

Game Board 1

After you have played a couple of games on Game Board 1, write down some of the strategies you have developed. Some useful starting points for consideration:

Typical strategies should include:

- *Identify numbers with the least number of factors; this can be extended to identify numbers with the least number of factors remaining. The later strategy takes into consideration numbers that have already gone.*
- *Extending beyond 'least number of factors', students should consider the value of the factors, for example: 27 has factors 1, 3, 9 and 27. The sum of the relevant factors: $1 + 3 + 9 = 13$. Compare this result with selecting 26 with factors: 1, 2, 13 and 26. A smaller number, with the same number of factors but the sum of the relevant factors: $1 + 2 + 13 = 16$ is larger. As most factors occur in pairs, having a factor of 2 means another factor, half the selected number will also be included.*

What is the best starting number?

On the 30 square board, 25 is the best starting number. It creates the greatest difference between a player and their opponent. Note that 25 is a perfect square which results in only 1 and 5 as factors, a score of 6 for the opponent versus a score of 25.

What is the worst starting number?

On the 30 square board, 30 and 24 are the worst starting numbers. The factors of 30 sum to 42. The factors of 24 sum to 36. Both numbers result in the factor sum being worth 12 more points than the selected number.

What numbers need to be avoided?

In general, any number where the factors add up to more than the selected number. This often includes even numbers as the highest factor is always half the number selected. An extension to this question is to ask the students to rank the numbers in order of best to worst. Note that this ranking is only true for the first turn. Once other factors have been removed, the ranking changes.

Are there any times you might use a prime number or one with no remaining factors?

The only time when this strategy would be employed is when the remaining numbers have factor sums larger than the selected number. Note that no one scores when a prime number; or a number with no remaining factors is selected. The player effectively misses a turn.

What is the maximum number of combined points for a game?

*This is a complex problem for students to solve: 277. The numbers would need to be selected in a specific order. For example: If 30 and 27 are selected before 18, then 18 can no longer be selected as the factors: 1, 3, 6 and 9 have already been included. Students could be challenged to obtain their total score prediction and record the order in which numbers were selected. (This solves any disputes about the highest total and also encourages students to work on the game independently). Note also the assumption that the opponent identifies **all** the allowable factors.*

Are there any numbers that a player might choose which will result in more points going to his/her opponent?

Again students will need to recognise the assumption(s) that this question assumes all factors are available.

12 has a factor sum of 16. 18 has a factor sum of 21. 20 has a factor sum of 22. 24 has a factor sum of 36 and 30 has a factor sum of 42.

Game Board 2

The second game board has more numbers. After you have played a couple of games on this board, write down any significant strategy differences between board 1 and 2. The points below provide for useful consideration:

What is the best starting number?

On the 49 square board, the number 49 is the best choice, again a perfect square creating a score of $1 + 7 = 8$ for the opponent against a score of 49.

What is the worst starting number?

On the 49 square board, 48 is by far the worst number. The number 48 results in a factor sum of 76.

What numbers need to be avoided?

Any number where the factors sum is greater than the selected number.

What is the maximum number of points combined for a game?

The highest total score is 761.

Are there any numbers that a player might choose which will result in more points going to his/her opponent?

This answer should include all the previous answers for the 30 square board. In addition to these answers, 36 has a factor sum of 55, 40 has a factor sum of 5; 42 has a factor sum of 54 and 48 has a factor sum of 76.