Area of Rectangles With Constant Perimeter

Answer Key

1. What type of function is implied in table C ?
   Why ?

   The table implies a quadratic function since equal first difference in L1 results in equal second difference in L3.

2. Is it possible to have a width of 25 ? Why?

   No since any rectangle width a width of 25 will have a perimeter of more than 40.

3. What is the domain of the function? Use interval form in your answer.

   (0, 20)

   Is it correct to use brackets instead of parentheses? Why not?

   No since its not possible to have 0 or 20 in the width.

4. Find the functions that generates table C.

   $y = -x^2 + 20x$ where x is the defines L1 and y defines L3.

5. Sketch a graph of the function.

   Note: The graph must be a parabola concave downward that is exclusively located in first quadrant only.

6. Find the appropriate width that produces the area below.

   A) 50 ft^2. ***width = 2.93 or 17.07***

   B) 35 ft^2 ***width = 1.94 or 18.06***

7. Is it possible to construct a rectangle with an area of 101 ft^2?
   Use your answer in number 4 to justify mathematically.

   No, since if the area is 101 then from number 4, $-x^2 + 20x = 101$ and by rearranging terms we get $x^2 - 20x + 101 = 0$. The discrimant of this quadratic equation is equal to – 4. This implies that the equation has no real number solution.

8. Base on the table, what appears to be the area of the largest possible rectangle?

From the table it appears that the maximum area is 100 sq. ft.

9. Verify mathematically that your answer in number 8 is largest area using your answer in number 4.

From number 4, the function that generates the table is a quadratic function that is concave downward. As such, it has a maximum value which will occur at the vertex. The x coordinate of the vertex can be found by -b/2a. Using this formula, we

$$x = \frac{-(-20)}{2(1)} = 10$$

Using 10 as the value of x, we get y by substitution as 100 which is the maximum area.

10. What kind of rectangle is formed that results in a maximum area ?

Square

E) Extension

A) Prove that the largest rectangle that can be constructed using a fix perimeter is a square.

Proof: ***Let p be the perimeter of the rectangle and let x be the width. This implies that the length is -x + p/2. Thus if y is the area then y = x (-x + p/2) which equivalent to*** $y = -x^2 + (p/2)x$***, the x coordinate of the vertex of this parabola is x = p/4. Thus the maximum area will occur when the width is p/4. If the width is p/4 , then its length must be - p/4 + p/2 which can be simplified into p/4. Therefore, the width and the length are the same when the area is maximum. Thus, the rectangle is a square.***

B) Writing to learn.

Is a square a rectangle? Justify your answer.

Yes, since a square satisfies the definition of the rectangle namely that it is a parallelogram with at least one right angle.

Is a rectangle a square? Justify your answer.

No, not all rectangle are squares. Consider for example a 2 cm by 3cm rectangle, certainly it is not a square.

Any comment, corrections and/or suggestion to the activity sheets and to the answer key is highly appreciated. Please send to arnoldninal@yahoo.com. Thanks.