NUMB3RS Activity: Driving with Dijkstra
Episode: "Money for Nothing"

Topic: Graph Theory **Grade Level:** 9 - 12
Objective: Students will determine an optimal path through a network.
Time: 20 minutes

Introduction

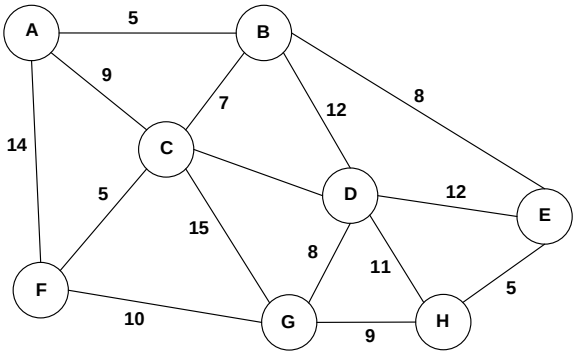
In "Money for Nothing," an armored truck carrying cash and medicine for African Relief is hijacked. Because the hijackers desire the fastest time to leave Los Angeles, Charlie sets about determining possible routes and the respective escape times. However, rather then calculating the times for every possible escape route, Charlie uses Dijkstra's Algorithm to calculate the most likely routes the hijackers might have used to escape Los Angeles.

Dijkstra's Algorithm is used in a branch of mathematics called graph theory. Graph theory reduces a situation to a series of vertices (or nodes) and edges (or line segments). For this activity, the streets are represented with edges and the locations where a car can change from one street to another with vertices. Because Charlie wishes to analyze time, the edges are labeled with the time (in minutes) required to travel between the two vertices it connects.

As Charlie explains, Dijkstra's Algorithm is a *greedy algorithm*. A greedy algorithm is an algorithm that, for each choice, picks the optimal choice in the desire that an optimal path will result. In other words, it will always choose the vertex that results in the optimal outcome. For this activity, that means it will always choose the vertex that results in the shortest total traveling time from the starting point.

Discuss with Students

Many students may not be familiar with this type of graph, so before using the activity, show the students a graph similar to the example below. Questions to ask: How many minutes is the trip from point A to point B? How long is the trip from point A to B to C? These questions will start the students' thinking about finding paths through the graphs.



Often, the space between the nodes is a result of organizing the nodes rather than representing the values of the edges. For example, the lengths of edge BE and edge DG represent 8 minutes.

To aid the students with establishing the total times, inform them that it is a good strategy to write the current time from the start to each circled vertex as they proceed through the algorithm. This will save time with the repeated addition.

Another good way to practice the algorithm is to use the Web site **http://links.math.rpi.edu/applets/appindex/graphtheory.html**. With the Web site, you can construct any graph used in this activity by creating vertices and undirected edges.

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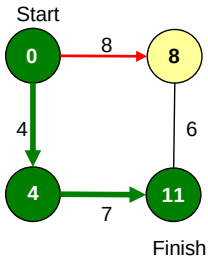
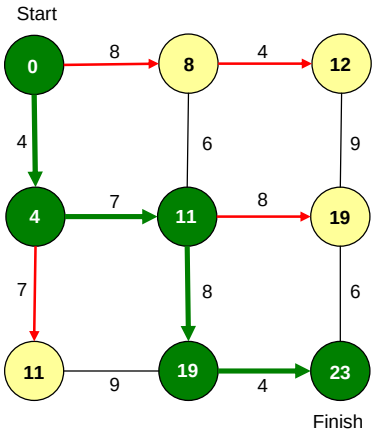
Also, this activity works extremely well if transparencies of the graphs are projected on a whiteboard. In this way, after the various vertices are circled and the edges are darkened, the projection can be turned off and the shortest route is displayed.

Student Page Answers:

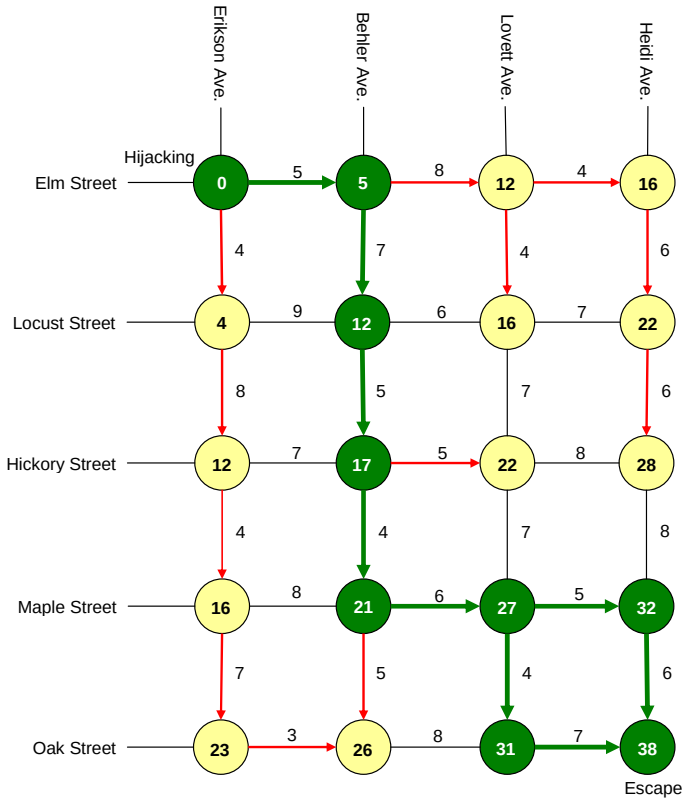
1.



2.

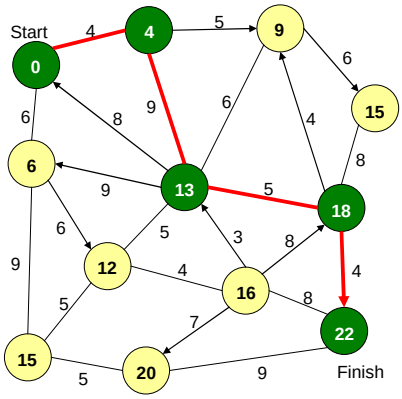


3.



4. There was more than one path with the same optimal time.

Extension Answers:



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Name: _____ Date: _____

NUMB3RS Activity: Driving with Dijkstra

In “Money for Nothing,” an armored truck carrying cash and medicine for African Relief is hijacked. Because the hijackers desire the fastest time to leave Los Angeles, Charlie sets about determining possible routes and the respective escape times. However, rather then calculating the times for every possible escape route, Charlie uses Dijkstra’s Algorithm to calculate the most likely routes the hijackers might have used to escape Los Angeles.

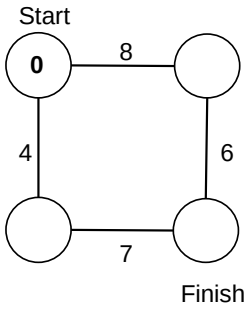
Dijkstra’s Algorithm is used in a branch of mathematics called graph theory. Graph theory reduces a situation to a series of vertices (or nodes) and edges (or line segments). For this activity, the streets are represented with edges and the locations where a car can change from one street to another with vertices. Because Charlie wishes to analyze time, the edges are labeled with the time (in minutes) required to travel between the two vertices it connects. Finally, an open vertex is one where time has yet to be written in the circle representing the vertex.

As Charlie explains, Dijkstra’s Algorithm is a *greedy algorithm*. A greedy algorithm is an algorithm that, for each choice, picks the optimal choice in the desire that an optimal path will result. In other words, it will always choose the vertex that results in the optimal outcome. For this activity, that means it will always choose the vertex that results in the shortest total traveling time from the starting point.

Beginning from the vertex labeled Start, the rules for the algorithm are as follows:

1) Circle the starting vertex.
2) Examine all vertices connected to the circled vertex. Circle the vertex that is the shortest time and darken the connecting edge. If more than one vertex is the same time from the circled vertex, choose either one as it will not impact the final path.
3) Write the total time from the start to the vertex you just circled.
4) Look at all open vertices connected to the circled vertices. Circle the vertex that has the shortest total time from the starting vertex and darken the connecting edge.
5) Write the total time from the starting vertex to the vertex you just circled.
6) Repeat steps 4 and 5 until all vertices have been circled.
7) Look for **all** paths connecting the start and finish with shaded edges. The sum of the times for these paths will be the same as the time written on the finish vertex.

1. Find the optimal path from Start to Finish in the graph to the right.



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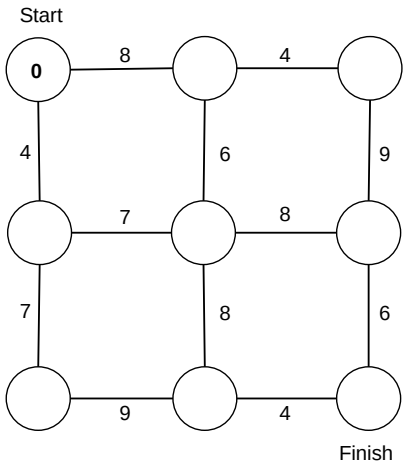
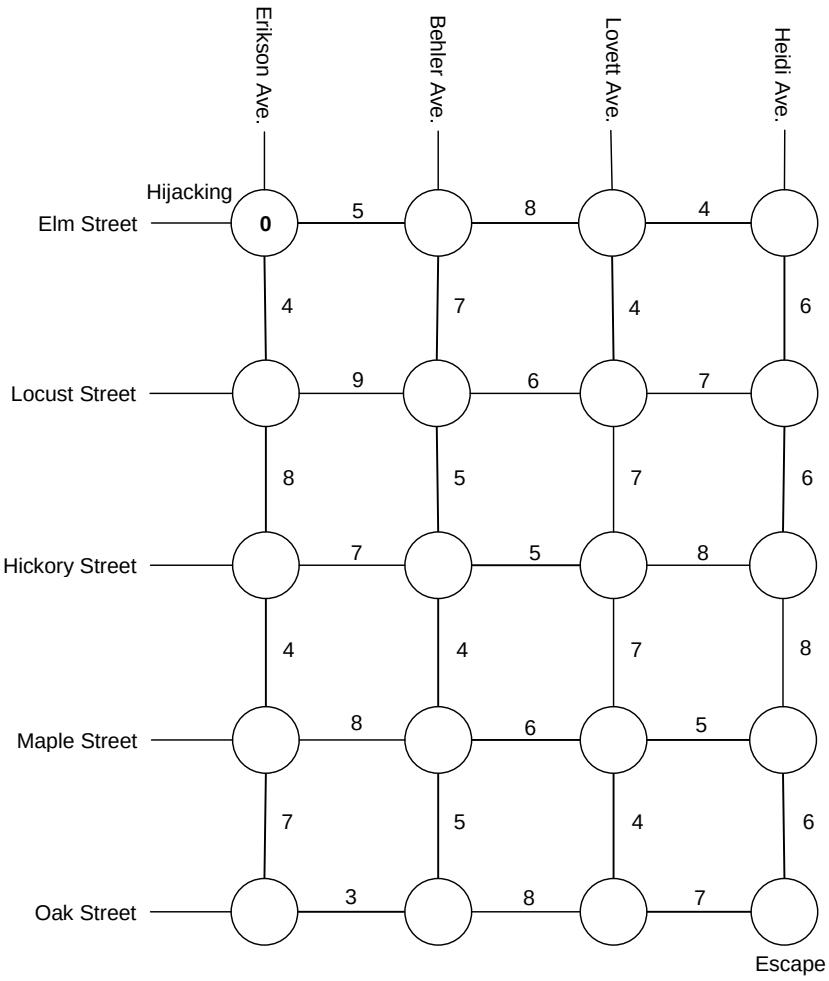
2. When we expand the number of streets we have a larger graph. Find the possible optimal path from Start to Finish.



3. The graph below represents a city map with the time ,in minutes, required to drive from intersection to intersection. Find a possible optimal path from the hijacking point to the escape point.



4. Why is there more than one optimal path?

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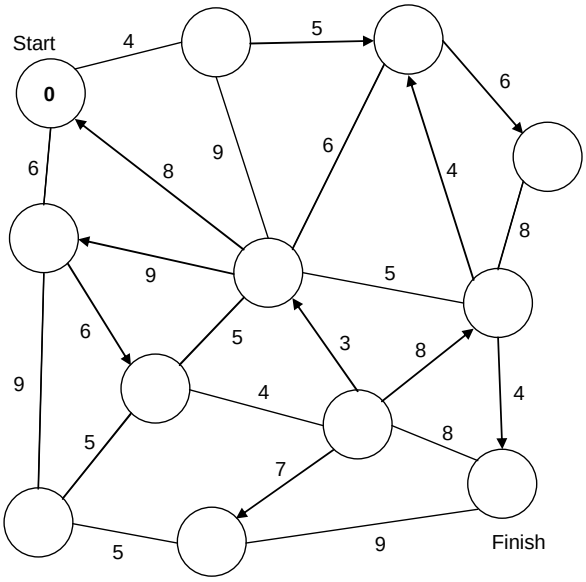
The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extension

Introduction

While the activity assumed that the car could travel in both directions on the graph, this is not always the case. In most cities, there are a number of one-way streets. To indicate this as a graph, we use arrows along the edges to show which direction the traffic travels. This type of graph is called a directed graph or digraph. Using Dijkstra’s Algorithm, find the optimal path for the digraph below. Remember that travel must follow the arrows.



Additional Resources

- An excellent Java™ applet illustrating Dijkstra’s algorithm can be found at the Web site **http://www.lupinho.de/gishur/html/DijkstraApplet.html**

- To learn how a modified Dijkstra’s algorithm and scoring rubric can be used to align DNA sequences, see the *NUMB3RS* Activity “DNA Sequence Alignment.” To download this activity, go to **http://education.ti.com/exchange** and search for “6810.”

- Visit this Web site and use Dijkstra’s algorithm to find the shortest path between two cities: **http://www.unf.edu/~wkloster/foundations/DijkstraApplet/DijkstraApplet.htm**

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