Square Root Spiral and Function Graphs
Teacher Notes

Overview

In this activity, students will investigate the:

- spiral formed by square roots of consecutive numbers
- numerical approximations for square roots
- plot of the square root spiral arm lengths
- graph of the square root function

Supplies/Materials

TI-Nspire or TI-Nspire CAS handheld devices

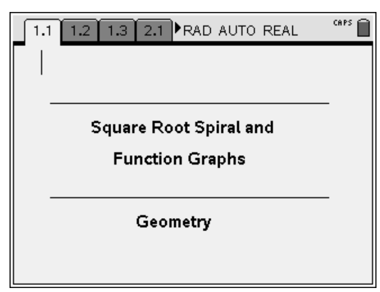
Pre-requisite Knowledge and Curricular Placement

Students should have a basic understanding of the Pythagorean Theorem and radicals. Possible curricular placements include middle school, informal and regular geometry in high school, or geometry courses for pre-service or in-service teachers at the college level.

Pedagogical Suggestions

This activity is designed to be student centered and requires minimal student experience with a TI-Nspire handheld device. Having students record their findings on the Student Worksheet will encourage engagement, retention, and mathematical communication. Anticipated timing for the activity is one class period of 50 minutes.

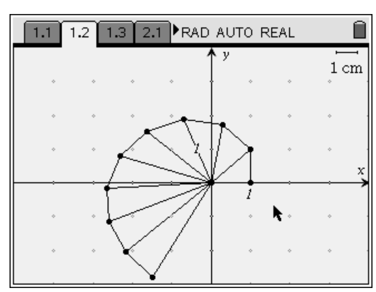
Instructions



Open the file **Sqrt Spiral Function_EN.tns** on your TI-Nspire™ handheld device and work through the activity. Use this document as a guide to the activity and to record your answers.

Problem 1 – Investigating the Square Root Spiral

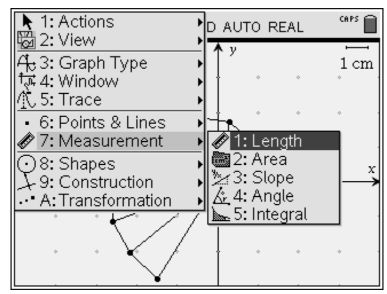
Advance to Page 1.2 by pressing ctrl and the right side of the NavPad. Examine the right triangles that form what looks like a spiral.



1. Based on the grid, what are the lengths of the outside leg and the base leg of the smallest right triangle? Record your answers in the table below on the Triangle Number 2 row. (Note that Triangle Number 1 is not seen since its outside leg length is 0.)

Triangle Number	**Outside Leg**	**Base Leg**	**Arm Hypotenuse Radical Form**	**Arm Hypotenuse Decimal Form**
1	0	1	$\sqrt{1}$	1.000
2	1	1	$\sqrt{2}$	1.414
3	1	$\sqrt{2}$	$\sqrt{3}$	1.732
4	1	$\sqrt{3}$	$\sqrt{4}$	2.000
5	1	$\sqrt{4}$	$\sqrt{5}$	2.236
6	1	$\sqrt{5}$	$\sqrt{6}$	2.449
7	1	$\sqrt{6}$	$\sqrt{7}$	2.646
8	1	$\sqrt{7}$	$\sqrt{8}$	2.828
9	1	$\sqrt{8}$	$\sqrt{9}$	3.000
10	1	$\sqrt{9}$	$\sqrt{10}$	3.162

2. Use the Measurement tool to find the lengths of the segments (legs) around the outside of the spiral. Record your answers in the Outside Leg column in the table above.

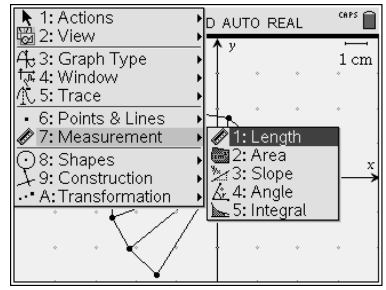


3. Using the Pythagorean Theorem, find the length in radical form of the spiral arm of the smallest right triangle (Triangle 2 in the table) and record the answer in the table above.

4. Examine the next larger triangle (Triangle 3 in the table) of the spiral. Explain what you observe about its base leg and one side of the smallest triangle (Triangle 2 in the table) in the spiral.
The hypotenuse of the smallest right triangle is the base leg of the next larger triangle.

5. Using the Pythagorean Theorem, find the length in radical form of the spiral arm of each of the other right triangles and record your answers in the table.

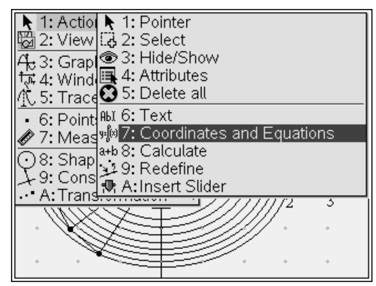
6. For each of the right triangles forming the spiral, use the Measurement tool to find the length of each spiral arm (hypotenuse) to the nearest thousandth. Record the results in the table.



Advance to Page 1.3.

7. Explain how the radius of each concentric circle is related to the length of each spiral arm.
   The length of the spiral arm (hypotenuse) of each right triangle is equal to the length of the radius of the circle that passes through the spiral arm endpoint and the x-axis.

8. Select menu, 1: Actions, 7: Coordinates and Equations, and examine the points on the positive x-axis. Explain how the x-coordinate of each concentric circle radius compares with the measurements in decimal form you recorded in the table.
   They are equal.



Problem 2 – Investigating the Graph of the Square Root Function
Advance to Page 2.1 by pressing ctrl and the right side of the NavPad.

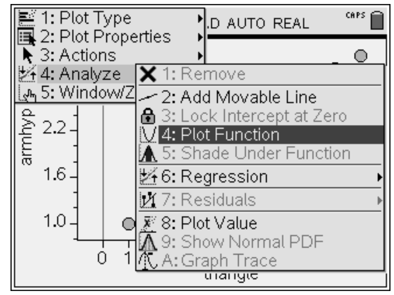
9. Enter your measurements of the outside leg, base leg (in radical form), and spiral arm (in radical form) in the lists on Page 2.1. The first two entries have been done for you.

Advance to Page 2.2. We now consider how the lengths of the spiral arms are related to the triangle number.

10. Examine the plot of the triangle number (triangle) and the spiral arm length (armhyp) that resulted from the data you entered. Explain whether the triangle-armhyp plot would be described as increasing or decreasing.
    The triangle-armhyp plot is increasing since the y-values increase as the x-values increase.

Advance to 2.3 and examine another plot of the triangle-armhyp data that does not show the square root spiral.

11. Decide on a likely function that fits the data and record it below.
    f(x) = $\sqrt{x}$



12. Select menu, 4: Analyze, and 4: Plot Function. Enter your likely function. How well does your likely function fit the plot? (Repeat with other functions if a good fit is not obtained.)
    The square root function fits perfectly.

13. Explain whether the shape of your best-fit function would be described as having a constant or variable slope. Justify your answer.
    The square root function has a variable slope since the slope between several pairs of points along the curve changes, unlike the constant slope of a linear function.

14. Explain how the square root spiral and the concentric circles constructed with radius equal to the spiral arm length could help you predict the shape of the square root function.
    As consecutive square roots are created from the spiral arm lengths, the radii of the concentric circles increases very slowly, which indicates that the slope of the square root function is very gradually increasing.