Parametric Races & Slopes

ID: 12515

Time Required
15–20 minutes

Activity Overview

Students investigate parametric equations. They will study the motion of two points and use calculus to determine the velocity and acceleration. The slope of parametric equations will be determined. As an extension, students will create a set of parametric equations for a given graph.

Topic: Parametric Equations

- *Parametric equations are used to investigate velocity and acceleration.*
- *Find the slope from parametric equations.*

Teacher Preparation and Notes

- *To graph a parametric equation on the TI-89 Titanium, select* ***MODE > Graph > Parametric****.*
- *Students begin the activity by running the text file param1.89t. Students should know how to execute each command line of the text file (press ƒ). Be sure to send both text files to students' graphing calculators prior to beginning the activity.*
- *Parametric equations are a Calculus BC topic. AB teachers may enjoy using this activity after the AP exam or using with students in your AB class who want to prepare for the BC exam. After completing the activity, students should be more successful with AP* multiple-choice questions like 03BC4,7,17,84, 98BC10,21,77, 88BC34, and free-response questions like 05formB BC1, 04BC3, 04 form B BC1, 03BC2, 03formB BC4, 02BC3, 02formB BC1, 01BC1, 00BC4, 99BC1, 97BC1, 95BC1.*
- ***To download the student TI-89 documents (.89t file) and student worksheet, go to education.ti.com/exchange and enter "12515" in the quick search box.***

Associated Materials

- *CalcWeek28_ParametricEq_Worksheet_TI89.doc*
- *param1.89t*
- *param2.89t*

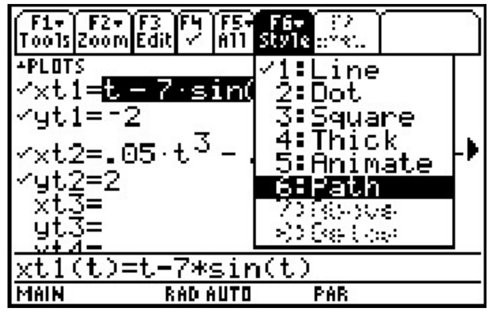
Suggested Related Activities

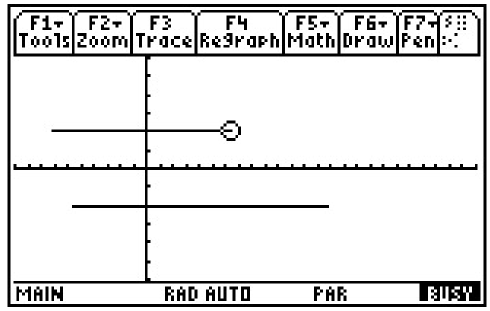
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Projectile Motion (TI-Nspire technology) — 10214*
- *Simultaneous Solutions (TI-Nspire technology) — 10092*
- *Gateway Arc Length (TI-89 Titanium) — 12439*
- *Warming Up to SimCalc MathWorlds (TI-84 Plus) — 7991*

Part 1 – Parametric Equations – Kinematics

Parametric equations are useful for comparing the motion of various objects. This has application in calculus by finding the *x* and *y* components of the velocity and acceleration. In this activity, two objects are considered to be racing linearly against one another. Have students use the worksheet and answer the questions to investigate the velocity and acceleration of the moving points.





<u>*Discussion Questions*</u>

- What is the acceleration when a point is changing direction? *Answer: Although many students will think the acceleration is zero, it is only the instantaneous velocity that is zero. If the acceleration was zero at that instant it would not be changing its velocity.*

Student Solutions

1. The two objects do not start at the same location. They both begin by going backwards, or in the negative *x* direction. At the end, they are both going forward, but the upper point goes further. In between, they both speed up and slow down, but the lower point's change in velocity is more dramatic. They pass each other once, when the time is about 7.5 units. This could be solved by setting the two equations equal to each other.

2. **x1**(t) = t – 7sin(t) represents the lower point; **x2**(t) = $0.05t^3 - 0.2t^2 - 0.5t - 5$ represents the upper point

3. Let **x1** model the motion of the lower point and **x2** model the upper point.

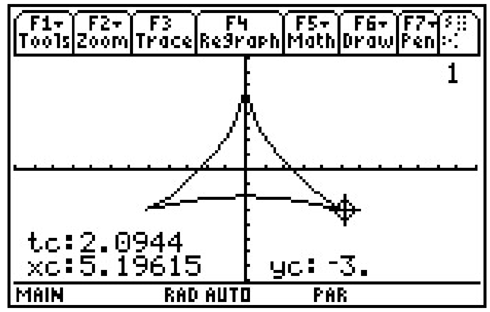
x1′(t) = 1 – 7cos(t)	**x1′**(3.5) = 7.555	**x1′**(8) = 2.019
x2′(t) = $0.15t^2 - 0.4t - 0.5$	**x2′**(3.5) = –0.0625	**x2′**(8) = 5.9

   a. The upper point moves faster at 3.5 units.
   b. The lower point moves faster at 8 units.

4. | | | |
   |---|---|---|
   | **a1**(t) = **x1″**(t) = 7cos(t) | **x1″**(3.5) = –2.455 | **x1″**(8) = 6.926 |
   | **a2**(t) = **x2″**(t) = $0.30t - 0.4$ | **x2″**(3.5) = 0.65 | **x2″**(8) = 2 |

5. At **x1**(3.5), the first derivative is negative and the second derivative is positive. This means it is slowing down as it travels in the negative direction. Similarly, **x2** at t = 3.5 has a positive velocity and negative acceleration, indicating it is slowing down as it moves forward.

Part 2 – Parametric Equations – Slope



Students will explore a graph that is not a function but can be graphed with parametric equations. They will use the TRACE feature to explore what happens at different 'times' on the graph.

Students learn about the vertical and horizontal slope of the tangent by answering the questions on the worksheet.

Student Solutions

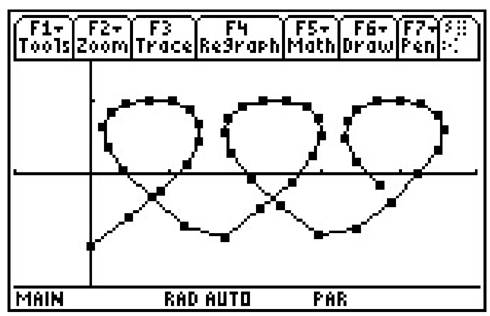
6. There is a cusp at 0, $\frac{2\pi}{3}$, and $\frac{4\pi}{3}$ (or at approximately 0, 2.094, and 4.189). The derivative is undefined at a cusp. This implies the derivative of **x1**(*t*) is zero at these times.

7. A horizontal tangent occurs at (0, –2).

8. This can be found when $y'(t) = 0$, $x'(t) \neq 0$.

9. $\frac{dy}{dx} = 1$ when $\frac{dy}{dt} = \frac{dx}{dt} \neq 0$
When $t = 4.712388$, $x(t) = -4.000$ and $y(t) = -2.000$.

Part 3 – Homework/Extension



Students will explore interesting graphs on their own and report that they understand the calculus. In the extension, students will try to create a set of parametric equations to match the scatter plot. In order to find a solution, students will need to study the slope of the vertical and horizontal tangent lines. You may wish to have students come up with a different set of parametric equations and find the vertical and horizontal tangents of their graph.

The solution shown at the right is **x**(*t*) = $0.5t + \sin(2t)$ and **y**(*t*) = $-1 + |2\sin(t)|$.