NUMB3RS Activity: Matrix Operations
Episode: "Provenance"

Topic: Matrix Operations **Grade Level:** 9 - 12

Objective: In this activity, students will use matrix operations to determine the probabilities an object will end up at certain locations.

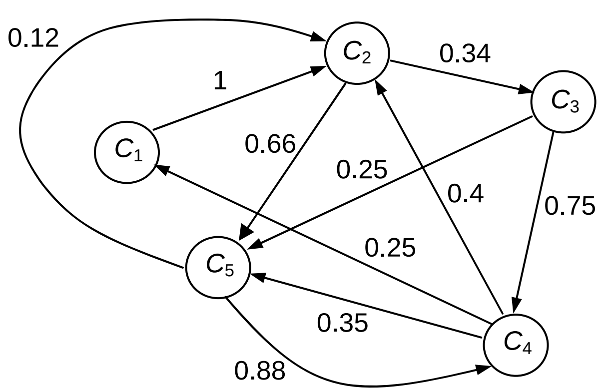
Time: 25 - 30 minutes

Materials: TI-83/TI-84 Plus graphing calculators, TI-Navigator™ system, and the following files: *Matrix1.act, Matrix2.act, Matrix-[A].8xm*
To download these files, go to **http://education.ti.com/exchange** and search for "7746."

Introduction

In "Provenance," Charlie uses a *Network Diffusion Probability Model* to help determine where a stolen Pissarro painting might have been shipped. Charlie explains that this model "helps track the flow of objects through networks." For example, a car can travel on almost any road whereas a truck (such as one carrying hazardous waste) can only travel on certain roads. Likewise, the Pissarro has only so many routes and destinations to which it can travel, based on a set of conditions and variables that Charlie has taken into consideration.

To understand this concept, look at the graph of cities C_1 through C_5 below. The vertices (cities) are connected by edges, with the direction of movement indicated by the arrows. Assume that city C_1 represents the city where the painting was initially stolen. Notice that each edge is assigned a weight, which represents the probability that the stolen painting will travel to that city. Furthermore, the sum of the probabilities of each route leaving a given city equals one. For example, there are three different destinations to reach from city C_4, with probabilities 0.4 + 0.25 + 0.35 = 1.



The following probability matrix *A* can be used to represent this situation.

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$$A = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & .34 & 0 & .66 \\ 0 & 0 & 0 & .75 & .25 \\ .25 & .4 & 0 & 0 & .35 \\ 0 & .12 & 0 & .88 & 0 \end{bmatrix} \end{array}$$

Now consider that the painting originates in City C_1. Use the 1×5 matrix

$$B = \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

to represent this fact by placing a 1 in the position for city C_1 and zeros elsewhere. If you multiply matrix B times matrix A, in that order, you will obtain another 1×5 matrix containing the probabilities associated with the painting's next location.

$$B \times A = \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

It should be no surprise that the location is in city C_2 since the graph indicates that there is a 100% chance that the location of the next city after C_1 is C_2. Each time you multiply this answer matrix and the original probability matrix, A, you obtain the probabilities associated with the next move. If you continue this process indefinitely, the probabilities will reach a "steady state matrix," which gives the probability, for each of cities C_1 through C_5, of the location of the stolen painting over time.

In Part I of this activity, students will inspect the graph given in this introduction and create the probability matrix A that represents this situation on their calculators. Students will then use the iterative process of matrix multiplication to determine the probable location of the painting over time. You will use **Screen Capture** check their work.

In Part II of this activity, students will be sent an 18×18 matrix similar to the one Charlie used in this episode. Students will select a starting point and repeat the process used in Part I to determine the probabilities associated with the objects eventual location.

Part I: Construct Matrices *A* and *B*

1. a. Students should work individually during this part of the activity.

b. Launch TI-Navigator™ on the computer and press **Begin Class** to start the session.

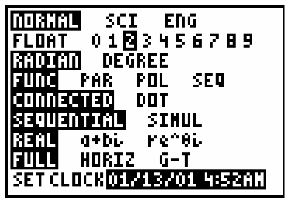
c. Have each student log into `NavNet` on their calculators.

d. Instruct students to press `4:EXIT APP` to exit back out of `NavNet` to the calculator's home screen.

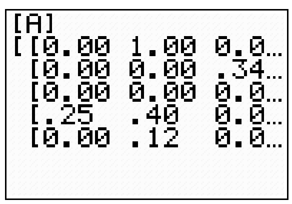
2. a. Load the ***matrix1.act*** activity settings file into Activity Center and click the ‘Graph’ tab. This will display the graph found in the introduction to this activity.

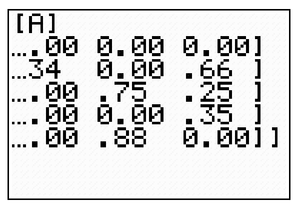
b. Instruct students to press [2nd] [MATRIX] to access the matrix editor for matrix A. Have students set up a 5×5 matrix in which each column represents C_1 through C_5 and each row represents C_1 through C_5. Explain to students that, as they fill in the first row, they are entering the probability of the painting traveling from City C_1 to each of the other cities. Because there is a 100% chance the painting will travel to City C_3, there will be a 1 under the third column and zeroes elsewhere. Students should continue filling in one row at a time until they complete the fifth row for City C_5.

c. When the students are finished, have them press [MODE] and set Float to 2 as shown below. This will round all decimals to the hundredths place, making it easier to view results.

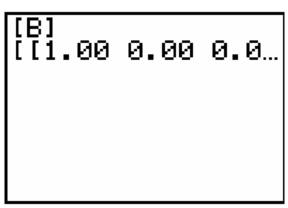


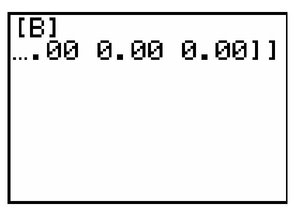
d. From the home screen have students press [2nd] [MATRX], select **1:[A]**, and press [ENTER] to display their matrices on the home screen. Use **Screen Capture** and verify that all students have the same screen as the one shown at left below. Then have students press the [▸] key repeatedly until the right side of the matrix is in view. Use **Screen Capture** again to view the results. If students made mistakes, discuss their errors, and then have them re-enter the matrix editor and fix their errors.





e. Now instruct students to go back to that matrix editor and construct matrix $B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$. Explain to students that this matrix is used to represent a starting point at city C_1. When complete, have students press [2nd] [MATRX], select **2:[B]**, and press [ENTER] to display this matrix to the home screen. Use **Screen Capture** to view the results as shown below.





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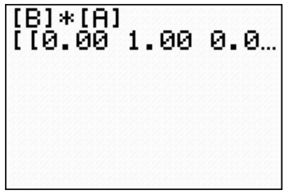
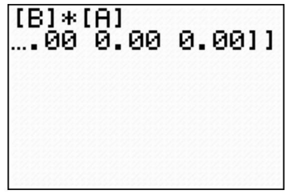
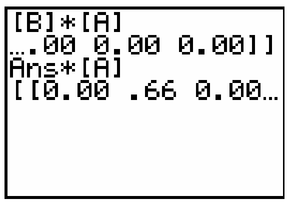
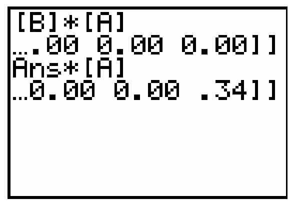
3. a. Tell students to find the product **[B]** * **[A]** on the home screen. Use **Screen Capture** to review their results. Ask students why this result should represent the location of the painting after it moves from City C_1 to the next city. (Note: because there is a 100% probability that the painting will travel to City C_3, it makes sense that there should be a 1 in the third column.)



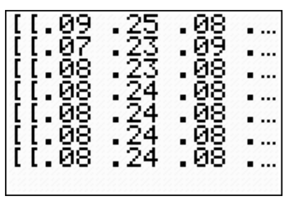


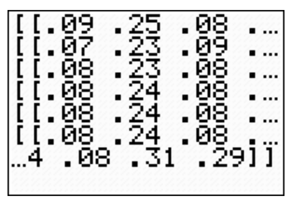
b. Now tell students to enter the command **Ans** * **[A]**. This will multiply the previous answer and matrix *A*. Use **Screen Capture** and discuss with students the fact that, after the next move, there is a 66% chance the painting will go to City C_2 and a 34% chance it will go to City C_5.





c. Have students press ENTER several more times until the results reach some limiting value. Explain to students that, over time, there is an 8% chance the painting will end up in City C_1, a 29% chance it will end up in City C_2, a 24% chance it will end up in City C_3, a 31% chance it will end up in City C_4, and an 8% chance it will end up in City C_5.





Part II: Investigate a Very Large Matrix

4. a. Load the ***matrix2.act*** activity settings file into Activity Center and click the 'Graph' tab. This will display part of an 18 × 18 matrix. Each row and column is designated with a letter, which can be used to represent each destination (or city).

b. Use **Send to Class** to send this 18 × 18 matrix (***Matrix-[A].8xm***) to each calculator. This matrix is similar to the one Charlie actually uses in this *NUMB3RS* episode.

c. Tell students to create a 1 × 18 matrix [B] in which all elements are zero except one. Encourage students to select different starting cities by assigning each student a city from C_1 to C_{18}. If, for example, a student is assigned City C_4, then their matrix will have a one under the C_4 column and zeroes elsewhere.

d. Have students use the iterative methods from Part I to determine the "steady-state matrix" that yields the probable locations of the painting over time.

e. Ask students to identify the top three most probable locations and their respective probabilities. Use **Screen Capture** to review students' results. Discuss differences in student answers based on the cities of origin.

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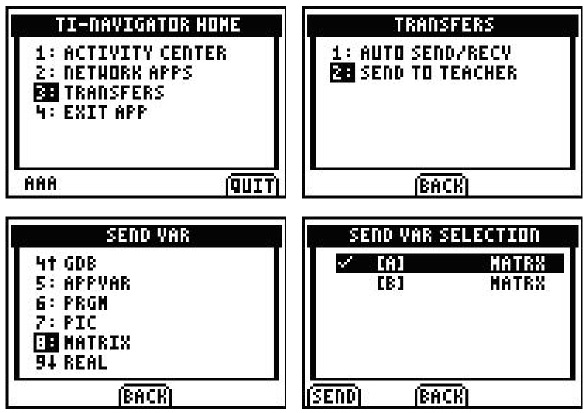
The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

- Have students form small groups and instruct them to draw a graph similar to the one investigated in Part I of this activity. For simplicity, restrict the number of vertices to between 5 and 10. Furthermore, encourage students to consider the possibility of the painting traveling *back* to a previous city.
- Next, ask students to make square Matrix [A] that represents this graph and use the **Send to Teacher** option to send it to the teacher’s computer. The sequence of screens below show how to send Matrix [A] to the teacher.



- The teacher can now send these matrices to other groups of students. Students should attempt to draw the graph represented by the matrix and compute a “steady-state matrix” as was done during Parts I and II of this activity.

<u>Related Topics</u>

- There are many interesting applications that can be modeled and solved using matrix operations. For example, go to the Web site below to see an application of how a *Food Web* can be investigated using matrices.
http://www.colorado.edu/education/DMP/activities/matrices/jaeact05.html
- An additional *NUMB3RS* activity for “Provenance” that involves probability matrices can be downloaded for free by going to **http://education.ti.com/exchange** and searching for “7494.”
- If you would like to learn more about TI-Navigator™, visit **http://education.ti.com/navigator**.

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