Law of Large Numbers

TIMATH.COM: STATISTICS



Math Objectives

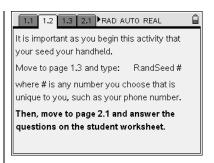
- Compare the observed probability of randomly selected outcomes to the expected probability from that sample space.
- Identify the pattern in the differences of observed and expected probabilities.
- Recognize that relative frequencies of particular outcomes after few observations are more likely to deviate substantially from expected values than are relative frequencies based on more observations.

Vocabulary

- distribution
- variability
- sample space
- outcome
- relative frequency

About the Lesson

- This lesson is a follow-up lesson to the activity *Probability Distributions*.
- Students will generate single outcomes from a given sample space and observe how the difference in probabilities changes.
- They will observe the generation of a very large number of outcomes to visualize the law of large numbers, namely, that as the number of observations increases, the relative frequency for any particular outcome approaches the underlying probability of that outcome's occurring.
- Students will also use a scatter plot to observe how the probability of a specific outcome stabilizes as the number of observations increases.



TI-Nspire™ Technology Skills:

- Download TI-Nspire document
- Open a document
- Move between pages

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- Have students "seed" their random number generators with different numbers before starting this activity, in order to avoid having all students' data be identical. (Syntax is RandSeed #, where # should be a number unique to each student.)

Lesson Materials:

Student Activity

Law_of_Large_Numbers_Student.PDF Law_of_Large_Numbers_Student.DOC

TI-Nspire document Law_of_Large_Numbers.tns

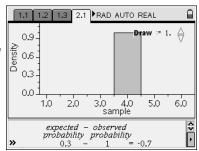


Discussion Points and Possible Answers:

TI-Nspire Problem/Page 2.1

Teacher Tip: Prior to beginning this activity, each student should seed the random number generator by moving to the Calculator page on 1.3 and following the instructions on 1.2. The entire command may be typed directly, using the alpha keys on the keyboard.

Students need only click on either of the "arrowheads" on page 2.1 to add samples to the plot. Each click is one more sample. **Note that clicking down does not reverse the prior count.**



 The plot on page 2.1 represents the result of selecting one block from the bag and recording the number it is labeled with.

The first draw is already shown. How does the observed probability compare to the expected probability?

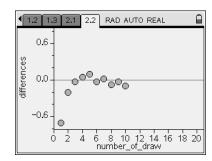
The observed probability is 1, whereas the expected probability is 0.3, which is a big difference in probability.

Click the up arrow on the slider nine times (so that a total of exactly 10 outcomes are displayed).

How did the difference in probability at the bottom of the screen change as the number of draws increased? The difference between the observed probability and the expected probability became smaller.

Discussion Points and Possible Answers: *TI-Nspire Problem/Page 2.2*

Teacher Tip: To see the points for the number of draws greater than 20, students need to hover the cursor over the number on the right end of the bottom axis until it changes to + . Grab the axis and move the cursor to the left.



 The scatter plot shows the differences of probability vs. the number of draws.
 Describe the pattern of the scatter plot.

4. How does the scatter plot support your answer from Question 2?

The points start off far away from the horizontal line 0, but then gradually move closer to the line.

The differences in probability looked like they were getting closer to zero, which is supported by the scatter plot because the points are getting closer to zero.

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5. Move back to page 2.1. Continue to click the slider until you have generated a distribution that has 50 sample elements. View the scatter plot again. (You may need to adjust the plot to see all 50 draws.) This plot demonstrates the law of large numbers. Explain in your own words what the definition of the law of large numbers is.

Sample answer: As the number of draws or trials increases, the observed probability gets closer to the expected probability.

Teacher Tip: After completing the activity, discuss with students the actual definition of the law of large numbers.

Wrap Up:

Upon completion of the discussion, the teacher should ensure that students:

- Understand the law of large numbers as a fundamental concept in statistics and probability that
 describes how the average relative frequency of an event is likely to be close to the actual probability
 of that event.
- Recognize that relative frequencies of particular outcomes after few observations are more likely to deviate substantially from expected values than are relative frequencies based on more observations.