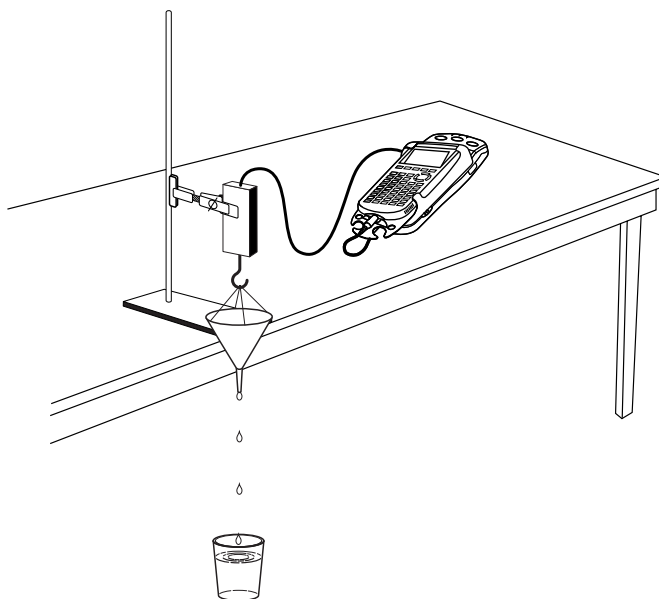


Funnel Volumes: Volume and Weight

Water drains from an open funnel. How is the volume of water changing with time? At what rate is the water level decreasing? How long will it take for the funnel to drain completely? These are a few questions that can be investigated by collecting weight data for a draining funnel, and then developing a mathematical model to describe the data.

In this activity, you will use a force sensor to measure the weight of a water-filled funnel as it drains. The weight versus time data can be graphed and analyzed using the calculator's statistical features.



OBJECTIVES

- Record weight versus time data for a draining funnel.
- Describe the data using concepts of intercept and slope of a linear function.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator
EasyData application
data-collection interface
Vernier Force Sensor

funnel
string
bucket
water

PROCEDURE

1. Make two small holes on opposite sides of the funnel near the top rim. Thread a piece of string through the holes, and then tie the ends of the string together to create a handle as shown in the drawing.
2. Place a cup or bucket on the floor to catch water as it drains from the funnel.
3. Block the funnel hole with your finger and fill the funnel completely with water. How long will it take for the funnel to drain? Remove your finger from the funnel hole and estimate the time required for the funnel to empty. Make a note of the time in seconds; you will use this value later.
4. Prepare the Force Sensor for data collection.
 - a. Turn on your calculator.
 - b. Set the Force Sensor range switch to 10 N.
 - c. Connect the Force Sensor, data-collection interface, and calculator.
5. Set up EasyData for data collection.
 - a. Start the EasyData application, if it is not already running.
 - b. Select **[File]** from the Main screen, and then select **New** to reset the application.
6. In this activity you only want to measure the weight of the water in the funnel. To do this, you need to reset the force scale so that the Force Sensor reads zero when supporting the weight of the empty funnel and string.
 - a. Hold the sensor so that the funnel can hang downward from the hook. Place the bucket under the funnel. Allow the funnel to stop swinging.
 - b. Select **[Setup]** from the Main screen, and then select **Zero...**
 - c. Ensure that the funnel is not swinging and select **[Zero]** to zero the Force Sensor.
7. If the drain time is less than three seconds, skip to Step 8. If the estimated time to drain the funnel is more than three seconds, adjust the data collection time.
 - a. Select **[Setup]** from the Main screen, then select **Time Graph...**
 - b. Select **[Edit]** on the Time Graph Settings screen.
 - c. Enter **0.1** as the time between samples in seconds if your funnel's emptying time is less than ten seconds. Use **0.2** if it is more than ten seconds.
 - d. Select **[Next]**.
 - e. Enter **100** as the number of samples and select **[Next]**.
 - f. Select **[OK]** to return to the Main screen.
8. You are now ready to collect weight versus time data.
 - a. Block the funnel hole with a finger, and fill the funnel completely with water.
 - b. Select **[Start]** to begin data collection.
 - c. When the first data point appears on the screen, remove your finger from the funnel and allow it to empty. Take care to not touch the funnel while it is emptying, and make sure that it does not swing.
 - d. After data collection is complete you will see a graph of the weight of the water in the funnel versus time.

9. Your graph should be a uniformly decreasing function, with a horizontal section at the end. If you are satisfied with your data, proceed to step 10. If you need to repeat data collection, select **(Main)** to return to the Main screen, and repeat Step 8.
10. Since data collection probably did not exactly coincide with the emptying of the funnel, you need to remove the data taken before you released the water and after the funnel was empty.
 - a. Select **(Anlyz)**, and then select **Select Region...** from the menu.
 - b. A warning will be displayed on the screen; select **(OK)** to begin the region selection process.
 - c. Use the **(D)** key to move the cursor to the left edge of the smoothly decreasing portion of the graph. (You may not need to move the cursor at all if data collection started after the water started to flow.) Select **(OK)** to mark this position.
 - d. Use the **(D)** and **(L)** keys to move the cursor to the right edge of the smoothly decreasing portion of the graph. Select **(OK)** to select the region of data between the left and right bounds.
 - e. The calculator will remove data outside the region you just marked. A new graph will be displayed showing only the linearly decreasing weight of the water remaining in the funnel as a function of time.
11. For further analysis, exit EasyData by selecting **(Main)**, selecting **(Quit)**, and then selecting **(OK)**.

ANALYSIS

1. Redisplay the graph and prepare to trace values from it.
 - a. Press **(2nd)** [STAT PLOT].
 - b. Press **(ENTER)** to select Plot1 and press **(ENTER)** again to select On.
 - c. Press **(ZOOM)**.
 - d. Press **(V)** until ZoomStat is highlighted; press **(ENTER)** to display a graph with the x and y ranges set to fill the screen with data.
 - e. Press **(TRACE)** to determine the coordinates of a point on the graph using the cursor keys.
2. Trace across the graph to choose two well-separated points on the graph. Round the x and y coordinates to 0.01, and record the values in the Data Table on the *Data Collection and Analysis* sheet.
3. Use the two points to find the slope of the weight versus time graph. Record the slope, with units, in the Data Table.

⇒ Answer Question 1 on the *Data Collection and Analysis* sheet.
4. You still need to find the y -intercept of this segment, but you cannot just trace to the y axis because the line segment does not necessarily go that far. However, using the definition of slope we can write the point slope form of a line, or $y - y_1 = m(x - x_1)$. Here y and x are variables, m the slope, and x_1 and y_1 are the values of a point on the line. Record the y -intercept in the Data Table.

⇒ Answer Question 2 on the *Data Collection and Analysis* sheet.

Activity 4

5. Next, plot the line whose equation you just determined. You can plot the line on the same graph as your data.
 - a. Press Y= .
 - b. Press CLEAR to remove any existing equation.
 - c. Enter the equation for the line you determined in the Y_1 field. For example, if your expression is $y = 4x + 3$, enter $4*x+3$.
 - d. Press 2nd until the icon to the left of Y_1 is blinking. Press ENTER until a bold diagonal line is shown in order to display your model with a thick line.
 - e. Press GRAPH to see your data with the model graph superimposed.

⇒ Answer Question 3 on the *Data Collection and Analysis* sheet.

6. Determine the x intercept for the linear equation you have found. Record the value, including units, in the Data Table.

⇒ Answer Question 4 on the *Data Collection and Analysis* sheet.

7. You can use the calculator to do a linear least-squares fit to the data. Instead of using only two points to determine the slope, the calculator can automatically determine the slope and intercept for the line that bests fits your data.

- a. Press STAT and use the cursor keys to highlight CALC.
- b. Press the number adjacent to LinReg(ax+b) to copy the command to the home screen.
- c. Press 2nd [L1] , 2nd [L2] , to enter the lists containing your data.
- d. Press VARS and use the cursor keys to highlight Y-VARS.
- e. Select Function by pressing ENTER .
- f. Press ENTER to copy Y_1 to the expression.

On the home screen, you will now see the entry LinReg(ax+b) L1, L2, Y1. This command will perform a linear regression with L1 as the x -values and L2 as the y -values. The resulting regression line will be stored in equation variable Y_1 .

- g. Press ENTER to perform the linear regression.

⇒ Answer Question 5 on the *Data Collection and Analysis* sheet.

- h. Press GRAPH to see your graph.

⇒ Answer Questions 6 and 7 on the *Data Collection and Analysis* sheet.

EXTENSION

Determine the radius-to-height ratio for the funnel you used. Write an equation relating volume, V and height, h . Differentiate this equation with respect to time. Use your weight versus time graph to find dW/dt . Convert this to dV/dt using the specific gravity of water. Substitute this value into the differential equation. Solve for h as a function of t .

Use this relationship to determine when the water level will be at half its initial value. To test your answer, plug the hole and re-fill the funnel. Unplug the hole and activate a stopwatch simultaneously. Re-plug the hole when the stopwatch reading matches the time you computed above. Is the height of the water in the funnel approximately half its initial value?

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLE

x_1		x_2	
y_1		y_2	
slope			
x-intercept		y-intercept	

QUESTIONS

1. Why is the value of the slope negative?

$$m = \frac{y_2 - y_1}{x_2 - x_1} =$$

2. Use this relation to write the equation of the line fitting the data in the traditional $y = mx + b$ form, and record it here.
3. Is your line a good fit for the data? Does the line pass directly through any particular points? Why?
4. What is the physical interpretation of the x -intercept?
5. Use the parameters a and b that appear on the calculator screen to write the equation of the calculator's best-fit line.

