

Exploring the Area Under a Curve

ID: 8268

Time required
45 minutes




Activity Overview

Students will explore Riemann sums to find the approximate area under the graph of $y = x^2$ on the interval $[0, 1]$. They will use the Rectangle tool to draw five left-endpoint, right-endpoint, and midpoint approximating rectangles. Additionally, students will learn to write formulas in expanded form and using summation notation. Finally, they will be asked to summarize their findings, including an analysis of when these approximations produce overestimates or underestimates and the idea that an infinite number of approximating rectangles will yield the exact area under a curve.

Topic: The Fundamental Theorem of Calculus

Approximate the area under a curve by constructing a Riemann sum and calculating its sum.

Teacher Preparation and Notes

- *It is important to review the concept of summation notation with students beforehand.*
- *Students should know how to write an arithmetic sequence which will be part of the summation expression they will use to find approximate areas.*
- *The heights of the rectangles drawn in this activity will not be exactly equal to the heights that are found analytically (The widths will ‘snap’ into place, whereas the heights will not.). However, when rounded to the nearest hundredth, each area approximation should have minimal error associated with it.*
- *On self-check questions, students can get immediate feedback by selecting  **Check Answer** (or by pressing  + ). With TI-Nspire Teacher software, teachers can change self-check questions to exam mode so students cannot check their answer. On any question, click the Teacher Tool Palette and select Question Properties. Change the Document Type from Self-Check to Exam. Also, teachers should use the Teacher Software in TI-SmartView Emulator view so the answers do not show as they would if the left panel was in Document Tools.*
- **To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter “8268” in the keyword search box.**

Associated Materials

- *ExploringAreaUnderCurve_Student.doc*
- *ExploringAreaUnderCurve.tns*
- *ExploringAreaUnderCurve_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

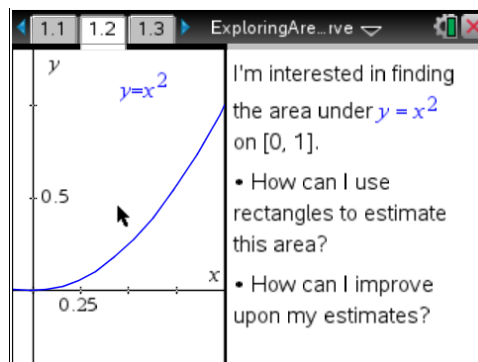
- *Sum Rectangles (TI-Nspire technology) — 12100*
- *Riemann Rectangle Errors (TI-Nspire technology) — 16107*
- *Summing it Up (TI-Nspire CAS technology) — 10094*
- *Riemann’s Sums (TI-Nspire technology) — 9578*

Problem 1 – Explore and discover

Two focus questions define this activity:

- How can you use rectangles to approximate the area under the curve $y = x^2$ and above the x -axis?
- Is there a way to use rectangles to find the exact area under the curve?

Some discussion should follow the posing of these questions, mainly related to how these rectangles should be constructed. Students will also want to know how many rectangles should be used. Prior experience with limits will suggest that a large number of rectangles will produce a more accurate estimate. Tell students that, for this activity, they will look at exactly five rectangles.



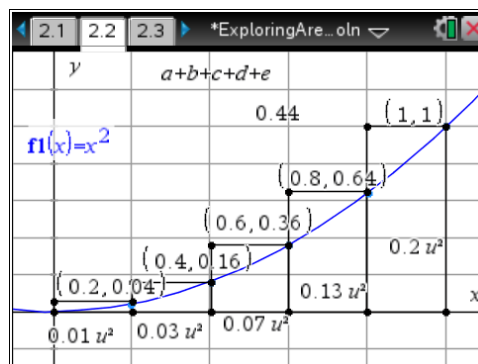
The goal of this problem is to give students an opportunity to discover for themselves how to use rectangles to approximate the area under a curve. Students might have some difficulty with the second condition, that is, that all rectangles must be built using the same methods. The purpose of this condition is to force students to find a systematic way of constructing their rectangles, such as making sure that the right endpoint of each rectangle touches the curve. Encourage discussion amongst students and provide support and advice as needed.

TI-Nspire Navigator Opportunity: Quick Poll, Live Presenter
See Note 1 at the end of this lesson.

After students have had sufficient time to explore, have them move on to Problem 2, which begins to address three methods for using rectangles to approximate the area under a curve.

Problem 2 – Using five right-endpoint rectangles

Discuss with students how they think they can construct their 5 right-endpoint rectangles. Make sure that everyone is in agreement before proceeding. This will make the remainder of the activity run much more smoothly. The screen image shown gives an estimate for Problem 2. If students make a mistake, encourage them to press **ctrl** + **esc** to undo and correct. Notice that all measurements have been rounded to the nearest hundredth.



TI-Nspire Navigator Opportunity: Screen Capture, Live Presenter
See Note 2 at the end of this lesson.

It is important that students understand the symbolic calculation that will produce an answer that is close to the one found in Step 1. For example, students need to recognize that 0.2 represents the width of each rectangle and is found by dividing the width of the entire interval by the number of approximating rectangles. The expression $f(0.2 \cdot i) = (0.2 \cdot i)^2$ represents the height of each approximating rectangle. These calculations are also called the Riemann sum with right-endpoints. You may want to spend some additional time covering this concept as it will be used in subsequent problems.

Problem 3 – Using five left-endpoint rectangles

Discuss with students how they think they can construct their 5 left-endpoint rectangles. It may trouble students to see that the first rectangle has a height of zero. Remind them to stick with their definition of what they think comprises a left-endpoint rectangle. Students may also notice that the heights of the rectangles are determined by $f(0)$, $f(0.2)$, ..., $f(0.8)$. Encourage them to notice that each value of x is shifted down by 0.2 when compared with what was used for the right-endpoint approximation.

The symbolic calculations for the Riemann sum with left-endpoints are shown at right. Emphasize the similarities and differences between these calculations and the calculations from Problem 2.

Problem 4 – Using five midpoint rectangles

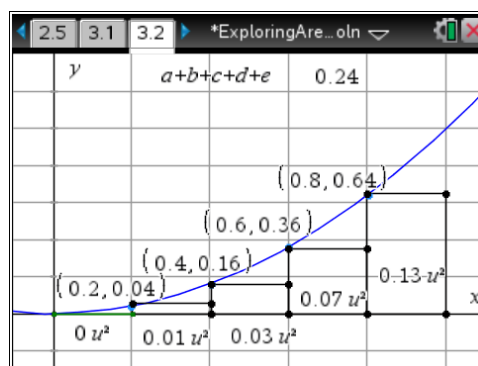
Students will be very accepting of the fact that midpoint approximating rectangles produce the best estimate. As an extension to this activity, students could be asked to determine if this approximation is an *overestimate* or an *underestimate*. This is a good time to discuss this if they bring up the subject.

The formula that expresses this area is:

$$R_5 = 0.2 \cdot (f(0.2) + f(0.4) + f(0.6) + f(0.8) + f(1)) = 0.44$$

$$\sum_{i=1}^5 (0.2 \cdot f(0.2 \cdot i)) = 0.44$$

Move the cursor into the math box that is after R_5 and press **enter** to evaluate the formula. Check to see if this result is

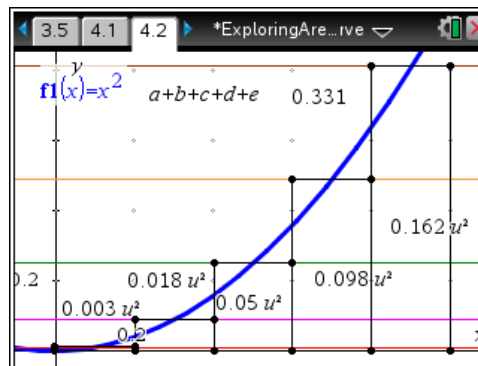


Press **enter** in the math box below to evaluate the formula for the area of left Riemann sums.

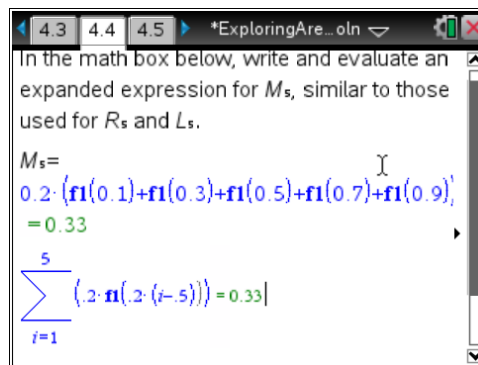
$$R_5 = 0.2 \cdot (f(0) + f(0.2) + f(0.4) + f(0.6) + f(0.8)) = 0.24$$

$$\sum_{i=1}^5 (0.2 \cdot f(0.2i - 0.2)) = 0.24$$

Check if this result is consistent with your rectangle.



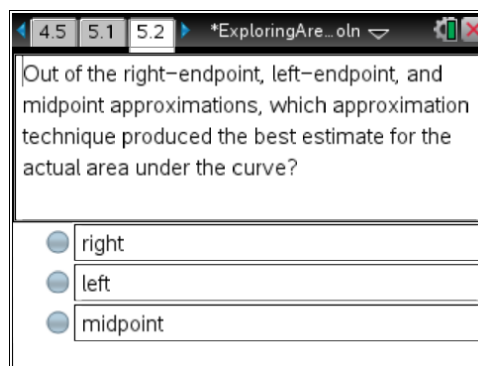
The symbolic calculations for the Riemann sum with midpoints are shown at right. Emphasize the similarities and differences between these calculations and the calculations from Problems 2 and 3.



Problem 5 – Summarize your findings

Students should readily agree that the midpoint approximation produces the best answer.

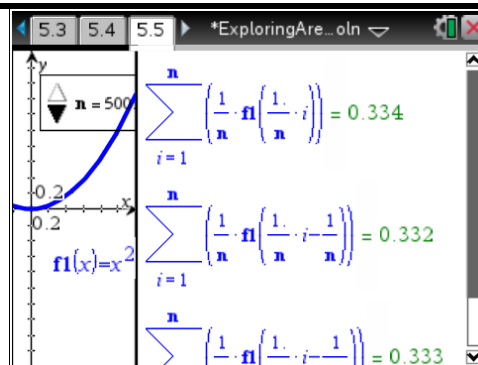
Furthermore, students should respond that the concavity of a function and whether it is increasing or decreasing will determine which estimates are *overestimates* versus *underestimates*. In this activity, students were working with a region that was concave up and increasing. In this type of situation, right-endpoint and midpoint approximations will always give *overestimates* and left-endpoint approximations will always yield *underestimates*.



TI-Nspire Navigator Opportunity: Quick Poll, Live Presenter

See Note 3 at the end of this lesson.

Finally, students will observe that a large number of approximating rectangles will improve upon their answers. Additionally, they will see that summation notation offers a convenient method of quickly evaluating a large number of rectangles. The screen at right show the results of using summation notation for each method with 500 approximating rectangles. Students can click the box and type in the desired value in the minimized slider on the left side of the screen.



As an extension to this activity, you may decide to have students evaluate these sums for an infinite number of rectangles. In doing so, all three approximations will give the exact area under the curve.

TI-Nspire Navigator Opportunity: Collect, Review, Save to Portfolio

See Note 4 at the end of this lesson.

TI-Nspire Navigator Opportunities**Note 1****Problem 1, *Quick Poll, Live Presenter***

Quick Polls can be used on any page that with a question, like 1.3, 1.4, etc. Especially use Quick Poll for page 5.2 which is a multiple choice Question application.

It is recommended to have a student demonstrate making a rectangle on page 1.5.

Note 2**Problem 2, *Screen Capture, Live Presenter***

Use Screen Capture to monitor students and see that they are progressing. Have different students model the process for finding the area and adding up the areas with the Calculate tool by using Live Presenter.

Note 3: *Quick Poll, Live Presenter*

Use Quick Poll on page 5.2 to have the students THINK. Then have them PAIR up to discuss their reasons, and then some can SHARE their rationale. Live Presenter can be used to have a student demonstrate some of the ease-of-use of a minimized slider. Click the slider and then use the arrows. Click the number in the slider and type the desired value.

Note 4**Problem 4, *Collect, Review, Save to Portfolio***

After the activity is completed, collect the file and recap what was learned by reviewing students' answers in the review workspace. This formative assessment can be documented by saving the file to the portfolio where the grade will automatically be calculated.