Exploring Space Through MATH

Applications in Algebra 1

EDUCATOR EDITION

Weightless Wonder

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, Evaluate) will be used to accomplish the following objectives.

Students will:

- solve quadratic equations and evaluate and graph quadratic functions;
- find the maximum, the *y*-intercept, the *x*-intercepts, and interpret their significance; and
- determine the effects of parameter changes on the graph of an quadratic equation.

Prerequisites

Students should:

- have experience using TI-Nspire graphing technology
- have prior experience working with quadratic equations and the properties of a parabola.

Background

This problem is part of a series that applies algebraic principles to the U.S. Space Exploration Policy.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

In our quest to explore, humans will have to adapt to functioning in a variety of gravitational environments. Earth, Moon, Mars and space all have different gravitational characteristics. Earth's gravitational force is referred to as one Earth gravity, or 1 *g*. Since the Moon has less mass than the Earth, its gravitational force is only one sixth that of Earth, or 0.17 *g*. The gravitational force on Mars is equivalent to about 38% of Earth's gravity, or

Key Concept
Quadratic Functions

Problem Duration
125 minutes

Technology
TI-Nspire

Materials
- Student Edition
- TI-Nspire

Degree of Difficulty
Moderate to Difficult

Skills
Analyze data to solve a real-world problem; connect quadratic equations and their parabolic graphs to a real-world situation

NCTM Standards
- Algebra
- Problem Solving
- Communication
- Connections
- Representation

0.38 *g*. The gravitational force in space is called microgravity and is very close to zero-*g*.

When astronauts are in orbit, either in the space shuttle or on the International Space Station, they are still affected by Earth's gravitational force. However, astronauts maintain a feeling of weightlessness, since both the vehicle and crew members are in a constant state of free-fall. Even though they are falling towards the Earth, they are traveling fast enough around the Earth to stay in orbit. During orbit, the gravitational force on the astronauts relative to the vehicle is close to zero-*g*.

Figure 1: C-9 jet going into a parabolic maneuver.

Figure 2: Astronaut crew training onboard the C-9 aircraft in preparation for the Microgravity Science Laboratory missions flown on the Space Shuttle Columbia in April and July of 1997.

The C-9 jet is one of the tools utilized by NASA to simulate the gravity, or reduced gravity, astronauts feel once they leave Earth (Figure 1). The C-9 jet flies a special parabolic pattern that creates several brief periods of reduced gravity. A typical NASA C-9 flight goes out over the Gulf of Mexico, lasts about two hours, and completes between 40 and 60 parabolas. These reduced gravity flights are performed so astronauts, as well as researchers and their experiments, can experience the gravitational forces of the Moon and Mars and the microgravity of space.

By using the C-9 jet as a reduced gravity research laboratory, astronauts can simulate different stages of spaceflight. This can allow crew members to practice what might occur during a real mission. These reduced gravity flights provide the capability for the development and verification of space hardware, scientific experiments, and other types of research (Figure 2). NASA scientists can also use these flights for crew training, including exercising in reduced gravity, administering medical care, performing experiments, and many other aspects of spaceflight that will be necessary for an exploration mission. A flight on the C-9 jet is the next best thing to blasting into orbit!

NCTM Principles and Standards

Algebra

- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil in simple cases and using technology in all cases.
- Draw reasonable conclusions about a situation being modeled.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (35 minutes)

- Have students read the Background section aloud to the class.
- Play the video NASA Reduce Gravity Student Flight Opp.(13 min.) at: http://www.nasa.gov/multimedia/videogallery/index.html?media_id=21393951
- Distribute the worksheet, **Weightless Wonder Video**.
- Instruct students to write their answers to the questions on the worksheet.
- Arrange students in groups of 3-4 and ask them to compare and discuss their answers to the questions.

2 – Explore (20 minutes)

- Distribute the worksheet, **Interpreting Graphs of Quadratic Functions.**
- Ask students to discuss the questions and answers as they work as a team.
- Call on students to give their answers and discuss as a class.

3 – Explain (30 minutes)

- Distribute the worksheet, **Weightless Wonder Problem.**
- Distribute the TI-Nspire. Have students open file *WeightlessWonder.tns.* Questions 4 -7 are embedded in the TI-Nspire file.
- Have a student read each of the paragraphs aloud to the class or students may read to themselves silently.
- Discuss the variables and expressions of the given function. Note: Using graphing technology y represents the altitude (h), and x represents the time (t).
- Have a student read the first question aloud to the class. Explain that the start of the parabola is at $t = 0$.
- Have the groups work together to answer the questions.
- Call on students to give their answers and discuss.

4 – Extend (20 minutes)

- Questions 8 -16 in the **Changing the Parameters of Quadratic Functions** section are embedded in the TI-Nspire file**.**
- Have a student read the paragraph aloud to the class.
- Discuss the variables and expressions of the given function.
- Have each group work together on the *Changes in Initial Altitude* section, recording their solutions in the TI-Nspire handheld.
- Have each group work together on the *Changes in Initial Velocity* section, recording their solutions in the TI-Nspire handheld. Discuss the results with the class if time permits.

5 – Evaluate (20 minutes)

- All questions in the, **Weightless Wonder: Wrap Up** are embedded in the TI-Nspire file and listed on paper so that educators have the option of using Nspire or assigning the wrap up as homework.
- Distribute **Weightless Wonder: Wrap Up** worksheet or complete using the TI-Nspire.
- Have students complete it individually.

ENGAGE

Weightless Wonder – Video

Solution Key

Please answer the following questions about the Weightless Wonder video.

1. In the flight of the C-9 what part of one maneuver is a true parabola and why?
   From the point on ascent where the pilot cuts thrust and the plane continues to rise, then noses over into descent (about 20 seconds), during which time the plane is in free fall simulating microgravity.

2. What else can this type of flight simulate besides zero-*g*?
   Gravity on the Moon. A lunar parabola is about one-sixth $g \approx 0.17\ g$.
   Gravity on Mars. A martian parabola is about one-third $g \approx 0.38\ g$.

3. What changes might occur to the body during this type of flight?
   Motion sickness, vomiting, hypoxia, lips and nails turn blue, feeling happy, dull or lightheaded.

4. What types of experiments do you think might be performed in the reduced gravity environment of a parabolic flight?
   Answers will vary.
   The effects of microgravity on fluid physics, combustion, material science, life sciences, new technologies, and the human body, i.e. breathing, blood pressure, heart rate, bone loss, buoyancy.

5. What do the students do to prepare themselves for the reduced gravity flight?
   Answers will vary.
   Plan the experiment, prepare the experiment hardware, test the experiment, participate in the orientation and safety review at Ellington Field. Pass the flight physical, the NASA Physiological Training course, and the Hyperbaric Chamber training (simulates air at 25,000 ft.). Put on their Air Force flight suits and receive their motion sickness meds and bags.

6. What are other instances where one might feel reduced gravity on Earth?
   Answers will vary.
   Roller coaster, bungee jumping, elevator ride, speeding over a hill in a car, bull riding.

EXPLORE

Interpreting Graphs of Quadratic Functions

Solution Key

The graph below shows the altitude of a C-9 jet during one parabolic maneuver. Use this graph to answer the questions below:

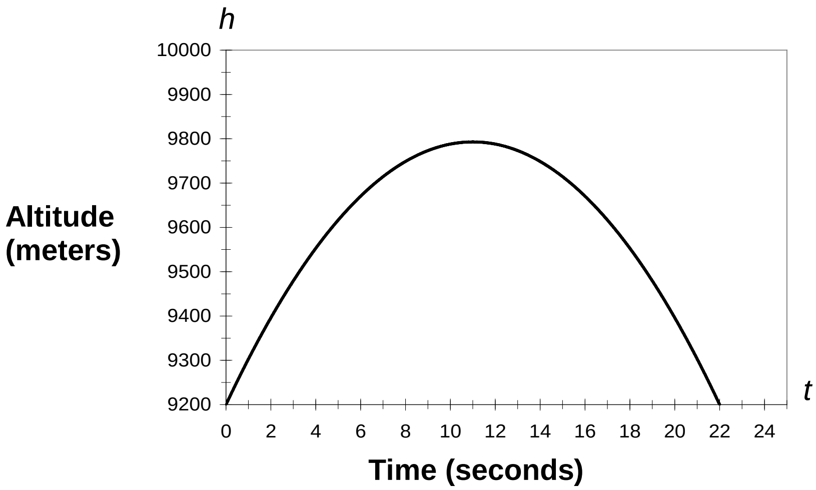


Figure 3: Altitude of C-9 During one parabolic maneuver

1. What does 9200 meters represent in this situation?

   The altitude where the plane started the parabolic maneuver.

2. When does the C-9 first reach an altitude of 9400 meters? How long does the plane remain above 9400 meters? Justify your answer.

   The plane first reaches an altitude of 9400 meters at 2 seconds. It continues to rise and then starts to fall. The altitude decreases to 9400 meters at 20 seconds. The C-9 remains above 9400 meters for the difference between 2 and 20 seconds, or 18 seconds.

3. Between what two whole number seconds was the plane at 9600 meters?

   Between 4 and 5 seconds and again between 17 and 18 seconds.

4. What is the approximate vertex of the parabola? What does this vertex tell you about this part of the flight?

   The vertex is at approximately (11, 9800). This tells you that the maximum altitude the plane reached during this parabolic maneuver was about 9800 meters and it occurred at 11 seconds.

5. What is a reasonable domain for this part of the flight? What does the domain tell you about the flight?

   The domain is $0 \le t \le 22$. This domain tells you that the parabolic maneuver lasted 22 seconds.

6. What is a reasonable range for this part of the flight? What does the range tell you about the flight?

   The range appears to be $9200 \le h \le 9800$. This tells you that the plane flew between 9200 meters to 9800 meters during this part of the flight.

EXPLAIN

Weightless Wonder Problem

Solution Key

To prepare for an upcoming mission, an astronaut participated in a C-9 flight simulating microgravity, or close to zero-*g*. The pilot flew out over the Gulf of Mexico, dove down to increase to a maximum speed then climbed up until the nose was at a 45° angle with the ground. To go into a parabolic maneuver, the pilot then cut the thrust of the engine letting the nose of the plane continue to rise then come back down at a -45° angle with the ground. Ending the maneuver, the pilot throttled the engine back up and began another dive to prepare for the next parabola. The pilot completed 50 parabolas during the 2 hour flight.

The figure below shows the movement of the plane during a typical flight. The parabolic maneuver, where microgravity is felt, is highlighted. This is the part of the flight that you will focus on for the following questions.

The function $h = -4.9t^2 + 87.21t + 9144$ describes the altitude (h) in meters (m) of the plane in relation to the time (t) in seconds (s) after it started the parabolic maneuver. You will use this function to analyze the parabolic flight of the C-9. Round all answers to the nearest tenth.

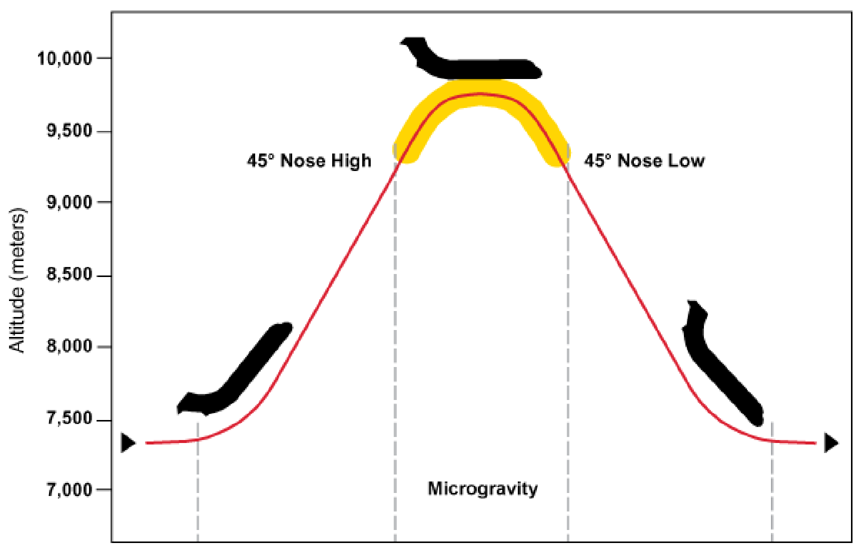


Figure 4: A typical microgravity maneuver.

1. Using the defined function, at what altitude did the astronaut first start to feel microgravity?

   Let $t = 0$

   $$h = -4.9t^2 + 87.21t + 9144$$
   $$h = 9144$$

   The altitude would be 9144 meters.

2. Consider the function $h = -4.9t^2 + 87.21t + 9144$. Use algebra to find the times when microgravity began and ended during this one maneuver.

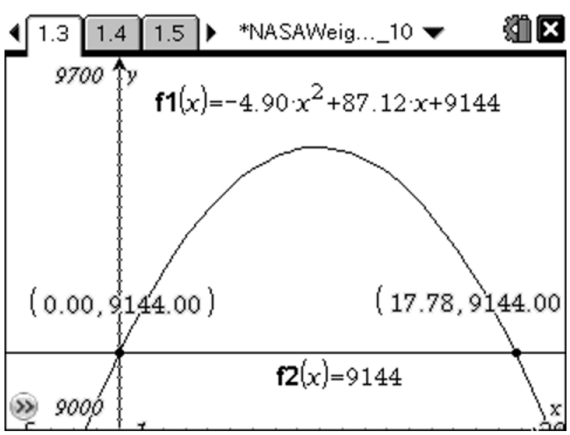
$$h = -4.9t^2 + 87.21t + 9144$$
$$9144 = -4.9t^2 + 87.21t + 9144$$
$$0 = -4.9t^2 + 87.21t$$
$$0 = t(-4.9t + 87.21)$$
$$t = 0\,\text{s}, 17.8\,\text{s}$$

3. What was the length of time the astronaut experienced microgravity during this one maneuver? Explain your answer.

   The astronaut experienced 17.8 seconds of microgravity, since the plane began the parabolic maneuver at 0 seconds and ended at 17.8 seconds.

View the graphs of the parabolic maneuver and altitude 9144 on page 1.3 in the TI-Nspire file *WeightlessWonder.tns* and answer the following questions.

4. How many times do the graphs intersect? Find the *x* and *y* values of the point(s) of intersection. Explain what these *x* and *y* values represent.



   The graphs intersect twice. They first intersect at (0, 9144). This represents the beginning of the parabolic maneuver. The second time they intersect is (17.8, 9144) which represents the end of the parabolic maneuver. The *x* values represent time the *y* values represent the altitude of the plane.

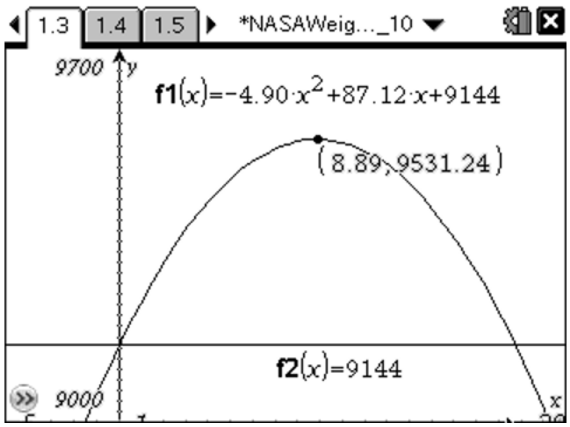
5. Use algebra to find the maximum altitude of the plane during this one parabolic maneuver. Explain your procedure.

   The maximum would occur halfway between the two points of intersection. The average of the two *x* values, or *x* = 8.9 seconds, would be the time at which the plane reaches its maximum altitude. Substituting this value in the equation gives:

$$h = -4.9(8.9)^2 + 87.21(8.9) + 9144$$
$$h = 9532.0 \text{ m}$$

6. Find the maximum altitude and when it occurs using the graph on page 1.3 in the TI-Nspire file.

   The maximum altitude is 9531.2 meters and it occurs at 8.9 seconds.



7. What percent of the astronaut's total flight was spent in microgravity?

   The trip lasted for 2 hours which is 7200 seconds. Each parabola lasted for 17.8 seconds and there were 50 parabolas flown.

$$\frac{17.8 \times 50}{7200} \times 100 = 12.4\%$$

EXTEND

Changing the Parameters of Quadratic Functions

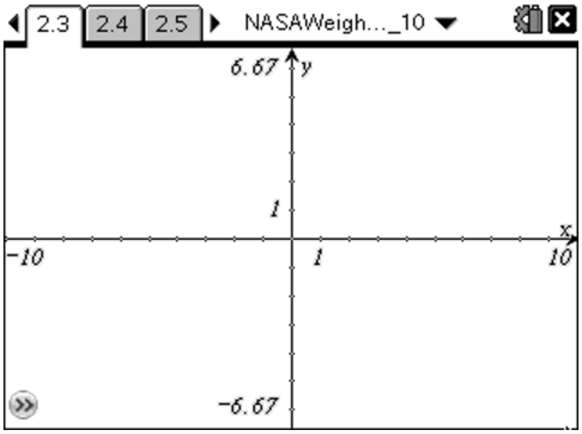
Solution Key

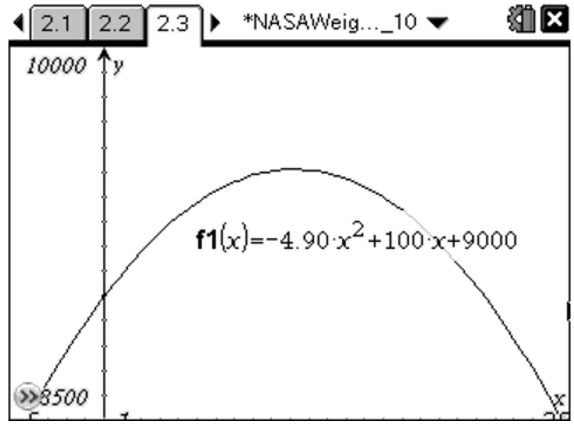
The function $f(t) = -4.9\, t^2 + v_0 t + h_0$ describes the altitude of the C-9 plane during one of its parabolic maneuvers with respect to the time (t) in seconds.. The coefficient v_0 is the vertical velocity of the airplane when it starts the parabolic maneuver. The coefficient h_0 *is* the altitude of the airplane when it starts the maneuver.

Changes in initial altitude:

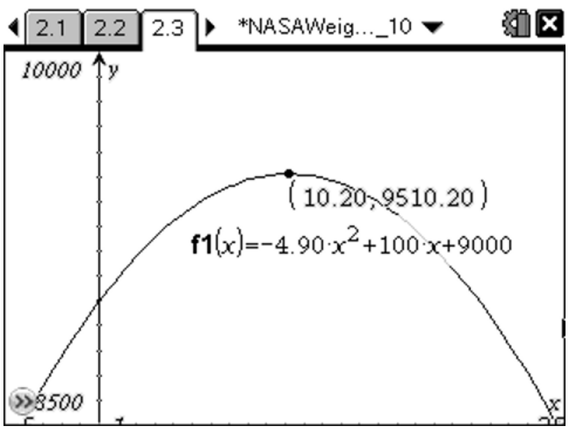
8. During another reduced gravity flight, the C-9 plane starts a parabolic maneuver at a velocity of 100 m/s and an altitude of 9000 meters. Write an equation that models the new parabolic maneuver. Then graph the equation on page 2.3 in the TI-Nspire file. Note: Using a graphing calculator y represents the altitude (h), and x represents the time (t).

$$y_1 = -4.9x^2 + 100x + 9000$$





9. What is the maximum altitude reached?



9510 meters, maximum altitude reached.

10. How long does the parabolic maneuver last?

20.4 seconds, time the parabolic maneuver lasts.

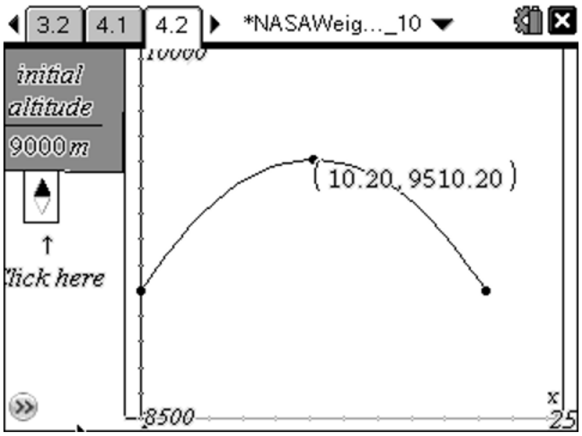
11. For the next parabola the initial velocity is the same but the altitude is 9200 meters when the plane starts the maneuver. Write a new equation to describe this flight.

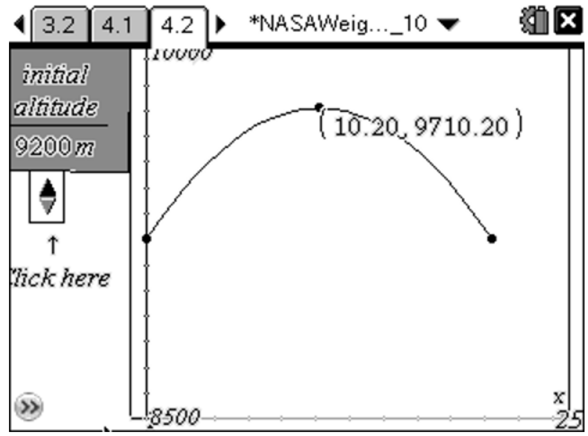
$y = -4.9x^2 + 100x + 9200$

12. Predict how the graph of this equation will change. How will the maximum altitude be affected?

The maximum altitude will be 200 meters above the last.

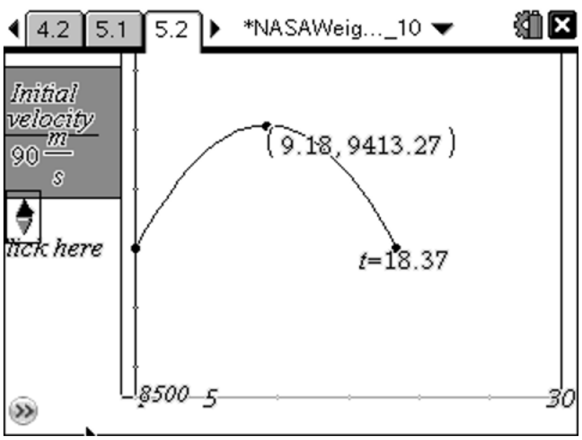
13. Verify your predictions using the graph on page 4.2 in the TI-Nspire file.

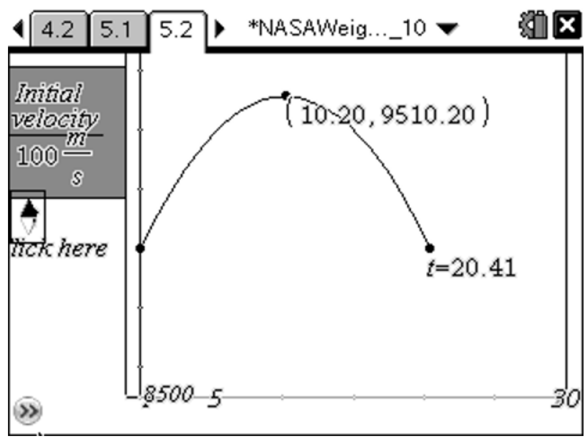




Changes in velocity:

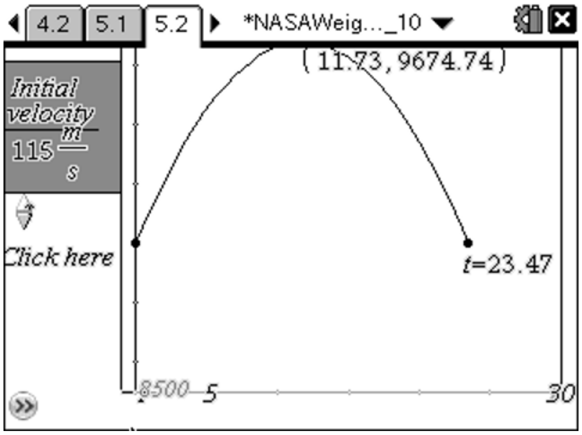
14. If the plane's starting velocity is 90 m/s when it performs the maneuver, explain how the plane's flight path changes. Use the graph on page 5.2 in the TI-Nspire file.





The plane does not fly as high and the maneuver is shorter.

15. If the plane's starting velocity is 115 m/s when it performs the maneuver, explain how the plane's flight path changes. Use the graph on page 5.2 in the TI-Nspire file.



The plane would fly higher and the maneuver would be longer.

EVALUATE

Weightless Wonder: Wrap Up

Solution Key

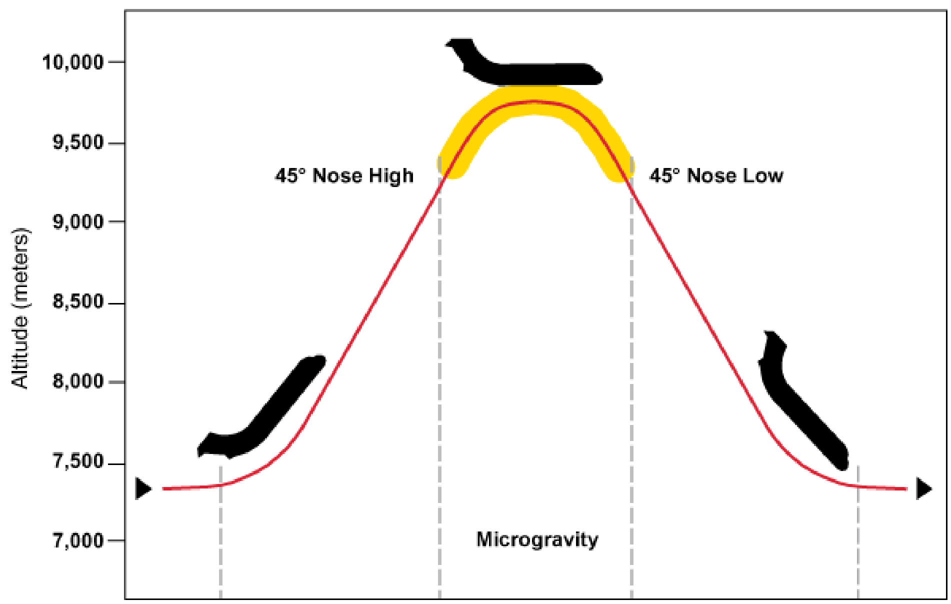


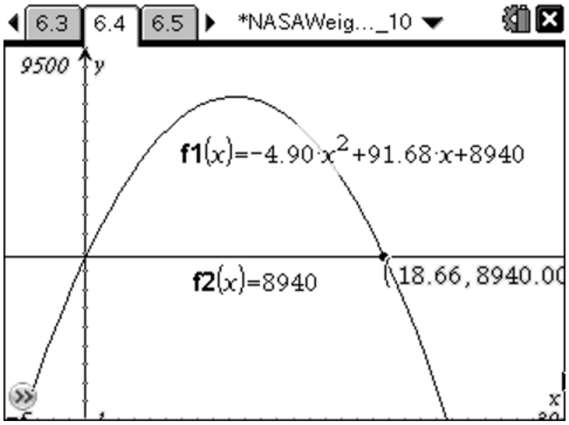
Figure 4: A typical microgravity maneuver.

The C9-jet is preparing for a parabolic maneuver. The flight crew has planned an initial velocity of 91.68 m/s and the airplane's initial altitude is 8940 meters.

1. Write an equation that represents the maneuver.

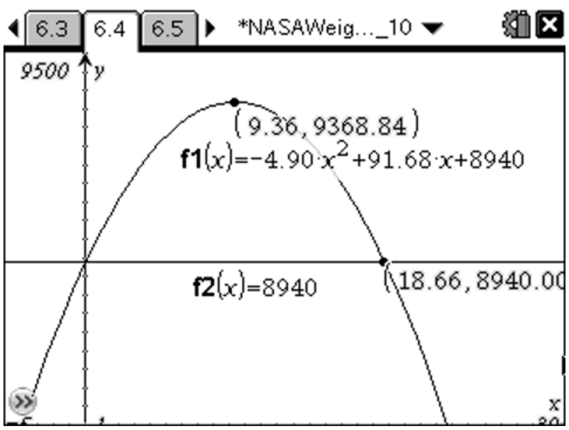
   $y_1 = -4.9x^2 + 91.68x + 8940$

2. Determine the length of one parabolic maneuver. Graph the equation for help. Round to the nearest tenth.



18.7 seconds

3. Find the maximum altitude of the plane during one parabolic maneuver and when it occurs. Round to the nearest tenth.



The maximum altitude is 9368.8 meters and it occurs at 9.4 seconds

4. The scientists aboard this flight must have a total of 15 minutes of microgravity to complete their experiments. Will the scientist have enough time if the plane completes 40 maneuvers using this flight path? Explain

$(40)(18.7) = 748\sec onds$

$(15)(60) = 900\sec onds$ No the scientists would not have enough time.

5. What parameter could be changed to increase the time of each maneuver?

The velocity could be increased to extend the time of each maneuver.

Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

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