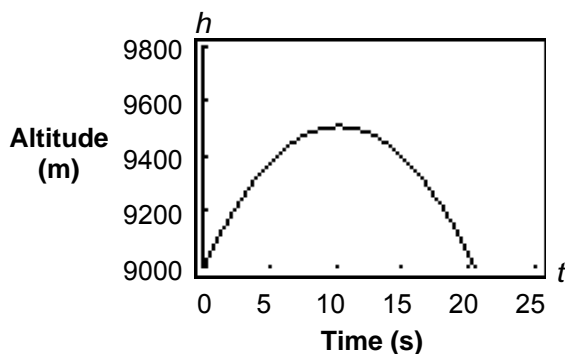
- The acceleration due to gravity is approximately  $-9.8 \text{ m/s}^2$ .
- The vertical velocity the C-9 plane travels when starting a parabolic maneuver ranges between 90 and 115 m/s.
- The altitude at which the plane starts the maneuver ranges between 8600 and 9200 meters.

#### Changes in initial altitude:


- During another reduced gravity flight, the C-9 plane starts a parabolic maneuver at a velocity of 100 m/s and an altitude of 9000 meters.
  - Write an equation to describe its altitude after  $t$  seconds. Note: Using a graphing calculator  $y$  represents the altitude ( $h$ ), and  $x$  represents the time ( $t$ ).

$$y_1 = -4.9 x^2 + 100 x + 9000$$

- Graph the equation with a graphing calculator and sketch it in the space provided.






2. Refer to the graph.
  - a. Describe what is happening with the plane.  
The plane's altitude increases rapidly at first, then more slowly, and then the altitude decreases.
  - b. Estimate the maximum altitude reached.  
Approximately 9500 meters.
  - c. Approximately how long does the parabolic maneuver last?  
Approximately 20 seconds.
  
3. For the next parabola the velocity is the same but the altitude is 9200 meters when the plane starts the maneuver.
  - a. Write a new equation to describe this flight.  
$$y = -4.9x^2 + 100x + 9200$$
  - b. Predict how the graph of this equation will change. How will the maximum altitude be affected? How will the time be affected?  
Predictions will vary.
  
4. Now check your predictions on the graphing calculator. How do your predictions compare with the changes you see on the graph?
 

The maximum altitude is approximately 9710 meters which is 200 meters higher than the original. The time it takes does not change. It is still approximately 20 seconds.
  
5. How will the equation and the graph change if the plane starts at 8700 meters?  
The equation will be  $y = -4.9x^2 + 100x + 8700$ . The maximum altitude will be approximately 9210 meters and the time it takes will remain the same, approximately 20 seconds.

### Changes in velocity:

6. If the plane's starting velocity is 90 m/s when it performs the maneuver, how will this change the equation in Question 1? Predict how this will affect the graph.  
$$y = -4.9x^2 + 90x + 9000$$
  
Predictions will vary.
  
7. Now graph the equation using a graphing calculator to check your prediction.
 
  - a. What is the maximum altitude reached?  
Approximately 9413 meters.
  - b. How long does the parabolic maneuver last?  
Approximately 18 seconds.



c. How does your prediction compare with the changes you see in the graph?

Students should notice that when the velocity decreased, the altitude and the time decreased as well.

8. If the plane is able to start the parabolic maneuver at 115 m/s, how will this change the graph? Check your answer by graphing the equation.



Since there is an increase in velocity the maximum altitude and the time it takes to complete the parabolic maneuver should increase. The graph shows the maximum is at approximately 9675 meters and it lasts for approximately 23 seconds.

9. Given the equation of the parabola is  $y = a x^2 + b x + c$ , summarize how changes in  $a$ ,  $b$ , and  $c$  affect the graph of a parabola.

a. What can you say about the value of  $a$ ? Explain your answer.

The value of  $a$  is negative because of the direction the parabola is opening downward.

b. How do changes in  $b$  affect the graph?

The value of  $b$  changes the location of the vertex of the parabola. In this case, as  $b$  is increased, the vertex moved up and to the right. When  $b$  decreased, the vertex moved down and to the left.

c. What does the value of  $c$  represent? Explain your answer.

The value of  $c$  represents the  $y$ -intercept of the graph and gives it a vertical shift.

## EVALUATE

### 5 – Weightless Wonder: Wrap Up

#### Solution Key

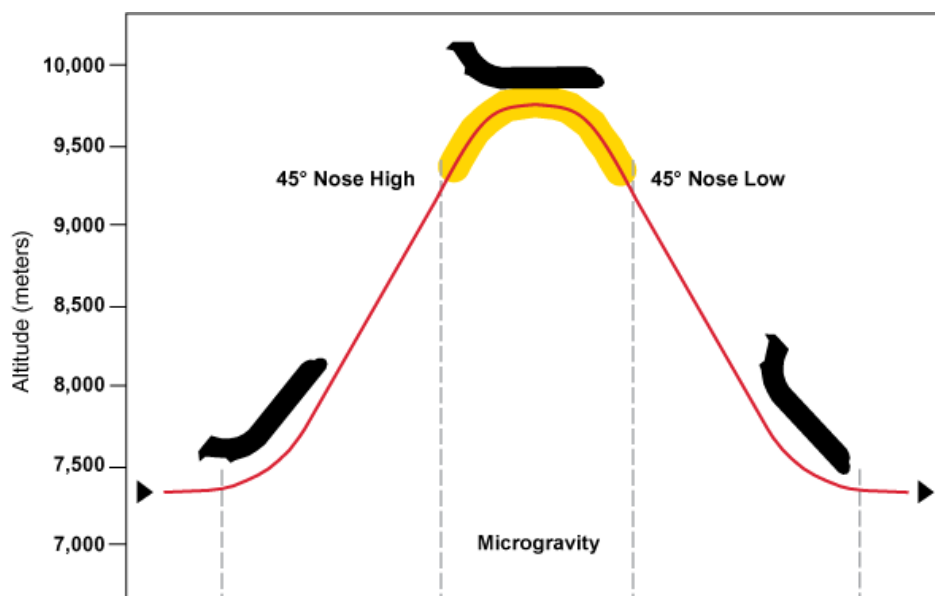


Figure 4: A typical microgravity maneuver.



1. Enter into the graphing calculator,  $y_1 = -4.9x^2 + 91.68x + 8940$  and  $y_2 = 8940$ . Set your **WINDOW** to show only the first parabolic maneuver. Note: Using a graphing calculator  $y$  represents the altitude ( $h$ ), and  $x$  represents the time ( $t$ ). Use the calculator to find the  $x$  and  $y$  values of the point of intersection that is on the right. Explain what these  $x$  and  $y$  values represent.



The point is (18.7, 8940). This point represents the end of 18.7 seconds of microgravity during the first maneuver when the value of  $y$  (altitude) is again 8940 meters.

2. Find the maximum altitude of the plane during one parabolic maneuver and when it occurs.



The maximum altitude is 9369 m and it occurs at 9.4 sec.

From the **CALC** menu Select **#4: maximum** and press **ENTER**. Move the cursor so that it is to the left of the maximum and press **ENTER**. You must now similarly move the cursor to the right of the maximum point when it displays **Right Bound?**. Move your cursor close to that point and press **ENTER**.

3. If the pilot flew for 2.5 hours and did 60 parabolas, what percent of the total flight was spent in microgravity?

The trip lasted for 2.5 hours which is 9,000 seconds. Each parabola lasted for 18.7 seconds and there were 60 parabolas flown.

$$\frac{18.7 \times 60}{9000} \times 100 = 12.5\%$$

4. If the equation changes to  $y_1 = -4.9x^2 + 91.68x + 8600$ , explain how the graph changes?  
The initial altitude at which microgravity begins is lower and occurs at 8600 meters.
5. In the previous question, if the velocity changes to 84.19 m/s, explain how the graph changes?  
The initial altitude at which microgravity begins is still at 8600 meters, but, since the velocity is less, the parabola does not go as high or as far. The vertex is to the left and down from the previous parabola.



### Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

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# Exploring Space through Math – Applications in Algebra 1

## Weightless Wonder

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**Please use the following scale to rank the attributes of this lesson:**

**5 = Excellent    4 = Very Good    3 = Good    2 = Fair    1 = Poor**

1. \_\_\_\_\_ It is appropriate for the identified target audience. (Algebra 1)
2. \_\_\_\_\_ It can be completed in the allotted time.
3. \_\_\_\_\_ It keeps the students engaged.
4. \_\_\_\_\_ It meets the stated instructional objectives.
5. \_\_\_\_\_ It addresses the identified NCTM Mathematics Standards.
6. \_\_\_\_\_ It promotes STEM careers (Science, Technology, Engineering, and Mathematics).
7. \_\_\_\_\_ The Student Edition is student friendly.
8. \_\_\_\_\_ The Educator Edition (solution key) is correct and complete.
9. \_\_\_\_\_ The Educator Edition provides adequate instructional support for the teacher.

Please provide suggestions for improvement of this product:

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Please provide suggestions for future mathematics problems, based on NASA topics, that you would like to see developed:

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Thank you for your participation.