

NUMB3RS Activity: A Mathematics Sampler Episode: "Calculated Risk"

Topic: Conditional Probability: sampling without replacement

Grade Level: 6 - 9

Objective: Understand how probabilities for an event may change depending on previous events.

Time: 10 - 15 minutes

Introduction

Students are probably familiar with finding the probability of simple events. For example, if you have a jar of 4 marbles, 2 yellow, 1 green, and 1 red, the probability of choosing a red marble is $\frac{1}{4} = 25\%$.

If a second marble is drawn and you want to know the probability of choosing the yellow marble, the probability depends on whether or not the first red marble has been replaced. If the red marble has been replaced, the probability of choosing a yellow is $\frac{2}{4} = 50\%$. If the first marble is not replaced, then the probability of choosing a yellow

marble changes to $\frac{2}{3} \approx 66.7\%$. Sampling with replacement will make each subsequent event independent of all others. Sampling without replacement will make each subsequent event dependent on the previous events.

Sampling with and without replacements is used everyday. For example, scientists take samples of animal populations and monitor their health through tagging. Whether to "put back" or replace an identified sick animal in an existing study would yield a different result than if they removed that animal from the sample. Whether the animal is returned to the entire population or not will affect the probability, but its effect diminishes as the size of the population increases.

Discuss with Students

Review with students the definition of probability and the fact that the probability of an event, denoted $P(E)$, must be $0 \leq P(E) \leq 1$. For example, the probability of 0 means an event cannot happen, while a probability of 1 means that it is certain to happen. Ask students to give examples where the probability is 0 or impossible, 1 or certain, likely, or unlikely. Then pose the questions, "Will there ever be a time where the probability of an event is dependent on the probability of another event? And under what conditions will this happen?"

NUMB3RS Example In "Calculated Risk," Charlie has a jar full of different colored gumballs. When Agent Colby Granger takes one of the gumballs, Charlie asks him the color. By knowing how many of each gumball the jar started with and by keeping track of the color of the gumballs as they are removed, Charlie can determine the chances of removing different colored gumballs in the future.

Student Page Answers: 1. $\frac{5}{25} = 20\%$ 2. $\frac{4}{24} \approx 16.7\%$ 3. 0 4. *It is more likely that you will not draw exactly one of each color. Explanations will vary* 5. *Answers vary. The theoretical probability is approximately .028 or 2.8%. We did 1000 trials of a simulation and had 30 with exactly one of each color gumball.* 6. 24. 7. *Answers will vary.*

Name: _____ Date: _____

NUMB3RS Activity: A Mathematics Sampler

The probability of drawing a certain color gumball from a jar can be shown as:

$$\frac{\text{number of that color of gumballs}}{\text{total number of gumballs}}.$$

For example, if Charlie's jar has 10 red, 5 yellow, 5 blue, 2 green, and 3 orange gumballs, the probability of choosing a red one is the number of red gumballs, 10, divided by the total number of gumballs, 25, or $\frac{10}{25} = 40\%$. If the chosen gumball is replaced with another red one, then the probability of drawing a red gumball will be the same for the next person. However, since gumballs are usually not replaced, the situation will be different the next time. If a red gumball is removed and not replaced, there are now only 9 red gumballs and 24 total gumballs so the probability of choosing a second red one would be $\frac{9}{24} = 37.5\%$.

For questions 1-6, use the information given in the example above.

1. What is the probability of randomly selecting a blue gumball? _____
2. Suppose you draw a blue gumball out of the jar and eat it. What is the probability of selecting another gumball that is blue?

3. Suppose Charlie draws a green gumball out of the original jar of 25 gumballs and then Colby draws a green gumball from the remaining 24 gumballs. What is the probability of selecting another green gumball?

4. Suppose you draw five gumballs from the jar without replacement. Is it more likely that you will draw exactly one marble of each color or is it more likely that you will **not** draw exactly one of each color? Explain your reasoning.

5. Use the Probability Simulation Application on a TI calculator, or design your own simulation, and simulate several draws of 5 gumballs without replacement. Based on your experimental results, what is the probability of drawing exactly one of each color when you draw 5 gumballs without replacement? _____

6. If you randomly drew gumballs out of the jar without replacement, what is the minimum number you must draw to **guarantee** that you would have one of each color? _____

7. Give another real-life example of sampling without replacement. Give a brief explanation to justify your answer. _____

