Point-Slope Form and Writing Linear Equations

Lesson Preview

6-4



OBJECTIVE

Interactive lesson includes instant self-check, tutorials, and activities.

Using Point-Slope Form

Suppose you know that a line passes through the point (3, 4) with slope 2. You can quickly write an equation of the line using the *x*- and *y*-coordinates of the point and using the slope.

y - 4 = 2(x - 3) \swarrow \downarrow \checkmark y-coordinate slope x-coordinate

You can use the definition of slope to verify that y - 4 = 2(x - 3) is the equation of the line through the point (3, 4) with slope 2.

$\frac{y_2 - y_1}{x_2 - x_1} = m$	Use the definition of slope.
$\frac{y-4}{x-3} = 2$	Substitute (3, 4) for (x_1, y_1) , (x, y) for (x_2, y_2) , and 2 for <i>m</i> .
$\frac{y-4}{x-3}(x-3) = 2(x-3)$	Multiply each side by $x - 3$.
y-4=2(x-3)	Simplify the left side of the equation.

The equation y - 4 = 2(x - 3) is in point-slope form.



Definition Point-Slope Form of a Linear Equation

The **point-slope form** of the equation of a nonvertical line that passes through the point (x_1, y_1) with slope *m* is

 $y - y_1 = m(x - x_1)$



 $y - 3 = \frac{8}{2}(x - 2)$

Check Understanding

- **3** a. Write an equation for the line in Example 3 in point-slope form using the point (-1, -5). $y + 5 = \frac{8}{3}(x + 1)$
 - **b.** Write the equation you found in part (a) in slope-intercept form. $y = \frac{8}{3}x 2\frac{1}{3}$
 - c. What is true about the equation you wrote in part (b) and the equation in Step 3 of Example 3? They are the same.

 $y - 3 = \frac{8}{3}x - 5\frac{1}{3}$

 $y = \frac{8}{3}x - 2\frac{1}{3}$

OBJECTIVE

Writing Linear Equations Using Data

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You can write a linear equation to model data in tables. Two sets of data have a linear relationship if the rate of change between consecutive pairs of data is the same. For data that have a linear relationship, the rate of change is the slope.

EXAMPLE

Writing an Equation Using a Table

Is the relationship shown by the data linear? If so, model the data with an equation.



Step 2 Use the slope and a point to write an equation. $y - y_1 = m \left(x - x_1 \right)$

> Substitute (5, 7) for (x_1, y_1) and $\frac{1}{2}$ for *m*.

$$y - 7 = \frac{1}{2}(x - 5)$$

Check Understanding (4) Is the relationship shown by the data at the right linear? If so, model the data with an equation.

> Yes; answers may vary. Sample: $y - 5 = \frac{2}{5}(x - 19)$

X	у
-11	-7
-1	-3
4	-1
19	5





Is the relationship shown by the data linear? If so, model the data with an equation.

Boiling Point of Water

	Altitude (1000 ft)	Temperature (°F)	
-3.5	8	197.6	6.
-15	4.5	203.9	K_2
	3	206.6	K_{0}^{2}
-0.5	2.5	207.5	$\mathcal{P}^{0.}$

Step 1 Find the rates of change for consecutive ordered pairs.



$\frac{6.3}{-3.5} = -1.8 \qquad \frac{2.7}{-1.5} = -1.8 \qquad \frac{0.9}{-0.5} = -1.8$

The relationship is linear. The rate of change is -1.8 degrees Fahrenheit per 1000 ft of altitude.

Step 2 Use the slope and a point to write an equation.

$$y - y_1 = m(x - x_1)$$
 Use the point-slope form.
 $y - 206.6 = -1.8(x - 3)$ Substitute (3, 206.6) for (x₁, y₁) and -1.8 for m.

The equation y - 206.6 = -1.8(x - 3) relates altitude in thousands of feet x to the boiling point temperature in degrees Fahrenheit.

Real-World < Connection At 5280 feet above sea level it

takes 17 minutes to hard-boil an egg. This is more than 40% longer than it takes the same egg to cook at sea level.

Check Understanding (5) Is the relationship shown by the data in the table linear? If it is, model the data with an equation. Yes; answers may vary. Sample: $y - 3030 = -\frac{50}{3}(x - 68)$

Working Outdoors

Temperature	Calories Burned per Day
68°F	3030
62°F	3130
56°F	3230
50°F	3330

In Example 5 you could rewrite y - 206.6 = -1.8(x - 3) as y = -1.8x + 212. This form gives you useful information about the y-intercept. For instance, 212°F is the boiling point of water at sea level.

Here are the three forms of linear equations you have studied.

Key Concepts	Summary Line	ar Equations	
-	Slope-Intercept Form	Standard Form	Point-Slope Form
Reading Math	y = mx + b	Ax + By = C	$(y - y_1) = m(x - x_1)$
For more help with the three forms of a linear equation, see page 310.	<i>m</i> is the slope and <i>b</i> is the <i>y</i> -intercept.	A and B are not both 0.	(x_1, y_1) lies on the graph of the equation, and <i>m</i> is the slope.
	Examples		
	$y = -\frac{2}{3}x + \frac{5}{3}$	2x + 3y = 5	$y - 1 = -\frac{2}{3}(x - 1)$

EXERCISES

Practice and Problem Solving

For more practice, see Extra Practice.

Practice by Example	Graph each equation. 1-9	9. See back of book.	
Example 1	1. $y - 2 = (x - 3)$	2. $y - 2 = 2(x - 3)$	3. $y - 2 = -\frac{3}{2}(x - 3)$
(page 305)	4. $y + 5 = -(x - 2)$	5. $y + 1 = \frac{2}{3}(x + 4)$	6. $y - 1 = -3(x + 2)$
	7. $y + 3 = -2(x - 1)$	8. $y - 4 = (x - 5)$	9. $y - 2 = 3(x + 2)$
Example 2 (page 305)	Write an equation in poin given slope. 10–18. See r	t-slope form for the line thro margin.	ugh the given point with the
	10. $(3, -4); m = 6$	11. (4,2); $m = -\frac{5}{3}$	12. (0,2); $m = \frac{4}{5}$
	13. $(-2, -7); m = -\frac{3}{2}$	14. (4,0); <i>m</i> = 1	15. $(5, -8); m = -3$
	16. $(-5,2); m = 0$	17. $(1, -8); m = -\frac{1}{5}$	18. $(-6, 1); m = \frac{2}{3}$
Example 3 (page 305)	A line passes through the slope form. Then rewrite t	given points. Write an equat he equation in slope-interce	ion for the line in point- pt form. 19–30. See margin.
	19. (-1, 0), (1, 2)	20. (3, 5), (0, 0)	21. (4, -2), (9, -8)
	22. (6, -4), (-3, 5)	23. (-1, -5), (-7, -6)	24. (-3, -4), (3, -2)
	25. (2,7), (1, -4)	26. (-2, 6), (5, 1)	27. (3, -8), (-2, 5)
	28. $(1, \frac{1}{2}), (3, 2)$	29. $\left(\frac{1}{2}, 2\right), \left(-\frac{3}{2}, 4\right)$	30. (0.2, 1.1), (7, 3)

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Example 4 (page 306)

- 31. Yes; answers may vary. Sample: y - 9 = -2(x + 4)
- 32. Yes; answers may vary. Sample: y - 40 = 3(x - 5)
 - Example 5 (page 306)

34.

34. Yes; answers may vary. Sample: y - 75 = 10(x - 10) Is the relationship shown by the data linear? If so, model the data with an equation.

31.	х	y	See left.
	-4	9	
	2	-3	
	5	-9	
	9	-17	

22		
32.	X	у
	-10	-5
	-2	19
	5	40
	11	58

See left. 33.	x	у
	3	1
	6	4
	9	13
	15	49

no

no

eed Over Posted Speed (mi/h)	Fine (\$)	35. See left.	Volume (gal)	Weight (lb)
10	75		0	0
12	95		2	16
15	125		4	33
19	165		6	50

Apply Your Skills

Write an equation of each line in point-slope form. 36–53. Answers may vary



Write one equation of the line through the given points in point-slope form and one in standard form using integers. 39–53. See margin.

39. (1,4), (-1,1)	40. (6, -3), (-2, -3)	41. (0,0), (-1, -2)
42. (0, 2), (-4, 2)	43. (-6, 6), (3, 3)	44. (2, 3), (-1, 5)
45. (5, -3), (3, 4)	46. (2, 2), (-1, 7)	47. (-7, 1), (5, -1)
48. (-8, 4), (-4, -2)	49. (2, 4), (-3, -6)	50. (5, 3), (4, 5)
51. (0,1), (-3,0)	52. (-2, 4), (0, -5)	53. (6, 2), (1, -1)

- **54.** Science At the surface of the ocean, pressure is 1 atmosphere. At 66 ft below sea level, the pressure is 3 atmospheres. The relationship of pressure and depth is linear.
 - **a.** Write an equation for the data. $y = -\frac{1}{33}x + 1$
 - b. Predict the pressure at 100 ft below sea level. about 4 atmospheres
- **55.** Environment Worldwide carbon monoxide emissions are decreasing about 2.6 million metric tons each year. In 1991, carbon monoxide emissions were 79 million metric tons. Use a linear equation to model the relationship between carbon monoxide emissions and time. Let x = 91 correspond to 1991.
 - y = -2.6x + 315.6
 56. a. Open-Ended Write an equation in point-slope form that contains the point (-4, -6). Explain your steps. a-b. See margin.
 - **b.** How many equations could you write in part(a)? Explain.
 - **57. Critical Thinking** How would the graph of y 12 = 8(x 2) change if all of the subtraction signs were changed to addition signs? *y*-intercept changes.
 - **58.** Reasoning Is y 5 = 2(x 1) an equation of a line through (4, 11)? Explain. Yes; the point satisfies the equation.



59. Open-Ended Write an equation in each of the following forms.

a. slope-intercept form y = x + 1

59a-c. Answers may vary.

- **b.** standard form -x + y = 1
- c. point-slope form y 1 = 1(x 0)

Sample is given.

60a. Answers may vary. Sample: $y - 332 = \frac{3}{5}(x - 0)$



- **60. Science** Use the scatter plot.
 - **a.** Write an equation to model the data. See left.
 - **b.** What is the speed of sound at 15°C? **341**
 - c. Predict the speed of sound at 60°C. 368

Write an equation in slope-intercept form of each line described below.

- **61.** The line contains the point (-3, -5) and has the same slope as y + 2 = 7(x + 3). y = 7x + 16
- **62.** The line contains the point (1, 3) and has the same y-intercept as y - 5 = 2(x - 1). y = 3
- 63. The line contains the point (2, -2) and has the same x-intercept as y + 9 = 3(x 4). $y = \frac{2}{5}x \frac{14}{5}$
- 64. The table shows data that you can model using a linear function.
 - **a.** Find the value of y when x = 6.14.75
 - **b.** Find the value of y when x = 120. **57.5**
 - **c.** Find the value of x when y = 11. -4
 - **d.** Find the value of x when y = 50. **100**

	on Speed of Sound									
	250	۱ 						-		
(m/s)	350									
peed	340		•							
SI	330	2							,	
6	0		10 Ter	2 npe	0 rat	3 ure	0 e (°	4 C)	0	

X	У		
4	14		
8	15.5		
12	17		
16	18.5		

Standardized Test Prep

Gridded Response	65. What is the slope of the graph of $y - 8 = \frac{1}{2}(x + 2)$? $\frac{1}{2}$
	66. Find the <i>y</i> -intercept of the line $y + 3 = 4(x + 3)$. 9
	67. What is the <i>x</i> -intercept of the line $y = 3x - 7? \frac{7}{3}$
	68. When $y - 1 = -\frac{4}{5}(x - 3)$ is written in standard form using positive integers, what is the coefficient of x? 4
	69. When $y = -\frac{5}{2}x + \frac{2}{3}$ is written in standard form using positive integers, what is the coefficient of y? 6

Mixed Review

Lesson 6-3	Graph each line. 70–75. See margin.			
	70. $6x + 7y = 14$	71. $-2x + 9y = -9$	72. $5x - 4y = 24$	
	73. $3x - 8y = 4$	74. $5x + 18y = 6$	75. $-7x + 4y = -21$	
Lesson 5-5	Find the common difference of each sequence. Then write the next two terms.			
	76. -12, -7, -2, 5; 3, 8	77. $\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \dots, \frac{2}{6}; \frac{3}{2}, \frac{11}{6}$	78. 2.45, 2.52, 2.59,	
	79. -3.2, -3.25, -3.3, -0.05; -3.35, -3.4	80. 18, 35, 52, 17; 69, 86	81. -7, -3, 1, 4; 5, 9	

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