Standard Form of Quadratic Functions

MATH NSPIRED

Math Objectives

- Students will be able to predict how a specific change in the value of *a* will affect the shape of the graph of the quadratic $\mathbf{f}(x) = ax^2 + bx + c$.
- Students will be able to predict how a specific change in the value of *c* will affect the position of the graph of the quadratic $\mathbf{f}(x) = ax^2 + bx + c$.
- Students will be able to describe how changes in *a* and *b* will affect the coordinates of the vertex of the quadratic $\mathbf{f}(x) = ax^2 + bx + c$ resulting in both horizontal and vertical shifts.
- Students will be able to utilize the values of *a* and *b* to predict the coordinates of the vertex and the axis of symmetry.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).

Vocabulary

- compression
- parameters
- standard form
- vertex

About the Lesson

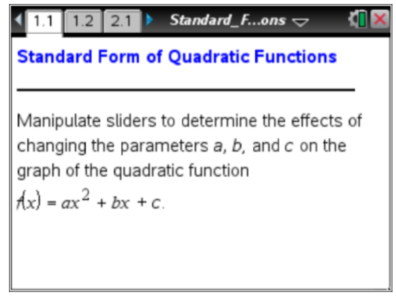
- This lesson involves utilizing sliders to determine the effect the parameters have on the graph of a quadratic in standard form.
- Students will manipulate sliders and make conjectures about the relationship between:
  - The value of *a* in the equation $\mathbf{f}(x) = ax^2 + bx + c$ and the shape of the graph.
  - The value of *c* and the position of the graph with respect to the horizontal axis.
  - The values of *a* and *b* and the coordinates of the vertex.

TI-Nspire™ Navigator™

- Use Live Presenter to demonstrate how to utilize sliders.
- Use Class Capture to monitor students' progress.
- Use Quick Poll to assess students' understanding.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Standard_Form_of_Quadratic_Functions_Student.pdf
- Standard_Form_of_Quadratic_Functions_Student.doc

TI-Nspire document

- Standard_Form_of_Quadratic_Functions.tns

Discussion Points and Possible Answers

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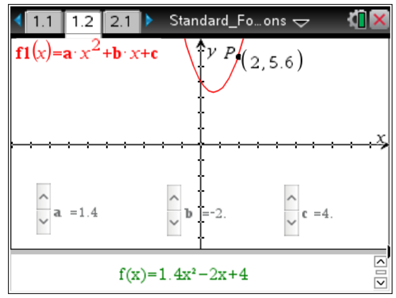

This page has the graph of a parabola in the standard form with a point *P* on the graph.

1. Set *a* = 1, *b* = 0, and *c* = 0. Note that point *P* has coordinates (2, 4). Click the slider to increase the value of *a*. Observe the effect on point *P* when *a* > 1. In the table below, list the coordinates of point *P* for four values of *a* > 1. Describe what happens to the *y*-value of point *P* as the value of *a* increases.

<u>**Sample Answers:**</u>

a =	1.2	1.5	2	2.5
P-coordinates	(2, 4.8)	(2, 6)	(2, 8)	(2, 10)

For $a > 1$, the parabola would open up and be vertically stretched in a positive direction.

Teacher Tip: The table above lists a sample of possible correct answers. Students' *P*-coordinates will vary depending upon their choices of values for *a.*

Tech Tip: Students will need to increase the size of the window to see some of these coordinates. In order to increase the window size, press **Menu > Window / Zoom > Zoom – Out**.

Tech Tip: To increase the window size, press 🔧 **> Window / Zoom > Window Settings...** and increase the window accordingly.

TI-Nspire Navigator Opportunity: ***Class Capture/Live Presenter***
See Note 1 at the end of this lesson.

2. a. Lisa says that when *a* < –1 she sees the *y*-values being vertically stretched away from the *x*-axis. Describe how it is similar yet different from the behavior of the function when *a* > 1. Use the slider to verify your answer.

Answer: The y-values are being stretched away from the x-axis both when $a < -1$ and when $a > 1$, but when $a < -1$, the y-values are negative. Thus, for $a < -1$, the parabola would open down and be stretched vertically away from the x-axis in a negative direction.

b. Write a sentence to explain the effect that a change in the value of a (for $a > 1$ or $a < -1$) has on the graph of the function $\mathbf{f}(x) = ax^2 + bx + c$. Explain why this happens.

Answer: When x is squared, it always becomes positive. That result is then multiplied by the value of a. If $|a| > 1$, the function is stretched vertically away from the x-axis.

> **Teacher Tip:** Students will not naturally work with the absolute value function here. You may have to help direct them toward the idea that if $|a| > 1$, the parabola is stretched vertically away from the x-axis.

TI-Nspire Navigator Opportunity: ***Quick Poll ((x,y) Numerical Input)***
See Note 2 at the end of this lesson.

3. a. Set $a = 1$, $b = 0$, and $c = 0$. Click the slider to examine the effect of values of a when $0 < a < 1$. What happens to the y-value of P?

Answer: The y-value of P decreases but is still positive. When $0 < a < 1$, the parabola opens up and is compressed toward the x-axis.

b. Set $a = -1$, $b = 0$, and $c = 0$. Click the slider to examine the effect of values of a when $-1 < a < 0$. What happens to the y-value of P?

Answer: At $a = -1$, the y-value of P is -4. When $-1 < a < 0$, the parabola opens down and is compressed toward the x-axis.

c. Explain why this is called a vertical shrink or compression.

Answer: The value of a determines whether the parent function, $y = x^2$, has been vertically stretched or compressed. If $|a| > 1$, the function is stretched away from the x-axis. If $0 < |a| < 1$, the function is compressed toward the x-axis.

4. Changing the value of a appears to change all of the points on the parabola except the y-intercept. Adjust each slider one at a time and observe the effect on the y-intercept. How is the location of the y-intercept related to the values of the three sliders?

Answer: Neither variable *a* nor variable *b* has any effect on the *y*-intercept. The value of parameter *c* exactly matches the y-coordinate of the *y*-intercept.

5. Given the parabola $\mathbf{f}1(x) = ax^2 + bx + c$, set *a* = 1 and *b* = 0. Adjust the slider to change the value of *c*. Explain why and how the graph is changing.

   Answer: Changes in the value of *c* shift the parabola vertically *c* units. Since $\mathbf{f}(x) = ax^2 + bx + c$, as you increase the value of *c*, the *y*-value also increases. When you decrease the value of *c*, the *y*-value also decreases.

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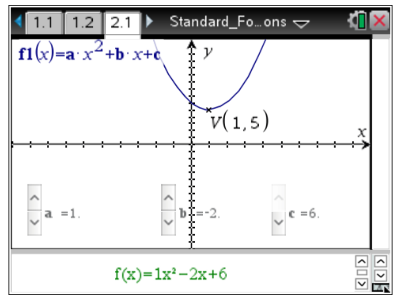

This page has the graph of a parabola in the standard form, $\mathbf{f}1(x) = ax^2 + bx + c$, with the coordinates of the vertex given.

6. Set *a* = 1, *b* = 0, and *c* = 0. Click the slider to change the value of the variable *b*. In the table below, fill in the coordinates of the vertex for the given parabolas.

Answers:

$\mathbf{f}(x) = x^2$	$\mathbf{f}(x) = x^2 + x$	$\mathbf{f}(x) = x^2 + 2x$	$\mathbf{f}(x) = x^2 + 3x$	$\mathbf{f}(x) = x^2 + 4x$
V: (0, 0) *a* = 1 *b* = 0	*V*: (–0.5, –0.25) *a* = 1 *b* = 1	*V*: (–1, –1) *a* = 1 *b* = 2	*V*: (–1.5, –2.25) *a* = 1 *b* = 3	*V*: (–2, –4) *a* = 1 *b* = 4
	$\mathbf{f}(x) = x^2 - x$	$\mathbf{f}(x) = x^2 - 2x$	$\mathbf{f}(x) = x^2 - 3x$	$\mathbf{f}(x) = x^2 - 4x$
	V: (0.5, –0.25) *a* = 1 *b* = –1	*V*: (1, –1) *a* = 1 *b* = –2	*V*: (1.5, –2.25) *a* = 1 *b* = –3	*V*: (2, –4) *a* = 1 *b* = –4

7. Using the information from the table above, write a rule to determine the *x*-coordinate of the vertex. Explain your reasoning.

   Answer: The x-coordinate of the vertex is $x = \frac{-b}{2a}$. The vertex of the parabola lies on the axis of symmetry, whose equation is $x = \frac{-b}{2a}$.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 3 at the end of this lesson.

Teacher Tip: Students might have difficulty determining the correct equation. The first guess might be $-\frac{b}{2}$. The next few questions will guide them to seeing the correct relationship. Be patient and help lead them to their discovery.

Teacher Tip: During this lesson, you may want to connect what students are seeing graphically with the algebra of completing the square. If you rewrite $\mathbf{f}(x) = ax^2 + bx + c$ in the form $y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$, students can see that a change in either *a* or *b* will shift the vertex both horizontally and vertically.

8. Set *a* = 2, *b* = 0, and *c* = 0. Click the slider to change the value of the variable *b*. In the table below, fill in the coordinates of the vertex for the given parabolas.

<u>**Answer:**</u>

$\mathbf{f}(x) = 2x^2$	$\mathbf{f}(x) = 2x^2 + x$	$\mathbf{f}(x) = 2x^2 + 2x$	$\mathbf{f}(x) = 2x^2 + 3x$	$\mathbf{f}(x) = 2x^2 + 4x$
V: (0, 0) *a* = 2 *b* = 0	*V*: (–0.25, –0.125) *a* = 2 *b* = 1	*V*: (–0.5, –0.5) *a* = 2 *b* = 2	*V*: (–0.75, –1.125) *a* = 2 *b* = 3	*V*: (–1, –2) *a* = 2 *b* = 4
	$\mathbf{f}(x) = 2x^2 - x$	$\mathbf{f}(x) = 2x^2 - 2x$	$\mathbf{f}(x) = 2x^2 - 3x$	$\mathbf{f}(x) = 2x^2 - 4x$
	V: (0.25, –0.125) *a* = 2 *b* = –1	*V*: (0.5, –0.5) *a* = 2 *b* = –2	*V*: (0.75, –1.125) *a* = 2 *b* = –3	*V*: (1, –2) *a* = 2 *b* = –4

9. Based upon the information from the table above, check question 7 to see if the rule you wrote is correct. If not, make the necessary changes and answer the following questions.

   a. Predict the *x*-value of the vertex of the parabola $f(x) = 3x^2 - 6x$. Use the sliders to check your answer.

   <u>**Answer:**</u> To find the *x*-coordinate of the vertex, use the fact that *a* = 3 and *b* = –6 to substitute into the equation $x = \frac{-b}{2a}$. You obtain $x = \frac{-(-6)}{2(3)} = 1$.

   b. Explain how to determine the *y*-value of the vertex without using the sliders.

   <u>**Answer:**</u> To obtain the *y*-value of the vertex, substitute *x* = 1 into the equation $y = 3x^2 - 6x$ to get the *y*-value of –3. Thus, the vertex is located at the point (1, –3).

10. The axis of symmetry is the line about which a parabola can be reflected without changing its position. Which point does the line of symmetry go through?

Answer: The axis of symmetry always goes through the vertex and has the equation $x = \frac{-b}{2a}$.

11. Write the equations, in standard form, for two parabolas that have the same axis of symmetry but different values of *a*. Check the work by adjusting the sliders.

Sample Answer: There are an infinite number of possible answers. For example $\mathbf{f}(x) = x^2 + 1$ and $\mathbf{g}(x) = 3x^2 + 2$ are both symmetrical with respect to the *y*-axis, $x = 0$.

12. Describe the difference between a vertical shift and a vertical stretch or compression.

Answer: A vertical shift slides the entire graph of the function up or down in a vertical direction. A vertical stretch or compression leaves the vertex in the same position but stretches (or compresses) the rest of the graph.

13. What effect does the value of *c* have on the *x*-coordinate of the vertex of the parabola $\mathbf{f}(x) = ax^2 + bx + c$? What effect does the value of *c* have on the *y*-coordinate of the vertex? Explain why this is so.

Answer: The value of *c* has no effect on the *x*-coordinate of the vertex of the parabola $\mathbf{f}(x) = ax^2 + bx + c$. This is because the *x*-value of the vertex is related only to the *a* and *b* values with the formula $x = \frac{-b}{2a}$. A change in the value of *c* vertically shifts the *y*-coordinate of the vertex. Substitute the *x*-value of the vertex into the equation $f(x) = ax^2 + bx + c$ to obtain the *y*-value.

14. a. Write an equation for a parabola that opens up with a vertex of (1, 4).

Sample Answer: A possible equation is $\mathbf{f}(x) = 2x^2 - 4x + 6$.

b. Explain how you obtained your answer.

Sample Answer: To write an equation for a parabola with a vertex whose *x*-coordinate is 1, you must find *a* and *b* values such that $\frac{-b}{2a} = 1$. There are an infinite number of solutions.

Since the parabola opens up, you know that *a* must be a positive number. You can choose $a = 2$, and then obtain $b = -4$.

Utilize the coordinates of the vertex, and the values of *a* and *b*. To obtain a *y*-value of 4, substitute $x = 1$ into the equation $f(x) = 2x^2 - 4x + c$, set the equation equal to 4, and solve for *c*.

In this case, you obtain $c = 6$.
Thus, an equation is $f(x) = 2x^2 - 4x + 6$. However, there are an infinite number of other possible equations with a vertex of (1, 4).

c. Why is there more than one possible correct equation?

Sample Answer: Since you only have the coordinates of the vertex, you know about the relationship between *a* and *b*, but not the specific values of *a* and *b*.

TI-Nspire Navigator Opportunity: *Quick Poll (Equations)*
See Note 4 at the end of this lesson.

15. a. Write an equation for a parabola that opens down with a vertex of (1, 4).

Sample Answer: A possible equation is $\mathbf{f}(x) = -x^2 + 2x + 3$.

b. Explain how you obtained your answer.

Sample Answer: To write an equation for a parabola with a vertex whose *x*-coordinate is 1, you must find an *a* and *b* value such that $\frac{-b}{2a} = 1$. There is an infinite number of solutions.

Since the parabola opens down, you know that *a* must be a negative number. You can choose $a = -1$and then obtain $b = 2$.

Utilize the coordinates of the vertex, and the values of *a* and *b*. To obtain a *y*-value of 4, substitute $x = 1$ into the equation $y = -x^2 + 2x + c$, set the equation equal to 4, and solve for *c*. In this case, you obtain $c = 3$.

Thus, an equation is $f(x) = -x^2 + 2x + 3$. However, there are an infinite number of other possible equations with a vertex of (1, 4).

c. If you knew that the *y*-intercept was (0, 2), would your answer to part 15a change? Why or why not?

Sample Answer: Yes, if you knew that the *y*-intercept was (0, 2), there would be only one quadratic function that could fit the data. Since you know that the graph of the quadratic function is symmetric over the line $x = 1$, and you know that the point (0, 2) lies on the graph, you also know that the point (2, 2) would lie on the graph.

You now have three points, (1, 4), (0, 2) and (2, 2), that lie on the graph of the parabola. These three points determine a unique quadratic function.

Extension: You may want to ask students to write the equation of the quadratic function containing the three points.

Since you are given that the point (0, 2) lies on the graph, you know that $c = 2$ by substituting $x = 0$ and $y = 2$ into the equation $\mathbf{f}(x) = ax^2 + bx + c$.

You can then substitute 2 for c into the equation $\mathbf{f}(x) = ax^2 + bx + c$ and substitute the coordinates of the other two points for x and y to produce the following system of two equations with two unknowns.

$$4 = a(1)^2 + b(1) + 2$$
$$2 = a(2)^2 + b(2) + 2$$

Simplifying, you obtain: $2 = a + b$

$0 = 4a + 2b$

By subtracting twice the first equation from the second equation, you get:

$$-4 = 2a$$
$$a = -2$$

When you substitute – 2 for a into the first equation, you see that $b = 4$.

Thus, the equation is $\mathbf{f}(x) = -2x^2 + 4x + 2$.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- That a specific change in the value of a will affect the shape of the graph of the quadratic $\mathbf{f}(x) = ax^2 + bx + c$ (vertical stretch or compression with respect to the horizontal axis, vertical reflection if the sign of a changes).
- That a specific change in the value of c will affect the position of the graph of the quadratic $\mathbf{f}(x) = ax^2 + bx + c$ (vertical translation).

- That changes in the values of *a* and *b* will affect the coordinates of the vertex of the quadratic $\mathbf{f}(x) = ax^2 + bx + c$ resulting in both horizontal and vertical shifts.
- That the *x*-coordinate of the vertex can be obtained by utilizing the formula $x = \frac{-b}{2a}$.

Assessment

Throughout the lesson, check that students understand the relationship between the form of the function and the graph of the function.

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Note 1

Question 1, *Class Capture/Live Presenter*: You may want to use *Live Presenter* to have a student demonstrate the procedure for using sliders to change the parameters in the equation. Use *Class Capture* to monitor the students' progress through the activity.

Note 2

Question 3, *Quick Poll ((x,y) Numerical Input):* Tell students that you are going to send an (*(x,y) Numerical Input) Quick Poll*. Ask them to type in one of their *P*-coordinates. After you collect students' responses, spend some time helping them discover the effect of changing the value of *a* on the graph of the parabola.

You may want to also use *Live Presenter* to have a student adjust the slider to increase the value of *a* to help the class see the vertical stretch of the parabola.

Note 3

Question 7, *Quick Poll (Open Response):* You may want to send students an *Open Response Quick Poll*. Ask them to type in their rule. You may want to check students' responses without sharing them with the class. This will enable you to see if students are able to determine the correct relationship.

You may want to send another *Open Response Quick Poll* after question 10 to check again.

Note 4

Question 14, *Quick Poll (Equations):* You may want to use an *Equations Quick Poll*. Ask students to type in their equation. Students should see a variety of equations that have the correct vertex. Lead them to discover the similarities and differences in the various correct equations.