Algebra 1   *TI-Nspire*™   **Matrix Multiplication**

Calculations at the Crazy Cookie Company
by Suzanne Moyers

Time required
45-55 minutes

Activity Overview
This activity is designed to begin with whole-class discussion. Teacher and students work together to do trial-and-error matrix multiplication. The intent is to generate matrix multiplication procedures and rules from student observations. Students then individually complete the step-by-step tutorial phase of the activity, multiplying each row by each column of a practice matrix. Finally, students work in pairs or small groups to solve the Crazy Cookie Company problem, generating their own matrices and explaining the resultant product matrix.

Concepts
- *Performing matrix multiplication for the first time, students are prompted to use reasoning skills to determine if matrix products are possible for given pairs of matrices. They are led through several matrix product operations to determine rules for matrix multiplication and how to predict new dimensions of matrix products.*
- *Application of matrix multiplication to a real-world problem is the goal of the Crazy Cookie Company problem in the latter part of the activity. Students work in pairs or groups to find the solution matrix to the Crazy Cookie Company problem.*

Teacher Preparation
Prior to the start of this activity, students should be proficient in matrix terminology. They should have already performed matrix addition, subtraction and scalar multiplication.
- *Load the **CrazyCookie.tns** file onto handhelds for student use.*
- *Each student is to work individually, but pairs or groups are recommended, particularly for work on **Problem Set 3.***
- *Screenshots of the **CrazyCookie.tns** file on the following pages are accompanied by descriptions of what students will be doing during the activity. Screenshots of the **CrazyCookieAns.tns** file complete the last pages*

Classroom Management
- *This activity is intended to be opened with whole-class discussion, followed by individual completion of **Problem Set 1** and **Problem Set 2**. During **Problem Set 3**, students should work in pairs or small groups to facilitate dialogue about the problem solving process.*
- *A separate student worksheet is provided (.doc and .pdf formats for editing if desired): **CrazyCookieWorksheet.doc** or **CrazyCookieWorksheet.pdf**, for students to record work as they progress through all parts of the activity. The document is four pages. The last two pages contain **Problem Set 3**, which can be worked separately.*

TI-Nspire™ Applications
Calculator, Notes
**Teacher Into:** We have been doing the

Suzanne

matrix operations of addition, subtraction and scalar multiplication. Remember that scalar multiplication is simply multiplying every matrix element by the same constant, changing the scale of its values. Today's activity sends us to work at the Crazy Cookie Company, where we must know how to do **matrix multiplication**. I have been telling you that it is different from scalar multiplication. The TI-Nspire handheld will show us just how different it is. (Distribute handhelds and worksheets.)

Problem Set 1 — Students begin by experimenting with a matrix that yields a valid product, and one that does not. Whole class discussion about dimensions helps students understand their importance. The second matrix gives a "Dimension error" as a result.

Next, students are asked to discuss the dimension error. Then they experiment to see if dimensions of the matrices used in the product must match. They discover that an exact dimension match is not necessary. Encourage discussion about why.

| 1.1 | 1.2 | 1.3 | 1.4 | ▶RAD AUTO REAL | 🔋 |

Why did the second product fail?

What does "dimension error" mean?

Must dimensions of matrices match in order to multiply them? Try this one:

$$\begin{bmatrix} 12 & 3 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ \end{bmatrix}$$

| 1.1 | 1.2 | 1.3 | 1.4 | ▶RAD AUTO REAL | 🔋 |

Notice that the product of a 1 by 2 matrix with a 2 by 1 yields a 1 by 1. How do we get 105?

Work in the lower right pane:

Then, students are prompted to enter their own computations (Page 1.4, lower right pane) of how they believe this matrix product was calculated.
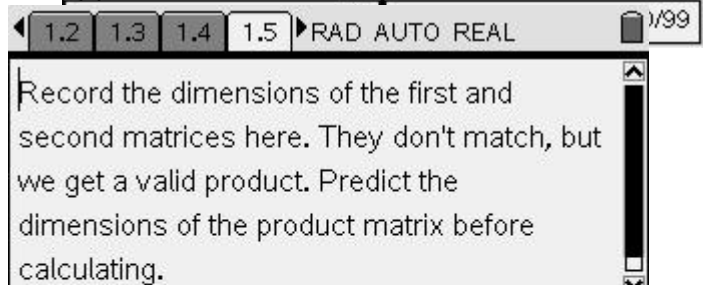
$$\begin{bmatrix} 12 & 3 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 105 \end{bmatrix}$$

$$[ ]$$

| ◀ | 1.2 | 1.3 | 1.4 | 1.5 | ▶RAD AUTO REAL | 🔋 |0/99

Record the dimensions of the first and second matrices here. They don't match, but we get a valid product. Predict the dimensions of the product matrix before calculating.

Subsequent problems in **Problem Set 1** prompt students to predict dimensions of the resultant matrix prior to attempting to find a product.
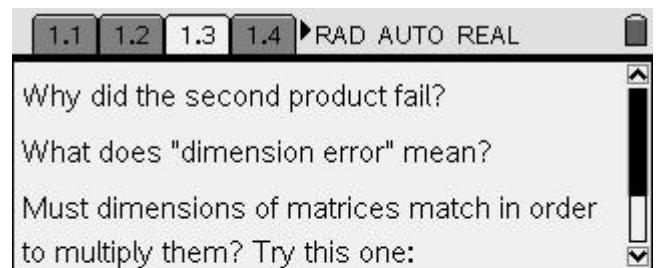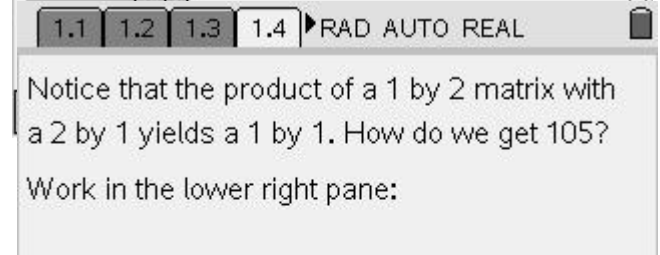
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$$\begin{bmatrix} -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} -3 & 0 \\ 5 & 0 \end{bmatrix}$$

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Additional practice guides students by directing that they multiply ROW by COLUMN elements, then ADD.

Test a few more samples. First, predict the dimensions of the new product. Find the PRODUCT of each ROW element by COLUMN element. Then ADD. Check your work below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

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On Pages 1.7 and 1.8, students answer questions about the dimensions of the left and right matrices, and the resultant product matrix.

Question

The ___ dimension of the left matrix must match the ___ dimension of the right matrix to find a matrix product.

Answer ⌄

Problem Set 2 – This is the "tutorial" problem set, where students must break matrices into row and column components. Page 2.2 is a Notes page, with no provision for calculation by the handheld. They are to fill in the predicted product dimensions.

| 2.1 | 2.2 | 2.3 | 2.4 | ▶RAD AUTO REAL |

Here's the problem:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{bmatrix}$$

Product dimensions will be: _____

Place blanks in the empty matrix on your worksheet for each of the product elements.

Begin on the next page.-->

Each successive page of the problem prompts students to compute products of corresponding row-column elements, then add to yield a result for the appropriate element in the result matrix. The student worksheet also has blanks for them to fill in. (There are two more pages like this, one for each row in the left matrix.)

| 2.1 | 2.2 | 2.3 | 2.4 | ▶RAD AUTO REAL |

$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 11 \end{bmatrix}$ ->

___ + ___ = ___

$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 12 \end{bmatrix}$ ->

___ + ___ = ___

$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 13 \end{bmatrix}$ ->

___ + ___ = ___

$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 14 \end{bmatrix}$ ->

___ + ___ = ___

The final step in **Problem Set 2** is for students to check their results, with this Calculator page that they must fill in with the matrices for the product.
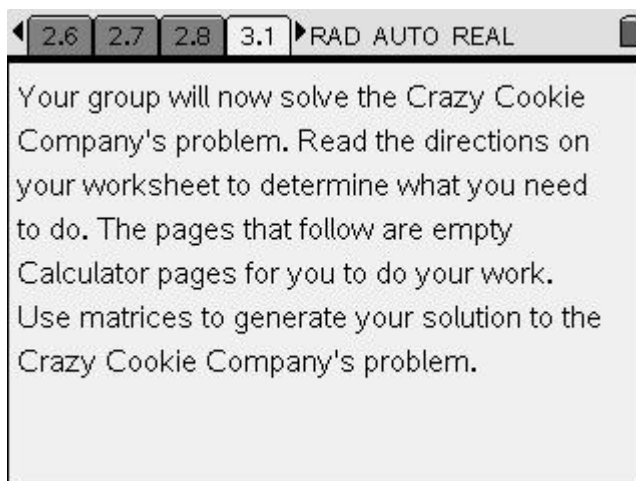
| 2.4 | 2.5 | 2.6 | 2.7 | ▶RAD AUTO REAL |

Now, check your results by filling in the matrices below. It's a Calculator pane (the previous were Notes panes), so you'll get immediate results.
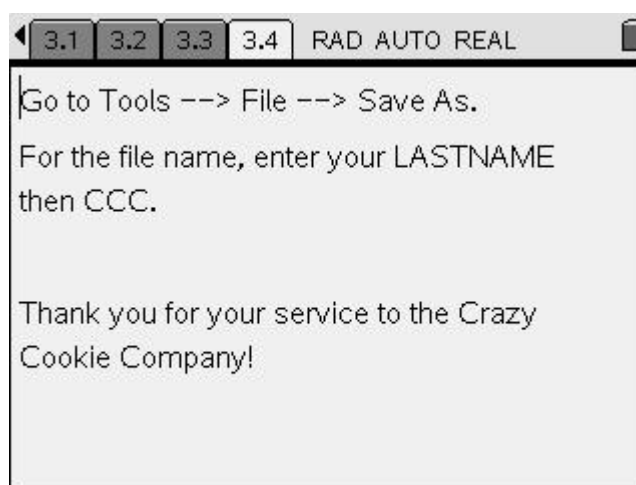
$$\begin{bmatrix} \Box & \Box \\ \Box & \Box \\ \Box & \Box \end{bmatrix} \cdot \begin{bmatrix} \Box & \Box & \Box & \Box \\ \Box & \Box & \Box & \Box \end{bmatrix}$$

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Problem Set 3 – The final set in the activity has students generate their own matrix products to solve the Crazy Cookie Company's problem.
They must use the information in the student worksheet to find the solution to the problem. Pages 3.2 and 3.3 are open Calculator pages for their use. The worksheet contains an empty table that they must complete, along with lines on which to write a description of the solution process.

| 2.6 | 2.7 | 2.8 | 3.1 | ▶RAD AUTO REAL |

Your group will now solve the Crazy Cookie Company's problem. Read the directions on your worksheet to determine what you need to do. The pages that follow are empty Calculator pages for you to do your work. Use matrices to generate your solution to the Crazy Cookie Company's problem.

Activity wrap-up prompts students to SAVE AS, using the filename: LASTNAMECCC. This prevents overwriting the original **CrazyCookie.tns** file on the handheld. Otherwise, student work with overwrite the file, which will then have to be reloaded on the handheld before reuse.

| 3.1 | 3.2 | 3.3 | 3.4 | RAD AUTO REAL |

Go to Tools --> File --> Save As.

For the file name, enter your LASTNAME then CCC.

Thank you for your service to the Crazy Cookie Company!

After the Activity: Assessment of student work can be carried out from completed worksheets, handheld files, or both. Students may download their files to a classroom computer, or to a single handheld used for assessment storage.

CrazyCookieAns.tns   Completed Student Document ~ File Screenshots

| 1.1 | 1.2 | 1.3 | 1.4 | RAD AUTO REAL |

Matrix multiplication is done differently from matrix addition. It is NOT multiplication of corresponding elements. Try these to see how it works.

$$\begin{bmatrix} -3 & 3 \\ 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 3 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 12 & -12 \\ -5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 10 \end{bmatrix} \cdot \begin{bmatrix} 12 & 3 \\ 0 & 0 \end{bmatrix}$$

"Error: Dimension e▸

1/1   1/1

| 1.1 | 1.2 | 1.3 | 1.4 | RAD AUTO REAL |

Why did the second product fail?

What does "dimension error" mean?

Must dimensions of matrices match in order to multiply them? Try this one:

$$\begin{bmatrix} 12 & 3 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 105 \end{bmatrix}$$

[ ]

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| 1.1 | 1.2 | 1.3 | 1.4 | RAD AUTO REAL |

Notice that the product of a 1 by 2 matrix with a 2 by 1 yields a 1 by 1. How do we get 105?

Work in the lower right pane:

$$\begin{bmatrix} 12 & 3 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} 105 \end{bmatrix}$$

$12 \cdot 10 + 3 \cdot -5$ 105

[ ]

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| 1.4 | 1.5 | 1.6 | 1.7 | RAD AUTO REAL |

Test a few more samples. First, predict the dimensions of the new product. Find the PRODUCT of each ROW element by COLUMN element. Then ADD. Check your work below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 50 \\ 122 \end{bmatrix}$$

1/1

| 1.4 | 1.5 | 1.6 | 1.7 | RAD AUTO REAL |

Question

The ___ dimension of the left matrix must match the ___ dimension of the right matrix to find a matrix product.

Answer   ⌄⌄

column

row

| 1.5 | 1.6 | 1.7 | 1.8 | RAD AUTO REAL |

Question

Product dimensions come from the number of ___ in the left matrix BY the number of ___ in the right matrix of the product.

Answer   ⌄⌄

rows, columns

◀ 1.7 | 1.8 | 2.1 | 2.2 ▶RAD AUTO REAL

Here's the problem:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 \end{bmatrix}$$

Product dimensions will be: _3 by 4_____

Place blanks in the empty matrix on your worksheet for each of the product elements. Begin on the next page.-->

◀ 1.8 | 2.1 | 2.2 | 2.3 ▶RAD AUTO REAL

$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 11 \end{bmatrix} ->$

_7_ + 22_ = _29

$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 12 \end{bmatrix} ->$

_8_ + 24_ = _32_

$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 13 \end{bmatrix} ->$

_9_ + _26 = _35

$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 14 \end{bmatrix} ->$

10_ + _28 = _38

◀ 2.2 | 2.3 | 2.4 | 2.5 ▶RAD AUTO REAL

$\begin{bmatrix} 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 11 \end{bmatrix} ->$

_21 + _44 = _65

$\begin{bmatrix} 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 12 \end{bmatrix} ->$

_24 + _48 = _72

$\begin{bmatrix} 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 13 \end{bmatrix} ->$

_27 + _52 = _79

$\begin{bmatrix} 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 14 \end{bmatrix} ->$

_30 + _56 = _86

◀ 2.3 | 2.4 | 2.5 | 2.6 ▶RAD AUTO REAL

$\begin{bmatrix} 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 11 \end{bmatrix} ->$

_35 + 66 = _101

$\begin{bmatrix} 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 12 \end{bmatrix} ->$

_40 + 72 = _112

$\begin{bmatrix} 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 13 \end{bmatrix} ->$

_45 + _78 = 123

$\begin{bmatrix} 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 14 \end{bmatrix} ->$

_50 + _84 = 134

◀ 2.4 | 2.5 | 2.6 | 2.7 ▶RAD AUTO REAL

Now, check your results by filling in the matrices below. It's a Calculator pane (the previous were Notes panes), so you'll get immediate results.

$\begin{bmatrix} 5 & 6 \end{bmatrix} \begin{bmatrix} 11 & 12 & 13 & 14 \end{bmatrix}$

$$\begin{bmatrix} 29 & 32 & 35 & 38 \\ 65 & 72 & 79 & 86 \\ 101 & 112 & 123 & 134 \end{bmatrix}$$

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◀ 2.7 | 2.8 | 3.1 | 3.2 ▶RAD AUTO REAL

$$\begin{bmatrix} .15 & .2 \\ .1 & .25 \\ .08 & .15 \\ .2 & .1 \end{bmatrix} \cdot \begin{bmatrix} 120 & 97 & 64 & 75 \\ 80 & 59 & 36 & 60 \\ 72 & 84 & 29 & 48 \end{bmatrix}$$

"Error: Dimension error"

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┌ 2.8 │ 3.1 │ 3.2 │ 3.3 ▶ RAD AUTO REAL          ▮

┌                    ┐ ┌ .15   .2  ┐
│ 120  97  64  75 │ │ .1    .25 │
│  80  59  36  60 │·│ .08   .15 │
│  72  84  29  48 │ │ .2    .1  │
└                    ┘ └           ┘
                        ┌ 47.82  65.35 ┐
                        │ 32.78  42.15 │
                        │ 31.12  44.55 │
                        └              ┘

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