

# Activity:

## Cooking the Thanksgiving Day Turkey

### Objectives:

- To find an equation using Newton's Law of Heating
- To find an exponential function
- To find the x value of a function, given the y value
- To find the y value of a function, given the x value
- To use technology to plot data

### Materials:

- TI-83 Plus family/TI-84 Plus family
- Data from cooking Turkey
- TI Interactive
- TI Navigator (optional)

### Introduction

Temperature variations for such cooling objects were summarized by Newton. He stated that the rate at which a warm body cools is approximately proportional to the temperature difference between the temperature of the warm object and the temperature of its surroundings. Stated mathematically:

$$T(t) = T_{\alpha} + (T_0 - T_{\alpha})e^{-kt}$$

where  $T(t)$  represents the object's temperature over time,  $t$  represents time,  $T_{\alpha}$  is the surrounding temperature,  $T_0 - T_{\alpha}$  is the initial temperature difference at  $t = 0$ , and  $k$  is a proportionality constant. Newton's Law of Heating is a corresponding principle which applies if an object is being warmed rather than cooled. The same formulas apply except the constant of proportionality is positive in the warming case.

$$T(t) = T_{\alpha} + (T_0 - T_{\alpha})e^{kt}$$

To explore Newton's Law of Heating we observe the temperature of a Turkey every hour as it is "heated" or cooked in the oven.

## Problem:

How can you use *Newton's Law of Heating* to find how long to cook objects that are impractical to measure using conventional techniques? Can you use this principal to find the time to cook a 40° F turkey in an oven kept at a constant temperature of 350° F, when you know after ten minutes the turkey's temperature is 65° F?

## Collecting the data

1. On Thanksgiving Day, students need to write down the temperature of the oven that they will be cooking the turkey at.
2. Starting at the time when the turkey is put in the oven ( $t = 0$ ) write down the temperature of the turkey every hour until the time that the turkey is taken out of the oven.

For example: Thanksgiving Day Turkey Data

Oven Temperature: 325 degrees F

Time	Temperature of Turkey
0	40
1	105
2	128
3	155
4	170

## Setting up the TI-83/TI-84

Before starting your data collection, make sure that the TI-83/TI-84 has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared.

## Entering the data in the TI-83/TI-84

1. Press **STAT** and select **1: Edit** by pressing **ENTER**.
2. Enter the data for the times that you took the temperature of the turkey in **L1**.
3. Enter the data for the temperature of the turkey in **L2**.
4. In **L3** arrow up to **L3** and type the temperature of the oven subtract **L2**.

```

2001 CALC TESTS
1:Edit
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor
  
```

L1	L2	L3	1
0	40	-----	
1	105		
2	128		
3	155		
4	170		
-----	-----		

L1(6)=

L1	L2	L3	3
0	40	-----	
1	105		
2	128		
3	155		
4	170		
-----	-----		

L3 = 325 - L2

L1	L2	L3	3
0	40	285	
1	105	220	
2	128	197	
3	155	170	
4	170	155	
-----	-----	-----	

L3(1)=285

## Setting up the window

1. Press **WINDOW** to set up the proper scale for the axes.
2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.
3. Set the **Xmax** value by identifying the maximum value in **L1**. Choose a number that is more than the maximum. Set the **Xscl** to 0.5.
4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.
5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is more than the maximum. Set the **Yscl** to 20.

```
WINDOW
Xmin=-1
Xmax=5
Xscl=.5
Ymin=-10
Ymax=180
Yscl=20
Xres=1
```

## Graphing the data: Setting up a scatter plot

Plot a scatter plot using the data in the table on the **Data collection and Analysis** page.

1. Press **2nd**[STAT PLOT] and select **1: Plot1** by pressing **ENTER**.

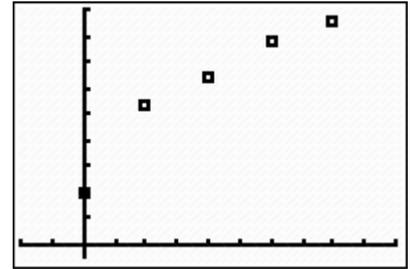
```
STAT PLOTS
1:Plot1...Off
   L4 1
2:Plot2...Off
   L1 L2
3:Plot3...Off
   L1 L2
4↓PlotsOff
```

2. Set up the plot as shown by pressing **ENTER**  
**▼**[ENTER]**▼**[2nd][L1]**ENTER**[2nd][L2]**ENTER**[ENTER]

NOTE: Press **▼****▼** if **L1** and **L2** are already displayed.

```
1:Plot1 Plot2 Plot3
Off Off Off
Type: [ ] [ ] [ ]
      [ ] [ ] [ ]
Xlist:L1
Ylist:L2
Mark: [ ] + .
```

3. Press **GRAPH** to see the plot.



4. Press **TRACE**  $\blacktriangleright$   $\blacktriangleright$   $\blacktriangleright$  to see times and temperatures.

5. Find a function to model the data that you plotted. (You might not want to include  $t = 0$ .)

Answer questions 1 through 4 on the **Data Collection and Analysis** page.

### Setting up the window

1. Press **WINDOW** to set up the proper scale for the axes.

2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.

3. Set the **Xmax** value by identifying the maximum value in **L1**. Choose a number that is more than the maximum. Set the **Xscl** to 0.5.

4. Set the **Ymin** value by identifying the minimum value in **L3**. Choose a number that is less than the minimum.

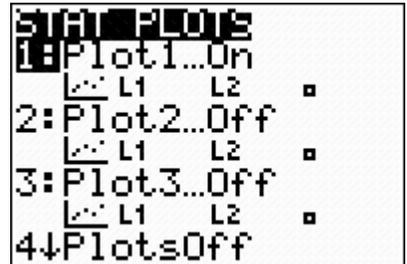
5. Set the **Ymax** value by identifying the maximum value in **L3**. Choose a number that is more than the maximum. Set the **Yscl** to 20.

```
WINDOW
Xmin=-1
Xmax=5
Xscl=.5
Ymin=-10
Ymax=300
Yscl=20
Xres=1
```

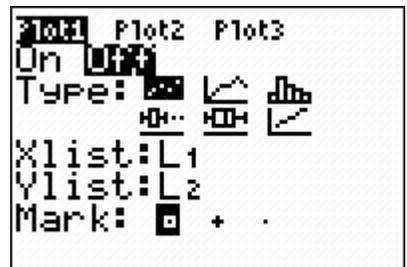
## Graphing the second set of data: Setting up a second scatter plot

Plot a scatter plot using the data in the table on the **Data collection and Analysis** page.

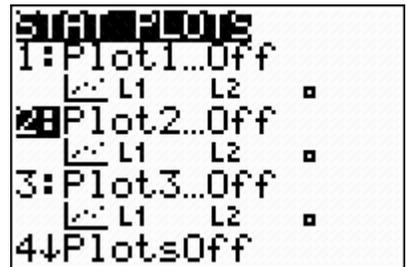
6. Press  $\boxed{2\text{nd}}\boxed{\text{STAT PLOT}}$  and select **1: Plot 1** by pressing  $\boxed{\text{ENTER}}$ .



7. Turn Plot 1 OFF by pressing  $\boxed{\blacktriangleright}\boxed{\text{ENTER}}$ .

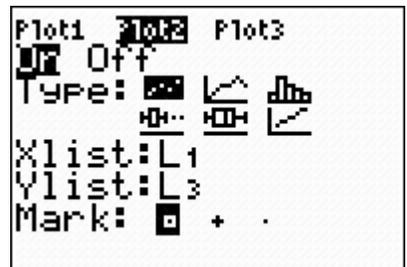


8. Press  $\boxed{2\text{nd}}\boxed{\text{STAT PLOT}}$  and select **2: Plot 2** by pressing  $\boxed{\blacktriangledown}\boxed{\text{ENTER}}$ .



9. Set up the plot as shown by pressing  $\boxed{\text{ENTER}}\boxed{\blacktriangledown}\boxed{\text{ENTER}}\boxed{\blacktriangledown}\boxed{2\text{nd}}\boxed{\text{L1}}\boxed{\text{ENTER}}\boxed{2\text{nd}}\boxed{\text{L3}}\boxed{\text{ENTER}}\boxed{\text{ENTER}}$

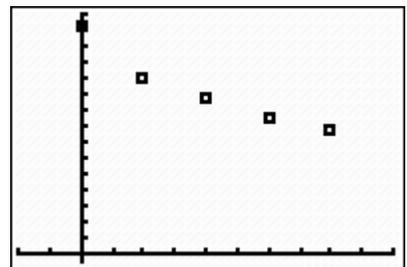
NOTE: Press  $\boxed{\blacktriangledown}\boxed{\blacktriangledown}$  if L1 and L3 are already displayed.



10. Press  $\boxed{\text{GRAPH}}$  to see the plot.

11. Press  $\boxed{\text{TRACE}}\boxed{\blacktriangleright}\boxed{\blacktriangleright}\boxed{\blacktriangleright}$  to see times and temperature difference between the turkey and the oven.

12. Find a function to model the data that you plotted.



Answer question 5 on the **Data Collection and Analysis** page.

## Analyzing the data: Finding a best fit line

For Newton's Law of Heating we do not have a line of best fit regression key. However, we do have a key for finding the line of best fit for an exponential regression. If we take Newton's Law of Heating and subtract the temperature of the oven from the temperatures of the turkey (the data collected in **L3**) the equation we are working with and the data we are working with becomes an exponential function.

$$T(t) = T_{\alpha} + (T_0 - T_{\alpha})e^{kt}$$

$$T_{\alpha} - T(t) = -(T_0 - T_{\alpha})e^{kt}$$

The line of best fit that models the data collected in **L1** vs. **L3** is an exponential regression (ExpReg). This model works well with the data starting at  $t=0$  (make sure to convert L1 back to its original time values).

Find the exponential regression for the data.

1. Press  $\boxed{2nd}\boxed{QUIT}\boxed{CLEAR}$  to return to the Home screen.
2. Press  $\boxed{STAT}$  and move the cursor to the **CALC** menu.

```
EDIT [2nd][QUIT] TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
```

3. Press the up arrow  $\boxed{\uparrow}$  or the down arrow  $\boxed{\downarrow}$  to select **0: ExpReg** and press  $\boxed{ENTER}$ .

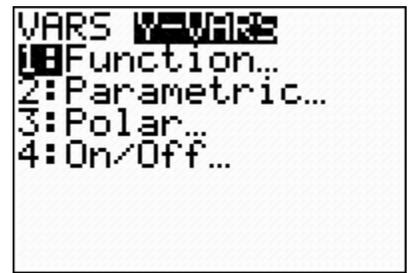
```
EDIT [2nd][QUIT] TESTS
7↑QuartReg
8:LinReg(a+bx)
9:LnReg
0:ExpReg
A:PwrReg
B:Logistic
C:SinReg
```

**NOTE:** Place the function in **Y4** so that you do not erase the function in **Y1, Y2, and Y3**.

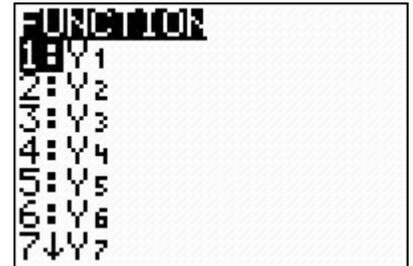
4. Press  $\boxed{2nd}\boxed{[L1]}\boxed{,}\boxed{2nd}\boxed{[L3]}\boxed{,}$ .

```
ExpReg L1,L3,
```

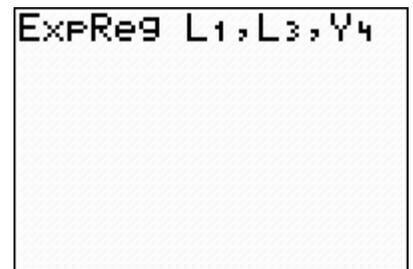
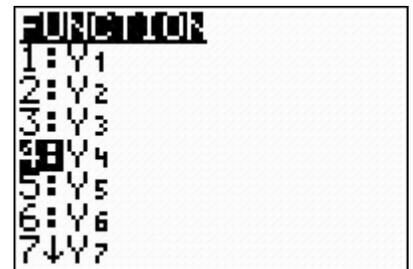
5. Press **[VARS]** and move the cursor to the **Y-VARS** menu.



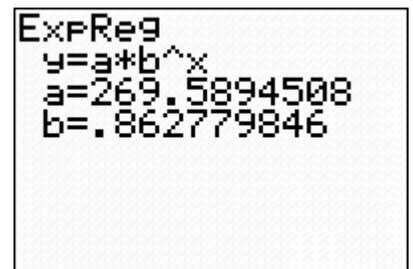
6. Select **1:Function** by pressing **[ENTER]**.



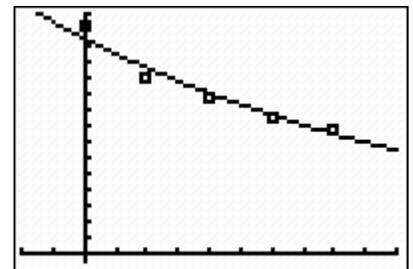
7. Select **4:Y<sub>4</sub>** and press **[ENTER]**.



8. Press **[ENTER]** to calculate the exponential regression and the function will be pasted in **Y<sub>4</sub>**.



9. Press **[GRAPH]** to see the exponential regression.



Answer question 6 through 10 on the **Data Collection and Analysis** page.

## Data Collection and Analysis

Name: \_\_\_\_\_

Date: \_\_\_\_\_

### Activity: Cooking the Thanksgiving Day Turkey

#### Collecting the data

Record your data in the table below.

Time (hours)	Temperature of Turkey (Fahrenheit)	Temperature difference between Turkey and Oven (Fahrenheit)
0		
1		
2		
3		
4		

#### Analyze the data

Use the equation you find in number 1 to answer questions 2 through 4.

1. Find an equation for the data in **L1** and **L2** using Newton's Law of Heating

$$y = \underline{\hspace{10em}}$$

Enter your equation in **Y<sub>1</sub>** in the TI-83 Plus.

2. What is the temperature of the turkey after one and a half hours? \_\_\_\_\_
3. How long has the turkey been in the oven when the temperature of the turkey is 100°F? \_\_\_\_\_
4. A turkey should be cooked until it reaches a temperature of 180°F. At what time should the turkey be taken out of the oven? \_\_\_\_\_
5. Find an equation for the data in **L1** and **L3**

$$y = \underline{\hspace{10em}}$$

6. Enter the equation from your calculator for the exponential regression  
 $y =$  \_\_\_\_\_.

7. Is this equation close to the equation you came up with? \_\_\_\_\_

What could you have done to come up with a more accurate equation?

\_\_\_\_\_

8. Knowing how we changed the data and equation to an exponential equation from Newton's Law of Heating, use your exponential equation from question 6 to write an equation for the data of time vs. temperature of the turkey (**L1** vs. **L2**).

$y =$  \_\_\_\_\_

9. How does this equation compare to your equation in question 1?

\_\_\_\_\_

10. Explain any discrepancies between your data points (**L1** vs. **L2**) and your equation in question 9.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_