Mr. Taylor, I Presume?

Student Activity

Name ___________________

Class ___________________

Introduction

Polynomials are easy to integrate, to differentiate, and even to tell jokes to (they always laugh!). Wouldn't it be nice if it were possible to transform a very difficult function into a nice, "easygoing" polynomial? Of course it would! But how?

Believe it or not, it is possible to determine any polynomial of the form $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_nx^n$ by just knowing the value of its derivatives at a point.

Taylor Polynomials Centered at Zero

For example, find a polynomial $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ if $P(0) = 1$, $P'(0) = 1$, $P''(0) = 6$ and $P'''(0) = 9$.

The first step is to find the derivatives:

$P'(x) = a_1 + 2a_2x + 3a_3x^2$ $\qquad P''(x) = 2a_2 + 6a_3x$ $\qquad P'''(x) = 6a_3$

Since we know the value of each derivative when *x* = 0, we can determine the a_n terms

$P'(0) = 1 = a_1 + 2a_2(0) + 3a_3(0)^2$ $\qquad P''(0) = 6 = 2a_2 + 6a_3(0)$ $\qquad P'''(0) = 9 = 6a_3$

$1 = a_1$ $\qquad 6 = 2a_2 \rightarrow \frac{6}{2} = a_2$ $\qquad 9 = 6a_3 \rightarrow \frac{9}{6} = a_3$

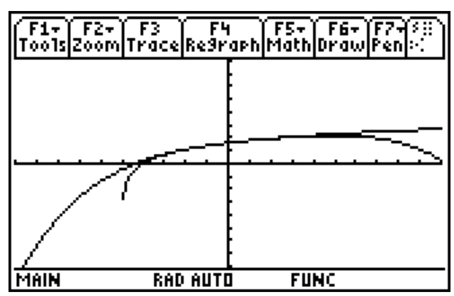
So the polynomial will be $P(x) = 1 + 1x + \frac{6}{2}x^2 + \frac{9}{6}x^3$.

Notice that the number in the numerator of the a_n term is $f^{(n)}(0)$ and the denominator is *n*!. This will come in handy in the next exercise.

While this is amazing, it is important to note that this polynomial is centered at the value *x* = 0 because that is where we calculated the derivative values.

Enough chatter, let's get to it.

Find a polynomial of degree four that approximates $f(x) = \ln(x + 5)$ when *x* = 0.



1. Find the values of $f(x)$, $f'(x)$, $f''(x)$, $f'''(x)$, $f^{(4)}(x)$ when *x* = 0:

$f(0) =$

$f'(0) =$

$f''(0) =$

$f'''(0) =$

$f^{(4)}(0) =$

2. Substitute the derivative values into the numerator and *n*! into the denominator of each term in *P*(*x*). Simplify the polynomial.

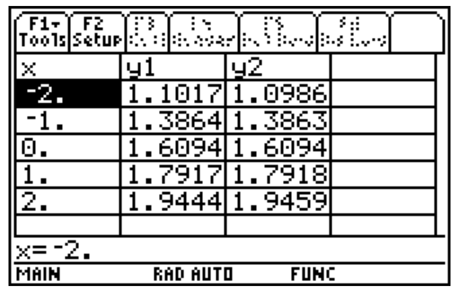
$P_4(x) =$

Check your answer with the **Taylor** command. We use the value of the center for **a** (in this case 0).

Home > F3:Calculus > 9:Taylor > (exp,var,power,a)

3. Set up a table for **y1(*x*) = *P*$_4$(*x*)** and **y2(*x*) = ln(*x* + 5)** where *x* starts with –4 and increases by 1 in the table.

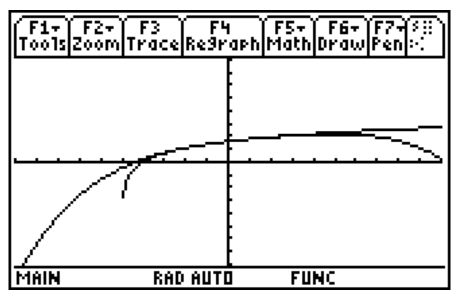
Decide where the polynomial and the function agree or nearly agree in value.



x	y1	y2
-2.	1.1017	1.0986
-1.	1.3864	1.3863
0.	1.6094	1.6094
1.	1.7917	1.7918
2.	1.9444	1.9459

Notice that the values of the Taylor polynomial and the values of the function do not agree everywhere on the graph. In fact, they are closest where the derivative was evaluated. This is called the *center* of the polynomial.

4. On what interval does the polynomial best approximate the original function?



5. Use the **Taylor** command and increase the power. Try several larger powers. What do you notice about the interval as the degree of the polynomial changes?

Taylor Polynomials Not Centered at Zero

Now it is time to leave the origin and find Taylor polynomials whose centers are not zero. The only adjustment is to change x^n to $(x - a)^n$ where a is the center of the approximation.

We want to find a 4th degree Taylor polynomial for $f(x) = \dfrac{1}{2 - x}$ centered at $x = 1$.

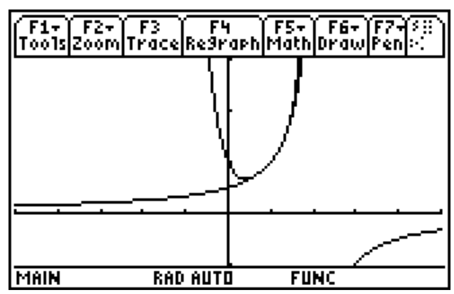
6. Fill in the following chart.

	Polynomial	**Value at *x* = 1**
$f(x)$		
$f'(x)$		
$f''(x)$		
$f'''(x)$		
$f^{(4)}(x)$		

Now substitute the derivative values into the numerator and n! into the denominator of each term in $P(x)$. Remember to write $(x – 1)^n$ instead of x^n. Simplify the polynomial.

$P(x)$ =

Graph the function and the Taylor polynomial you just found. Examine the graphs and pay close attention to where $x = 1$.



7. Your teacher will give you three new values of a center. Find the 4th degree Taylor polynomial using the **Taylor** command with these values as the center. Graph each Taylor polynomial and the original function on the same set of axes. What do you notice about the interval as the center is changed?