## 6-4

## Point-Slope Form and Writing Linear Equations

## Lesson Preview

## What You'll Learn



To graph and write linear equations using point-slope form


To write a linear equation using data
... And Why
To write an equation relating altitude and the boiling point of water, as in Example 5

## Check Skills You'll Need

(For help, go to Lessons 6-1 and 1-7.)
Find the rate of change of the data in each table.
1.

| $x$ | $y$ |
| ---: | ---: |
| 2 | 4 |
| 5 | -2 |
| 8 | -8 |
| 11 | -14 |

$-2$
2.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| ---: | :---: |
| -3 | -5 |
| -1 | -4 |
| 1 | -3 |
| 3 | -2 |

3. 

| $x$ | $y$ |
| :---: | ---: |
| 10 | 4 |
| 7.5 | -1 |
| 5 | -6 |
| 2.5 | -11 |

Simplify each expression.
4. $-3(x-5)-3 x+15$
5. $5(x+2) 5 x+10$
6. $-\frac{4}{9}(x-6)$
$-\frac{4}{9} x+\frac{8}{3}$

## New Vocabulary • point-slope form

Interactive lesson includes instant self-check, tutorials, and activities.

## Using Point-Slope Form

Suppose you know that a line passes through the point $(3,4)$ with slope 2 . You can quickly write an equation of the line using the $x$ - and $y$-coordinates of the point and using the slope.

$$
\begin{gathered}
y-4=2(x-3) \\
\swarrow \\
\downarrow \\
y \text {-coordinate } \\
\downarrow \\
\text { slope } \\
x \text {-coordinate }
\end{gathered}
$$

You can use the definition of slope to verify that $y-4=2(x-3)$ is the equation of the line through the point $(3,4)$ with slope 2 .

$$
\begin{aligned}
\frac{y_{2}-y_{1}}{x_{2}-x_{1}} & =m & & \text { Use the definition of slope. } \\
\frac{y-4}{x-3} & =2 & & \text { Substitute }(3,4) \text { for }\left(x_{1}, y_{1}\right),(x, y) \text { for }\left(x_{2}, y_{2}\right), \text { and } 2 \text { for } m . \\
\frac{y-4}{x-3}(x-3) & =2(x-3) & & \text { Multiply each side by } x-3 . \\
y-4 & =2(x-3) & & \text { Simplify the left side of the equation. }
\end{aligned}
$$

The equation $y-4=2(x-3)$ is in point-slope form.

## Definition Point-Slope Form of a Linear Equation

The point-slope form of the equation of a nonvertical line that passes through the point $\left(x_{1}, y_{1}\right)$ with slope $m$ is

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

## 1 EXADIPLE Graphing Using Point-Slope Form

Graph the equation $y-5=\frac{1}{2}(x-2)$.
1.

$\checkmark$ Check Understanding

## Reading Math

Square brackets, [ ], are grouping symbols commonly used when parenthesis, ( ), are inside.


The equation shows that the line passes through $(2,5)$ with a slope $\frac{1}{2}$.
Start at $(2,5)$. Using the slope, go up 1 unit and right 2 units to (4, 6). Draw a line through the two points.
(1) Graph the equation $y-5=-\frac{2}{3}(x+2)$. See left.

## 2) ExAluple Writing an Equation in Point-Slope Form

Write the equation of the line with slope -3 that passes through the point ( $-1,7$ ).
$y-y_{1}=m\left(x-x_{1}\right) \quad$ Use the point-slope form.
$y-7=-3[x-(-1)] \quad$ Substitute $(-1,7)$ for $\left(x_{1}, y_{1}\right)$ and -3 for $m$.
$y-7=-3(x+1) \quad$ Simplify inside the grouping symbols.

## Check Undersłanding

2 Write the equation of the line with slope $\frac{2}{5}$ that passes through the point $(10,-8)$. $y+8=\frac{2}{5}(x-10)$
If you know two points on a line, first use them to find the slope. Then you can write an equation using either point.

## 3 Exandple Using Two Points to Write an Equation

Write equations for the line in point-slope form and in slope-intercept form.

Step 1 Find the slope.

$$
\begin{aligned}
& \frac{y_{2}-y_{1}}{x_{2}-x_{1}}=m \\
& \frac{-5-3}{-1-2}=\frac{8}{3}
\end{aligned}
$$

The slope is $\frac{8}{3}$.

Step 2 Use either point to write the equation in point-slope form.
Use $(2,3)$.

$$
\begin{aligned}
y-y_{1} & =m\left(x-x_{1}\right) \\
y-3 & =\frac{8}{3}(x-2)
\end{aligned}
$$



Step 3 Rewrite the equation from Step 2 in slopeintercept form.

$$
\begin{aligned}
y-3 & =\frac{8}{3}(x-2) \\
y-3 & =\frac{8}{3} x-5 \frac{1}{3} \\
y & =\frac{8}{3} x-2 \frac{1}{3}
\end{aligned}
$$

3 a. Write an equation for the line in Example 3 in point-slope form using the point $(-1,-5) . \quad y+5=\frac{8}{3}(x+1)$
b. Write the equation you found in part (a) in slope-intercept form. $y=\frac{8}{3} x-2 \frac{1}{3}$
c. What is true about the equation you wrote in part (b) and the equation in Step 3 of Example 3? They are the same.

You can write a linear equation to model data in tables. Two sets of data have a linear relationship if the rate of change between consecutive pairs of data is the same. For data that have a linear relationship, the rate of change is the slope.

## 4 ExADJPLE Writing an Equation Using a Table

Is the relationship shown by the data linear? If so, model the data with an equation.

Step 1 Find the rate of change for consecutive ordered pairs.

2 \begin{tabular}{|r|r|}
\hline$x$ \& $y$ <br>
\hline-1 \& 4 <br>
\hline 3 \& 6 <br>
\hline 5 \& 7 <br>
\hline 11 \& 10 <br>
\hline

, 

2 \& $\frac{2}{4}=\frac{1}{2}$ <br>
1 \& $\frac{1}{2}=\frac{1}{2}$ <br>
3 \& $\frac{3}{6}=\frac{1}{2}$
\end{tabular}

Step 2 Use the slope and a point to write an equation.

$$
y-y_{1}=m\left(x-x_{1}\right)
$$

Substitute $(5,7)$ for $\left(x_{1}, y_{1}\right)$ and $\frac{1}{2}$ for $m$.
$y-7=\frac{1}{2}(x-5)$

4 Is the relationship shown by the data at the right linear? If so, model the data with an equation.
Yes; answers may
vary. Sample:
$y-5=\frac{2}{5}(x-19)$

| $x$ | $y$ |
| ---: | ---: |
| -11 | -7 |
| -1 | -3 |
| 4 | -1 |
| 19 | 5 |

## 5 ExANPLE Real-World Problem Solving

Is the relationship shown by the data linear? If so, model the data with an equation.

|  | Boiling | of Water |
| :---: | :---: | :---: |
|  | Altitude ( 1000 ft ) | Temperature ( ${ }^{\circ} \mathrm{F}$ ) |
|  | 8 | 197.6 |
|  | 4.5 | 203.9 |
|  | 3 | 206.6 |
|  | 2.5 | 207.5 |

Step 1 Find the rates of change for consecutive ordered pairs.

$$
\frac{6.3}{-3.5}=-1.8 \quad \frac{2.7}{-1.5}=-1.8 \quad \frac{0.9}{-0.5}=-1.8
$$

The relationship is linear. The rate of change is -1.8 degrees Fahrenheit per 1000 ft of altitude.

Step 2 Use the slope and a point to write an equation.

$$
\begin{aligned}
y-y_{1} & =m\left(x-x_{1}\right) & & \text { Use the point-slope form. } \\
y-206.6 & =-1.8(x-3) & & \text { Substitute }(3,206.6) \text { for }\left(x_{1}, y_{1}\right) \text { and }-1.8 \text { for } m .
\end{aligned}
$$

The equation $y-206.6=-1.8(x-3)$ relates altitude in thousands of feet $x$ to the boiling point temperature in degrees Fahrenheit.

Is the relationship shown by the data in the table linear? If it is, model the data with an equation.
Yes; answers may
vary. Sample:
$y-3030=-\frac{50}{3}(x-68)$

Working Outdoors

| Temperature | Calories Burned per Day |
| :---: | :---: |
| $68^{\circ} \mathrm{F}$ | 3030 |
| $62^{\circ} \mathrm{F}$ | 3130 |
| $56^{\circ} \mathrm{F}$ | 3230 |
| $50^{\circ} \mathrm{F}$ | 3330 |

In Example 5 you could rewrite $y-206.6=-1.8(x-3)$ as $y=-1.8 x+212$. This form gives you useful information about the $y$-intercept. For instance, $212^{\circ} \mathrm{F}$ is the boiling point of water at sea level.

Here are the three forms of linear equations you have studied.

For more help with the three forms of a linear equation, see page 310.

## Summary Linear Equations

Slope-Intercept Form
$y=m x+b$
$m$ is the slope and $b$ is the $y$-intercept.

## Examples

$$
y=-\frac{2}{3} x+\frac{5}{3} \quad 2 x+3 y=5
$$

## Standard Form

$A x+B y=C$
$A$ and $B$ are not both 0 .

## Point-Slope Form

$\left(y-y_{1}\right)=m\left(x-x_{1}\right)$
$\left(x_{1}, y_{1}\right)$ lies on the graph of the equation, and $m$ is the slope.

## EXERCISES

## Practice and Problem Solving

Practice by Example
Example 1
(page 305)

Example 2 (page 305)

Graph each equation. 1-9. See back of book.

1. $y-2=(x-3)$
2. $y-2=2(x-3)$
3. $y-2=-\frac{3}{2}(x-3)$
4. $y+5=-(x-2)$
5. $y+1=\frac{2}{3}(x+4)$
6. $y-1=-3(x+2)$
7. $y+3=-2(x-1)$
8. $y-4=(x-5)$
9. $y-2=3(x+2)$

Write an equation in point-slope form for the line through the given point with the given slope. 10-18. See margin.
10. $(3,-4) ; m=6$
11. $(4,2) ; m=-\frac{5}{3}$
12. $(0,2) ; m=\frac{4}{5}$
13. $(-2,-7) ; m=-\frac{3}{2}$
14. $(4,0) ; m=1$
15. $(5,-8) ; m=-3$
16. $(-5,2) ; m=0$
17. $(1,-8) ; m=-\frac{1}{5}$
18. $(-6,1) ; m=\frac{2}{3}$

Example 3 A line passes through the given points. Write an equation for the line in point(page 305)
slope form. Then rewrite the equation in slope-intercept form. 19-30. See margin.
19. $(-1,0),(1,2)$
20. $(3,5),(0,0)$
21. $(4,-2),(9,-8)$
22. $(6,-4),(-3,5)$
23. $(-1,-5),(-7,-6)$
24. $(-3,-4),(3,-2)$
25. $(2,7),(1,-4)$
26. $(-2,6),(5,1)$
27. $(3,-8),(-2,5)$
28. $\left(1, \frac{1}{2}\right),(3,2)$
29. $\left(\frac{1}{2}, 2\right),\left(-\frac{3}{2}, 4\right)$
30. $(0.2,1.1),(7,3)$

Example 4 (page 306)
31. Yes; answers may vary. Sample: $y-9=-2(x+4)$
32. Yes; answers may vary. Sample: $y-40=3(x-5)$

Example 5
(page 306)
34. Yes; answers may vary. Sample: $y-75=10(x-10)$

B Apply Your Skills

Is the relationship shown by the data linear? If so, model the data with an equation.
31.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| ---: | ---: |
| -4 | 9 |
| 2 | -3 |
| 5 | -9 |
| 9 | -17 |


| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| ---: | :---: |
| -10 | -5 |
| -2 | 19 |
| 5 | 40 |
| 11 | 58 |

See left. 33.

| $x$ | $y$ |
| ---: | ---: |
| 3 | 1 |
| 6 | 4 |
| 9 | 13 |
| 15 | 49 |

34. 

| Speed Over Posted <br> Speed (mi/h) | Fine <br> (\$) |
| :---: | :---: |
| 10 | 75 |
| 12 | 95 |
| 15 | 125 |
| 19 | 165 |

35. 

| Volume <br> (gal) | Weight <br> (lb) |
| :---: | :---: |
| 0 | 0 |
| 2 | 16 |
| 4 | 33 |
| 6 | 50 |

Write an equation of each line in point-slope form. 36-53. Answers may vary from point indicated by the equation.

$y-2=\frac{3}{4}(x-1)$

$y+3=\frac{2}{5}(x-1)$

$y=-\frac{5}{7}(x-5)$

Write one equation of the line through the given points in point-slope form and one in standard form using integers. 39-53. See margin.
39. $(1,4),(-1,1)$
40. $(6,-3),(-2,-3)$
41. $(0,0),(-1,-2)$
42. $(0,2),(-4,2)$
43. $(-6,6),(3,3)$
44. $(2,3),(-1,5)$
45. $(5,-3),(3,4)$
46. $(2,2),(-1,7)$
47. $(-7,1),(5,-1)$
48. $(-8,4),(-4,-2)$
49. $(2,4),(-3,-6)$
50. $(5,3),(4,5)$
51. $(0,1),(-3,0)$
52. $(-2,4),(0,-5)$
53. $(6,2),(1,-1)$

A scuba diver can
descend to about
131 feet.
The sub-
mersible
Alvin can
descend to
about 2.5 miles.
In 1960, the
submersible Trieste
descended to a record depth of 6.8 miles.

54. Science At the surface of the ocean, pressure is 1 atmosphere. At 66 ft below sea level, the pressure is 3 atmospheres. The relationship of pressure and depth is linear.
a. Write an equation for the data. $y=-\frac{1}{33} x+1$
b. Predict the pressure at 100 ft below sea level. about 4 atmospheres
55. Environment Worldwide carbon monoxide emissions are decreasing about 2.6 million metric tons each year. In 1991, carbon monoxide emissions were 79 million metric tons. Use a linear equation to model the relationship between carbon monoxide emissions and time. Let $x=91$ correspond to 1991. $y=-2.6 x+315.6$
56. a. Open-Ended Write an equation in point-slope form that contains the point $(-4,-6)$. Explain your steps. a-b. See margin.
b. How many equations could you write in part(a)? Explain.
57. Critical Thinking How would the graph of $y-12=8(x-2)$ change if all of the subtraction signs were changed to addition signs? $\boldsymbol{y}$-intercept changes.
58. Reasoning Is $y-5=2(x-1)$ an equation of a line through $(4,11)$ ? Explain. Yes; the point satisfies the equation.
59. Open-Ended Write an equation in each of the following forms.
a. slope-intercept form $y=x+1$
b. standard form $-x+y=1$

59a-c. Answers may vary.
c. point-slope form $y-1=1(x-0)$

60a. Answers may vary.
Sample:
$y-332=\frac{3}{5}(x-0)$
60. Science Use the scatter plot.
a. Write an equation to model the data. See left.
b. What is the speed of sound at $15^{\circ} \mathrm{C}$ ? 341
c. Predict the speed of sound at $60^{\circ} \mathrm{C} .368$

Write an equation in slope-intercept form of each line described below.
61. The line contains the point $(-3,-5)$ and has the same slope as $y+2=7(x+3) \cdot y=7 x+16$

62. The line contains the point $(1,3)$ and has the same $y$-intercept as $y-5=2(x-1) . y=3$
63. The line contains the point $(2,-2)$ and has the same $x$-intercept as $y+9=3(x-4) \cdot y=\frac{2}{5} x-\frac{14}{5}$
64. The table shows data that you can model using a linear function.
a. Find the value of $y$ when $x=6.14 .75$
b. Find the value of $y$ when $x=120.57 .5$
c. Find the value of $x$ when $y=11 .-4$
d. Find the value of $x$ when $y=50.100$

| $x$ | $y$ |
| ---: | :--- |
| 4 | 14 |
| 8 | 15.5 |
| 12 | 17 |
| 16 | 18.5 |

## Standardized Test Prep

Gridded Response

Take lt to the NET
Online lesson quiz at www.PHSchool.com
65. What is the slope of the graph of $y-8=\frac{1}{2}(x+2) ? \frac{1}{2}$
66. Find the $y$-intercept of the line $y+3=4(x+3) .9$
67. What is the $x$-intercept of the line $y=3 x-7 ? \frac{7}{3}$
68. When $y-1=-\frac{4}{5}(x-3)$ is written in standard form using positive integers, what is the coefficient of $x$ ? 4
69. When $y=-\frac{5}{2} x+\frac{2}{3}$ is written in standard form using positive integers, what is the coefficient of $y$ ? 6

## Mixed Review

## Lesson 6-3 Graph each line. 70-75. See margin.

70. $6 x+7 y=14$
71. $-2 x+9 y=-9$
72. $5 x-4 y=24$
73. $3 x-8 y=4$
74. $5 x+18 y=6$
75. $-7 x+4 y=-21$

Lesson 5-5 Find the common difference of each sequence. Then write the next two terms.
76. $-12,-7,-2, \ldots 5 ; 3,8$
77. $\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \ldots \frac{2}{6} ; \frac{3}{2}, \frac{11}{6}$
78. $2.45,2.52,2.59, \ldots$
79. $-3.2,-3.25,-3.3, \ldots$
80. 18, 35, 52, ...17; 69, 86
81. $-7,-3,1, \ldots 4 ; 5,9$ $-0.05 ;-3.35,-3.4$

