Math Objectives

- Students will make a connection between the sign of the derivative (positive/negative/zero) and the increasing or decreasing nature of the graph.

Activity Type

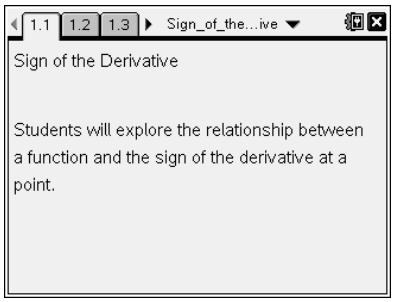
- Student Exploration

About the Lesson

- Students will move a point on a function and observe the changes in the sign of the derivative. Based on observations, students will answer questions on a worksheet. In the second part of the investigation, students will move the point and see a trace of the derivative to connect the positive or negative sign of the derivative with the location on the coordinate plane.

Directions

- Grab the point on the function and move it along the curve. Observe the relationship between the sign of the derivative and the direction of the graph.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Sign_of_the_Derivative_Student.pdf

Sign_of_the_Derivative_Student.doc

TI-Nspire document

Sign_of_the_Derivative.tns

Visit www.mathnspired.com for lesson updates.

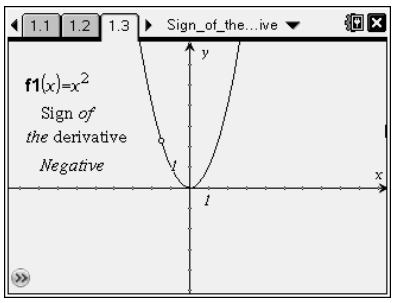
Discussion Points and Possible Answers

PART I:

Move to page 1.3.

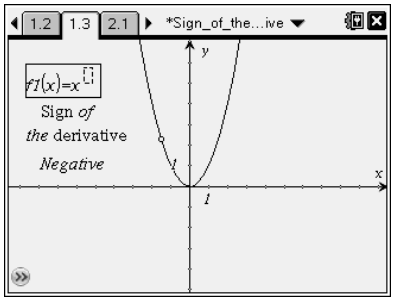
Tech Tips:

The sign of the derivative will indicate negative when the function is decreasing and positive when the function is increasing. The screen will also indicate a zero derivative.

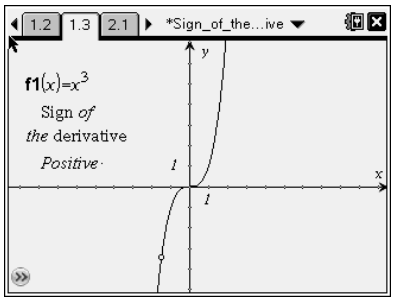


Students will need to edit the function on the screen in order to investigate other functions.

Note: Make sure students have double-clicked on the function to get a text box around the function. Students should use the backspace button to edit the function. Students should also be aware of the location of the cursor when using the exponent key.



If students do not see the open circle after changing the function, they should change the window settings, **Menu > Window > Window Settings,** to find the point.



For $f(x) = x^2$:

1. Describe the function (increasing, decreasing, minimum, maximum) when the sign of the derivative is positive. For what values of x is the sign of the derivative positive?

   Answer: When the sign of the derivative is positive, the graph is increasing. The sign of the derivative is positive for all values of $x > 0$.

2. Describe the function when the sign of the derivative is negative. For what values of x is the sign of the derivative negative?

   Answer: When the sign of the derivative is negative, the graph is decreasing. The sign of the derivative is negative for all values of $x < 0$.

3. When does the sign of the derivative for the function equal zero? For what value(s) of x is the derivative zero?

   Answer: The sign of the derivative for the function is equal zero at the minimum of the function. The derivative is zero when $x = 0$.

Change the function to $\mathbf{f}(x) = x^3$. Double-click on the function label and change the exponent to 3
For $f(x) = x^3$:

4. Describe the function (increasing, decreasing, minimum, maximum) when the sign of the derivative is positive. For what values of x is the sign of the derivative positive?

   Answer: The function is increasing when the sign of the derivative is positive. The derivative is positive for all values of x.

5. Describe the function when the sign of the derivative is negative. For what values of x is the sign of the derivative negative?

   Answer: The derivative of the function is always positive. There are no x values that yield a negative derivative.

6. When does the sign of the derivative for the function equal zero? For what value(s) of x is the derivative zero?

   Answer: The sign of the derivative for the function never equals zero.

Change the function to $\mathbf{f}(x) = -x^3$. Double-click on the function label and add a negative sign.
For $f(x) = -x^3$:

7. Describe the function (increasing, decreasing, minimum, maximum) when the sign of the derivative is positive For what values of x is the sign of the derivative positive?

   Answer: The derivative of the function is always negative. There are no x values that yield a positive derivative.

8. Describe the function when the sign of the derivative is negative. For what values of *x* is the sign of the derivative negative?

   Answer: The function is decreasing when the sign of the derivative is negative. The derivative is negative for all values of *x*.

9. When does the sign of the derivative for the function equal zero? For what value(s) of *x* is the derivative zero?

   Answer: The sign of the derivative for the function never equals zero.

Table for the given functions

Function	Sign of the Derivative		
	Positive	Negative	Zero
$\mathbf{f}(x) = -x^2$	$(-\infty, 0)$	$(0, \infty)$	0
$\mathbf{f}(x) = -x^2 + 2x$	$(-\infty, 1)$	$(1, \infty)$	1
$\mathbf{f}(x) = x^2 + 3x + 2$	$\left(-\frac{3}{2}, \infty\right)$	$\left(-\infty, -\frac{3}{2}\right)$	$-\frac{3}{2}$
$\mathbf{f}(x) = x^2 - 36$	$(0, \infty)$	$(-\infty, 0)$	0
$\mathbf{f}(x) = 3x^2$	$(0, \infty)$	$(-\infty, 0)$	0

10. How can you algebraically find the point(s) where the function has a zero for the value of the derivative?

    Answer: You can find the point(s) where the function has a zero for the value of the derivative by algebraically calculating the vertex of the parabola (since all of the functions are quadratics).

CLASSROOM DISCUSSION:

After students complete part I of the investigation, lead a discussion about the results. Students should use the vocabulary *increasing, decreasing, maximum,* or *minimum* when describing the function.

Part II:

Move to page 2.1.

Tech Tips: Students will now trace a new point on the Graphs page that will show the derivative function.

The derivative function will not stay permanently on the screen, but enough points should remain for students to see the function.

11. When the sign of the derivative is positive, where does the graph of the derivative lie in the coordinate plane?

    Answer: When the derivative is positive, the graph of the derivative is above the *x*-axis.

12. When the sign of the derivative is negative, where does the graph of the derivative lie in the coordinate plane?

    Answer: When the derivative is negative, the graph of the derivative is below the *x*-axis.

13. When the derivative has a value of zero, where does the graph of the derivative lie in the coordinate plane?

    Answer: When the derivative is zero, the graph lies on the origin.

14. Are your answers the same if $\mathbf{f}(x) = x^3$? Why or why not?

    Answer: If $\mathbf{f}(x) = x^3$ the derivative is always positive and the graph of the derivative is always above the *x*-axis.

CLASSROOM DISCUSSION:

After students complete the questions, lead a discussion relating the graph of the derivative and sign of the derivative. Students should realize that a positive derivative means the derivative function lies above the *x*-axis, a negative derivative means the derivative function lies below the *x*-axis, and a zero derivative means the derivative function is on the *x*-axis.