Derivatives of Trigonometric Functions

ID: 9290

Time required
45 minutes

Activity Overview

In this activity, students will use the graph of the sine function to estimate the graph of the cosine function. They will do this by inspecting the slope of a tangent to the graph of the sine function at several points and using this information to construct a scatterplot for the derivative of the sine. Students will then use this result to find the derivative of the cosine function. Finally, students will write the tangent function in terms of the sine and cosine and use the quotient rule to determine its derivative.

Topic: Formal Differentiation

- *Use **Limit** to show that* $\lim\limits_{h \to 0} \frac{\sin(a+h)-\sin(a)}{h} = \cos(a)$ *and verify the rule for differentiating f(x)=sin(x).*
- *Use **Limit** to show that* $\lim\limits_{h \to 0} \frac{\cos(a+h)-\cos(a)}{h} = -\sin(a)$ *and verify the rule for differentiating f(x)=cos(x).*
- *Use **Derivative** to verify the Generalized Rules for differentiating the sine and cosine functions.*

Teacher Preparation and Notes

- *Students should have some experience taking a graph and sketching its derivative.*
- *Students should know the definition of the derivative of a function,* $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ *and be familiar with the quotient rule.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- *As an extension, students can find the derivatives of the cosecant, secant and cosecant functions by writing them in terms of sine and cosine and then using the quotient (or reciprocal) rule. Their results can be checked using the **derivative** command.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "9290" in the keyword search box.***

Associated Materials

- *DerivativeTrig_Student.doc*
- *DerivativeTrig.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Derivatives with Piece-Wise Defined Functions (TI-Nspire technology) — 9269*
- *Derivative Function (TI-Nspire technology) — 16071*
- *Derivatives in Flight (TI-Nspire CAS technology) — 13367*
- *Sign of the Derivative (TI-Nspire technology) — 16084*

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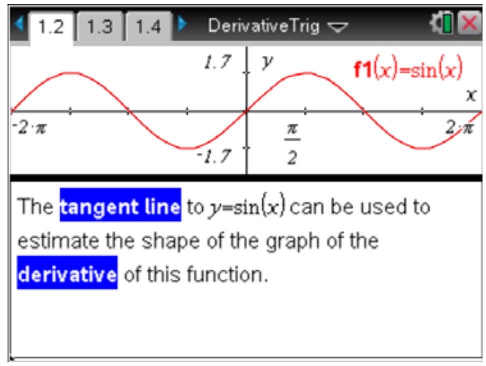
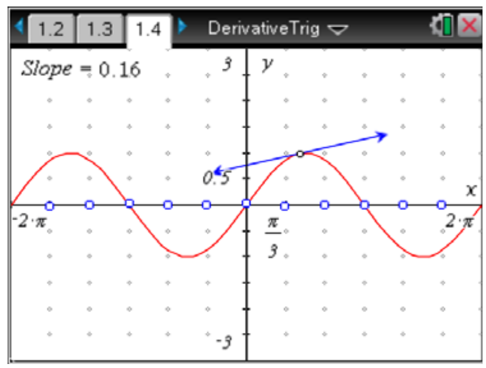
Introduction



One focus question defines this activity: *How can you use the graph of the sine function to determine the derivative of the cosine function?*

Use page 1.2 as a discussion point for this question. Explain to students that they will be graphing a scatterplot that will represent the derivative of the $y = \sin(x)$. Students will use the slope of the tangent to the graph of $y = \sin(x)$ as the y-coordinate of each point on the scatterplot. The x-coordinate will be the same as the x-coordinate of the point of tangency.

Problem 1 – Finding the derivative of the sine function

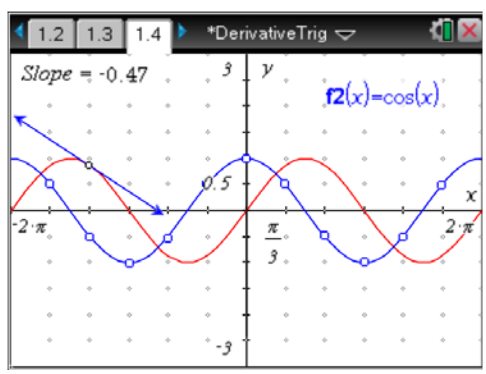


Students will drag the tangent line along the graph so that it is directly above or below each of the points on the x-axis. They will then move the corresponding x-axis point to a location such that the y-coordinate is approximately equal to the slope of the tangent. In the diagram at right, it can be seen that the third point on the x-axis will need to be moved such that it's y-coordinate is approximately equal to -0.6.

Note: The points on the x-axis have been configured such that they can only be moved vertically, not horizontally.

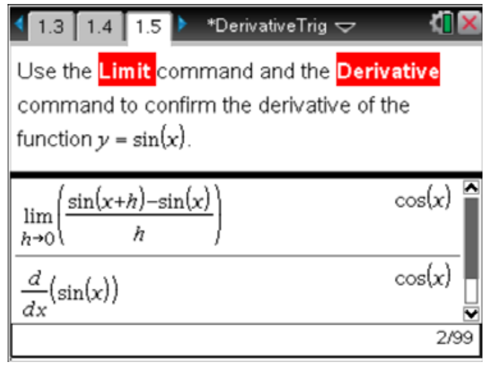
TI-Nspire Navigator Opportunity: *Live Presenter, Screen Capture, Quick Poll*

See Notes 1, 2 and 3 at the end of this lesson.



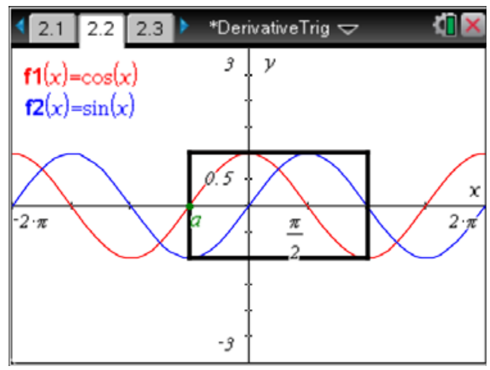
After completing the scatter plot, students should recognize that the derivative of $y = \sin(x)$ appears to be $y = \cos(x)$. The screen at right shows that the graph of $y = \cos(x)$ does in fact match the scatter plot very well.

Students will set up a limit on page 1.5, based on the definition of the derivative, to check their result. They can also use the **Derivative** command to verify that $\frac{d}{dx}(\sin(x)) = \cos(x)$.



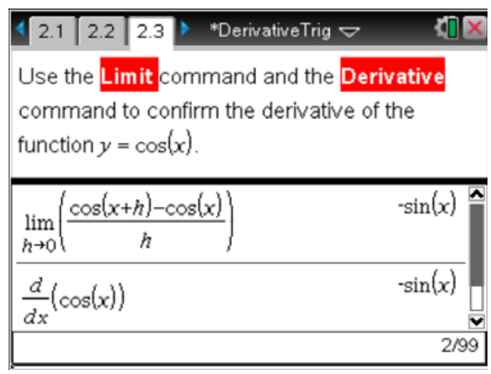
Problem 2 – Finding the derivative of the cosine function

Students should recognize that the one cycle shown in the box at right is one cycle of the sine function, shifted left $\frac{\pi}{2}$ units. Using what they learned in Problem 1, students should realize that the corresponding derivative should be one cycle of the cosine function. A complete sketch of the graph of the derivative reveals a sinusoidal graph that can be modeled with a number of equivalent functions.



Written strictly in terms of the sine function, the derivative can be expressed as $y = -\sin(x)$.

Students will confirm that $\frac{d}{dx}(\cos(x)) = -\sin(x)$ by using the **Limit** command and **Derivative** command as shown on the screen at right.



Problem 3 – Finding the derivative of the tangent function

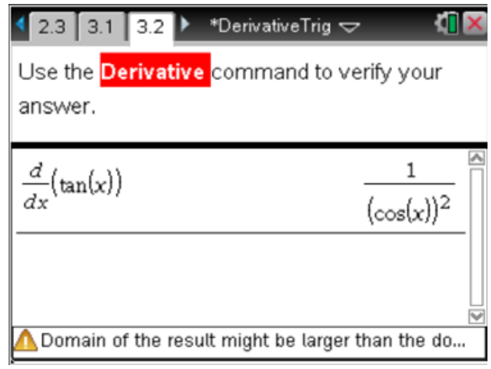
Students will write $\tan(x)=\dfrac{\sin(x)}{\cos(x)}$ and use the quotient rule to find the derivative of the tangent:

$$\frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right)=\frac{\cos(x)\cdot\frac{d}{dx}(\sin(x))-\sin(x)\cdot\frac{d}{dx}(\cos(x))}{\cos^2(x)}$$

$$=\frac{\cos(x)\cdot\cos(x)-\sin(x)\cdot(-\sin(x))}{\cos^2(x)}$$

$$=\frac{\cos^2(x)+\sin^2(x)}{(\cos(x))^2}$$

$$=\frac{1}{\cos^2(x)}$$



Students will confirm this result using the **Derivative** command as shown. Written in terms of the reciprocal function, the derivative of the tangent is $\frac{d}{dx}(\tan(x))=\sec^2(x)$.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Live Presenter*

On page 1.4, *Live Presenter* is an excellent way to demonstrate to students how to drag the tangent line as well as the plotted points. This is also an excellent opportunity to discuss locations where the derivative (slope of the tangent line) is zero, positive, negative, etc.

Note 2
Problem 1, *Screen Capture*

On page 1.4, use *Screen Capture* to monitor student progress as they drag the points vertically. This provides the teacher an opportunity to correct students who are having trouble.

Note 3
Problem 1, *Quick Poll*

On page 1.4, send a *Quick Poll* to students asking them what the graph of the plotted points formed when they were vertically dragged to the value of the derivative.