NUMB3RS Activity: Lost and Found
Episode: "Nine Wives"

Topic: Modeling a lost person's path **Grade Level:** 9 - 12
Objective: Use simulation and coordinate geometry to model possible paths of a lost girl
Materials: TI-83 Plus/TI-84 Plus graphing calculator, compass (optional)
Time: 20 - 25 minutes

Introduction

One morning, a young girl named Josephine is found beaten and unconscious on a desert road. She had escaped from her "husband" the previous night, and then walked barefoot through the desert most of the night before collapsing. The FBI wants to try to reconstruct Josephine's path in order to determine where she had been held. Unfortunately, she does not know what direction she walked. Don has some starting coordinates and asks Charlie to narrow down the possible 2,200-square-mile search area. Charlie tries to calculate the maximum possible distance she could have traveled in the night. It is also possible that he may (from the point of view of this activity) have used this information to estimate her possible starting location. He does know that she was driven east into the desert for an hour before escaping.

Discuss with Students

Charlie's math is based on an adaptation of a *Lévy flight path*, a variation of a random walk, used to model a search for objects with unknown locations. The searcher walks in what she believes is the correct direction for a while. If no destination is reached, she walks in a random direction for a short period in order to reorient, then repeats the process, starting by walking in the right direction again. This activity assumes that, given that Josephine was driven to the east, she was generally trying to make her way west.

Students will use a simulation to recreate possible paths Josephine could have followed to end up where she was found. Generating a set of three possible starting points, students estimate a center and radius for a circle containing them. The FBI could then translate the circle into the appropriate location, providing a much smaller search area.

The parameters for the simulation are:

1. The ending point of a path that generally runs from east to west is known.
2. To generate a possible path from the *ending* point, students generate a path that runs from west to east. This means that each segment of the path must be generated in the reverse order from how it actually happened.
3. The data used to create the path is a list of 12 sets of three random numbers to represent traveling for 30 minutes for each set. The three numbers indicate:
  - **a.** The random choice of a wrong direction (1 = from the south, 2 = backtrack from the west, 3 = from the north);
  - **b.** Distance walked in this random direction (0.25 – 0.5 mile);
  - **c.** Distance traveled from the east (0.25 – 1.5 miles).

4. The distances and directions from #3 are repeated to simulate Josephine traveling in the night for a total of six hours.

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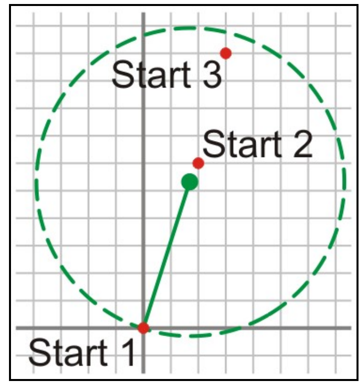
Student Page Answers:
Students' actual results may vary greatly from those shown. The sample paths and their corresponding starting points shown here are used to provide sample answers for the questions.



Here are the data sets used for the sample solution:

Path 1 (dotted):			Path 2 (solid):			Path 3 (dashed):		
L1	L2	L3	L1	L2	L3	L1	L2	L3
2	2	3	3	2	5	3	1	1
1	1	2	1	1	5	3	1	3
1	2	6	1	1	6	1	1	5
3	1	4	1	1	1	3	2	3
3	2	6	3	2	5	3	1	1
1	2	1	1	1	5	3	1	3
1	2	2	2	1	1	2	2	6
1	2	3	3	1	5	2	2	3
1	2	2	1	2	1	2	1	6
3	2	6	2	1	1	1	1	3
2	2	4	3	1	2	2	1	6
2	2	1	2	1	3	2	1	4

1. *Area* $= \pi r^2$*, so* $2{,}200 = \pi r^2$*,* $r^2 = \sqrt{\frac{2{,}200}{\pi}}$*, and* $r \approx 26.46$ *miles.* **2.** *From the sample, Start 1 could be the origin, which places Start 2 at (3, 10) and Start 3 at (2, 6). This puts the center point at C(1.67, 5.33); d(1, C) =* $\sqrt{(1.67)^2 + (5.33)^2} \approx 5.59$*, d(2, C) =* $\sqrt{(0.33)^2 + (1.33)^2} \approx 1.37$*, d(3, C) =* $\sqrt{(1.33)^2 + (4.67)^2} \approx 4.86$*; d(1, C) is clearly the largest. Converting to miles (1 square = 0.25 mile) yields a distance of 1.40 miles.* **3.** *A =* $(1.40)^2 \pi \approx 6.16$ *square miles.*
4. *Obviously, this search area is significantly smaller. 6.16* $mi^2 \div 2{,}200\ mi^2 \approx 0.0029$*, or approximately 0.3% of the original search area.*

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Name: ________________________________ Date: _______________

NUMB3RS Activity: Lost and Found

One morning, a young girl named Josephine is found beaten and unconscious on a desert road. She had escaped from her "husband" the previous night, and then walked barefoot through the desert most of the night before collapsing. The FBI wants to try to reconstruct Josephine's path in order to determine where she had been held. Unfortunately, she does not know what direction she walked. Don has some starting coordinates, and asks Charlie to narrow down the possible 2,200-square-mile search area. Charlie tries to calculate the maximum possible distance she could have traveled in the night. It is also possible that he may (from the point of view of this activity) have used this information to estimate her possible starting location. He does know that she was driven east into the desert for an hour before escaping.

In this activity, you will use a simulation to recreate possible paths Josephine could have followed to end up where she was found. By generating a set of three possible starting points, you can estimate a center and radius for a circle containing them. The FBI could translate the circle into the appropriate position, providing a greatly reduced search area.

The simulation starts at the point at which Josephine was found. By plotting the possible path in the reverse direction, and in the reverse order in which she would have chosen her directions, you can determine possible starting points for her path. This activity makes the assumption that she was generally trying to make her way to the west.

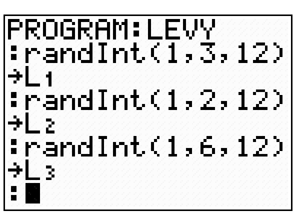
The math Charlie uses is an adaptation of what is called a *Lévy flight path*, which operates according to the following algorithm:

1. Josephine traveled to the west for a while, before losing her confidence that she was approaching a desirable destination.
2. She then chose a random direction, and walked it for a short distance until she got reoriented.
3. She then walked to the west again, before repeating the process. This activity models her walking for a total of six hours.

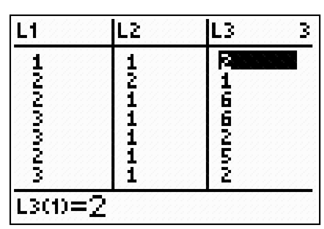
The data used to create a path is a list of 12 sets of three random numbers to represent traveling for 30 minutes for each set. *Keep in mind that the order here is in reverse order of what she actually did, with the directions reversed, too*. The three numbers in each of the 12 sets indicate:

   a. The random choice of a wrong direction (1 = from the south, 2 = backtrack from the west, 3 = from the north).
   b. Distance walked in this random direction (0.25 – 0.5 mile).
   c. Distance traveled from the east (0.25 – 1.5 miles).

Enter this program on your calculator (Do not add "End"):



Execute the program, press STAT and then select **1:Edit**. Here is a sample of what the data might look like (your lists will have different numbers):



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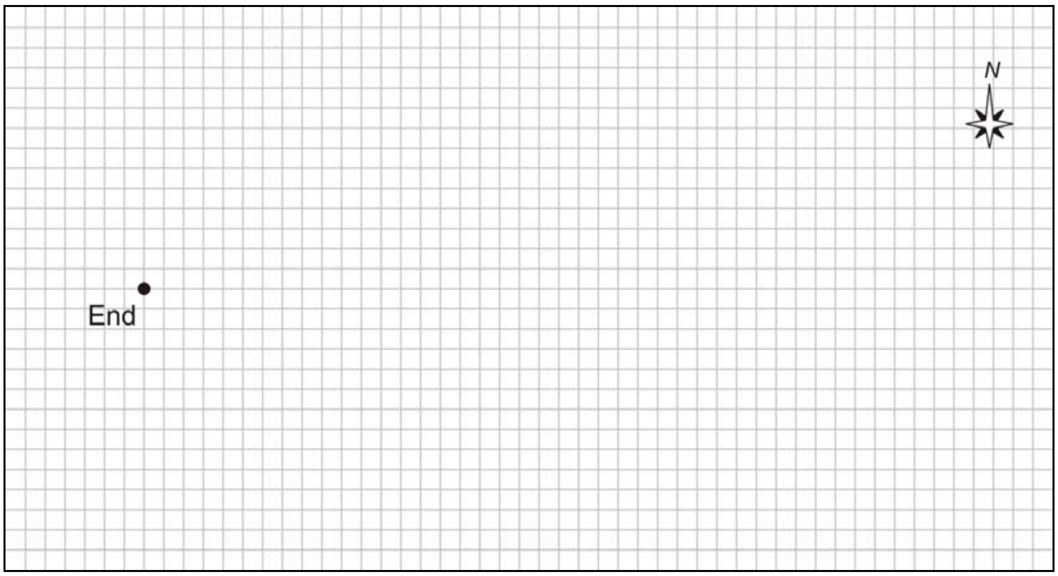
Use the grid below, starting at “End.” Use your data to plot Josephine’s path. Each side of a square represents 0.25 mile.

Example for the sample data set:

Row of Data	Data	Corresponding Movement
First Row	1, 1, 2	south 1 square, east 2 squares
Second Row	2, 2, 1	west 2 squares, east 1 square
Third Row	2, 1, 6	west 1 square, east 6 squares

After you have completed one path and marked the starting location, press 2nd [QUIT] and then press ENTER to execute the program again. Construct a second path, and then a third path.



1. Initially, Don tells Charlie that the search area covers 2,200 square miles. Assuming that this area is roughly a circle, what is its radius?

Place the origin of a coordinate plane near your three starting points so that they all lie in the first quadrant. (Perhaps one of the points could even *be* the origin.) Average the coordinates of your three starting points and plot this “center” point. Use the Pythagorean Theorem (distance formula) to find the distance from the center to each starting point.

2. The largest distance between your center point and one of the three starting points will be the radius of your search area. What is this distance (in miles)?

Use a compass or simply sketch the circle, which will contain the three starting points, using your center point. This is your calculated search area.

3. What is the area of the circle to search?

4. Make an observation comparing your calculated search area to the one originally proposed in question 1. What percent of the original search area is yours?

Congratulations! If the FBI is able to find the right road in your circle, they can shift its location to account for Josephine’s ride to the east before escaping. This will give them a much better chance of finding the Prophet’s trailer hideout.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

The purpose of this activity has been to capture the spirit of the use of mathematical modeling in a unique setting. Obviously, a set of only three possible starting points will not produce a very accurate calculated search area. With the use of a computer, a very large set of paths could be determined. Although the resulting area would probably be larger than that produced by a smaller set of paths, the mathematical model would produce results of great help to the FBI.

For the Student

(Directed by the teacher) Use the original “End” point from this activity as the origin of a Cartesian plane. Determine the coordinates and plot all of the students’ sets of starting points on a single graph (on a transparency, on the board, etc.). Repeat questions 2 – 4 from the activity to calculate a more feasible calculated search area based on a large sample of mathematical models. Alternately, make a transparency of the grid on the student page and have all of the students plot their starting points on it.

Related Topic

Lévy flights have been used to describe characteristics of objects generated using a branch of mathematics called chaos theory. This mathematics is in turn related to fractal geometry. In 2000, *Plus*, an Internet mathematics magazine from England, published an article that discusses the abstract art of Jackson Pollock, including some mathematical properties. Along with chaos and fractals, there is a reference to Lévy flights. See:
http://plus.maths.org/issue11/features/physics_world

Additional Resources

For a lesson on how to generate a true Lévy flight, go to:
http://classes.yale.edu/fractals/RandFrac/Levy/Levy4.html

A Lévy fight path is a variation of a random walk. For a *NUMB3RS* activity on random walks, see “Walkabout” for the Season 2 Episode “Mind Games.” To download this activity, go to: **http://education.ti.com/exchange** and search for “6641.”

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