Stacking Bricks

ID: 9090

Time required
45 minutes

Activity Overview

This activity presents a real-world situation—stacking bricks in a pile—that can be modeled by a polynomial function. Students create a small table to show how the number of bricks relates to the number of rows, then calculate the first, second, and third differences of the data in order to determine what degree of polynomial model to use. Next they use the handheld's statistical calculation functions to perform the correct regression. Finally, they evaluate the model using a variety of methods: by graphing the model and the data together, by examining the value of R^2, and by discussing the model's applicability to the real-world situation.

Topic: Polynomials & Polynomial Equations

- *Use finite differences to find the degree of a polynomial that will fit data*
- *Use a polynomial function to model data*

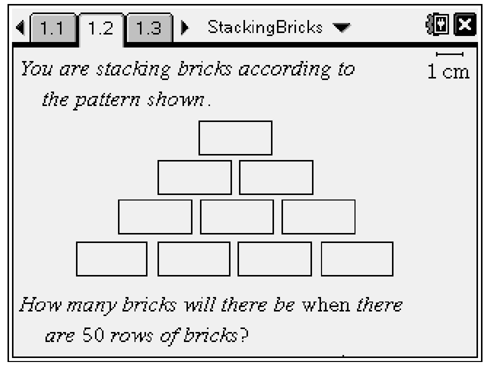
Teacher Preparation and Notes

- *This activity is designed to be used in an Algebra 2 or Precalculus classroom.*
- *Prior to beginning this activity, students should have an introduction to basic polynomial (linear, quadratic, cubic, and quartic) functions, their graphs, and the concept of degree.*
- *This activity requires students to graph functions, plot data in scatter plots, perform statistical calculations on a dataset, use simple formulas in a spreadsheet and apply them with the* ***FillDown*** *command. If students have not experience with these functions of the handheld, extra time should be taken to explain them.*
- *This activity is intended to be mainly* ***student-centered****, and could be performed in pairs or small groups with some periods of teacher-led discussion. The TI-Nspire document helps guide students through the activity.*
- *If time considerations require, Problem 1 can be performed in class and Problem 2 assigned as homework.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9090" in the quick search box.***

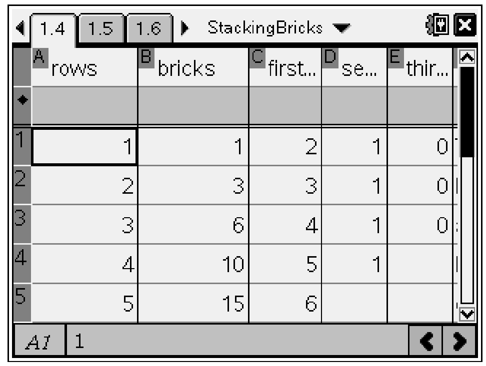
Associated Materials

- *StackingBricks_Student.doc*
- *StackingBricks.tns*
- *StackingBricks_Soln.tns*

Problem 1 – A flat triangular stack

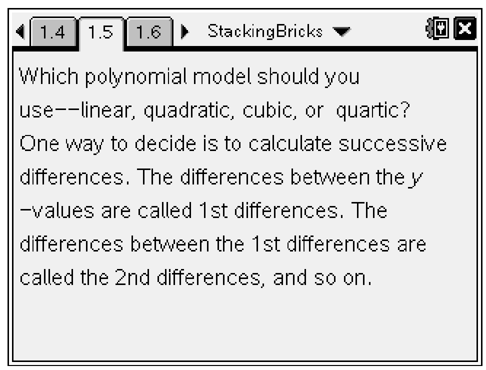


This problem presents a real-life situation that can be perfectly modeled by a polynomial function. Page 1.2 presents the problem: bricks are stacked to form a triangle. Each row contains one more brick than the row above it. How many bricks will there be in the stack when it has 50 rows? You may wish to discuss various methods for solving this problem with students before proceeding to the next page. (Draw a picture, make a table, use a spreadsheet, look for a pattern.)



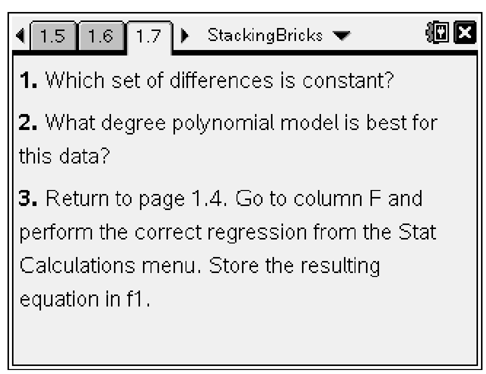
	A rows	B bricks	C first...	D se...	E thir...
◆					
1	1	1	2	1	0
2	2	3	3	1	0
3	3	6	4	1	0
4	4	10	5	1	
5	5	15	6		

Page 1.3 suggests using a polynomial model to solve this problem and prompts students to complete the table on page 1.4. If students have difficulty completing the table, suggest that they draw a picture. Pages 1.5 and 1.6 discuss successive differences and their connection to the degree of a polynomial model.



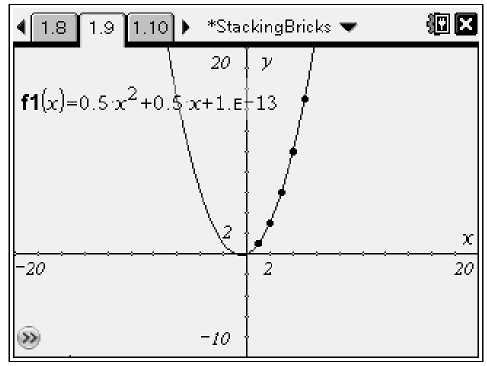
Students may need assistance calculating the first differences. One simple way to do so is to move to cell C1, press = (to begin a formula), move to cell B2, press −, then move to cell B1 and press enter. Then the **FillDown** command is used to copy this formula into cells C2–C5. Repeat the process in column D to find the second differences, the differences between the first differences (D1=C2-C1). Finally, repeat the process in column E to find the third differences.

Discuss with students how many first, second, and third differences there should be (5 first differences, 4 second differences, and 3 third differences). A common mistake is to **FillDown** too far.



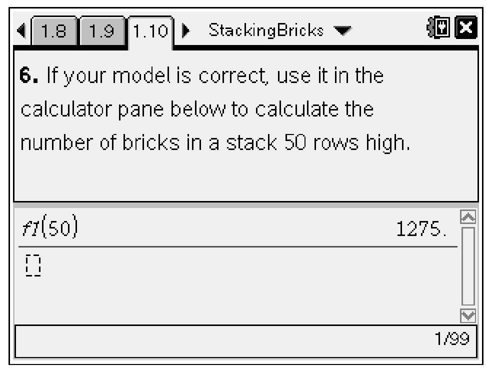
On page 1.7, students are prompted to choose a polynomial model based on the successive differences. They should find that the second differences are constant, so a second degree or quadratic model is most appropriate. Returning to page 1.4, they should move to cell F1 and choose **Quadratic Regression** from the Stat Calculations menu (**MENU > Statistics > Stat Calculations > Quadratic Regression**). Students may need assistance choosing the correct XList and YList (XList: rows, YList: bricks). They should store the regression equation in ***f1***.

Page 1.8 begins the process of evaluating the model. It asks students about the value of R^2 for their model. Students should find that because this data is perfectly quadratic, $R^2 = 1$. Next students are directed to graph their model together with the data points from the table they created. To graph the model, students should go to function ***f1*** in the formula bar on page 1.9 and press enter. To graph the data points, they must first change the Graph Type to Scatter Plot, then choose rows as the XList and bricks as the YList. Page 1.9 is already set to an appropriate viewing window for them.

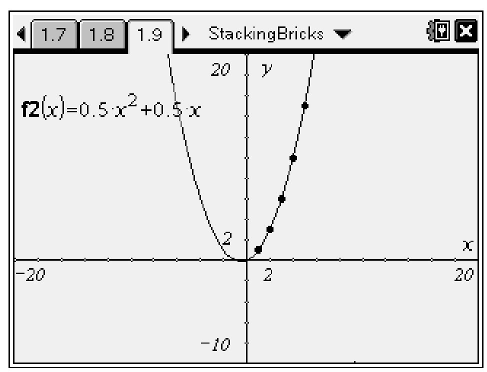


This step provides a control of error. If students have not performed the regression correctly, they will know immediately, because the model will not pass through the points.

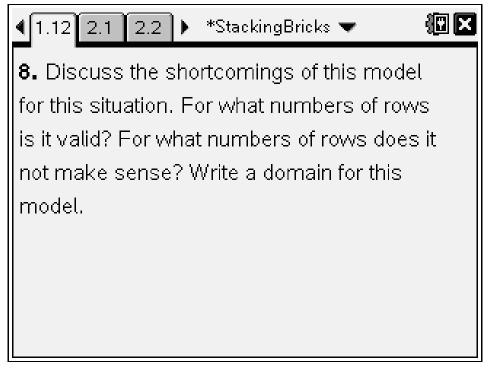
On page 1.10, students should evaluate their model for $x = 50$ to answer the original question (1,275 bricks).



Page 1.11 continues the evaluation of the model. The degree 0 term, 1 E –13, is very close to 0 and has no effect on the model. This step encourages students to examine the handheld's statistical calculation, rather than simply accepting the output.

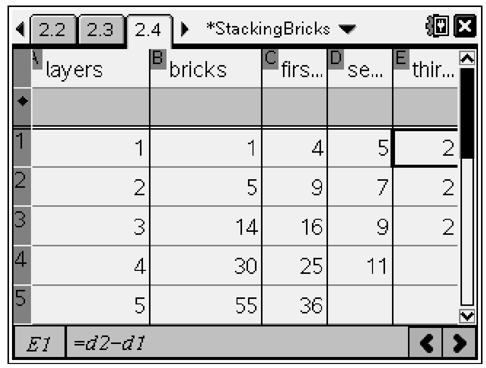


Finally, page 1.12 prompts students to think about the validity of this model in the real-world situation. There will never be a negative number of rows or a negative number of bricks, so the domain of the model is $\{x \mid x \geq 0\}$.



Problem 2 – A pyramidal stack

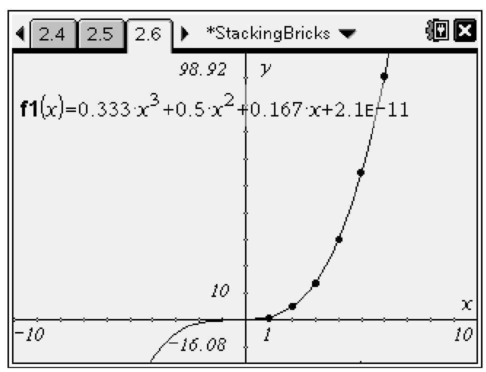
In this problem, students use the same process they used in Problem 1 with a different type of stack: a pyramid with made up of square layers of bricks of increase size: 1 on the top layer, 4 on the second layer, 9 on the third, and so on. (Note that the shape of the bricks has changed. In problem 1 they were rectangular, and in this problem they are square.)



	A layers	B bricks	C firs...	D se...	E thir...
◆					
1	1	1	4	5	2
2	2	5	9	7	2
3	3	14	16	9	2
4	4	30	25	11	
5	5	55	36		

E1 =d2−d1

Pages 2.1 and 2.2 introduce the problem and show two views of the stack. Page 2.3 reviews the method introduced in problem 1. Page 1.4 provides students a place to record their small table and calculate successive differences. In this case, students should find that the third differences are constant, so a third degree or cubic regression is most appropriate. Pages 2.5 and 2.6 direct students to check their model using the R^2 coefficient and a graph. On page 2.7, they answer the original question (42,925 bricks). Pages 2.8 and 2.9 prompt them to further evaluate the model. (The degree 0 term, 2.1 E –11, can be omitted without affecting the model; the domain of the model is $\{x \mid x \geq 0\}$.



The number of bricks in problem 1 can also be represented by the triangular numbers. The number of bricks in problem 2 can also be represented by the pyramidal numbers. Optionally, you may extend this problem to discuss square and cubic stacks of bricks.

