Xtreme Calculus: Part 2

ID: 11487

Time Required
40–45 minutes

Activity Overview

In this activity, students will explore distance-time and velocity-time graphs to deepen their understanding of the application of the derivative. Students will investigate kinematic relationships—graphically, numerically, and algebraically—using animations.

Topic: Derivative Applied to Motion

- *Interpret the derivative as a rate of change in various applied contexts*
- *Average velocity, instantaneous velocity, speed, acceleration*

Teacher Preparation and Notes

- ***The program that is used to complete this activity (tanimat2.89p) is the same program that was used to complete the Extreme Calculus: Part 1 activity.***
- *The activity is designed to be a student-centered discovery and instruction of differential calculus as it relates to motion. Teaching the basics before beginning the activity is optional. After completing the activity, students should be more successful with AP questions like 2002AB3, 2003AB2, 2004AB3, 2004formB AB3&BC3, 2005B AB3, 2005AB5&BC5, mc1998BC90, mc1998AB24, and mc 2003BC84.*
- ***To download the calculator program and student worksheet, go to education.ti.com/exchange and enter "11487" in the quick search box.***

Associated Materials

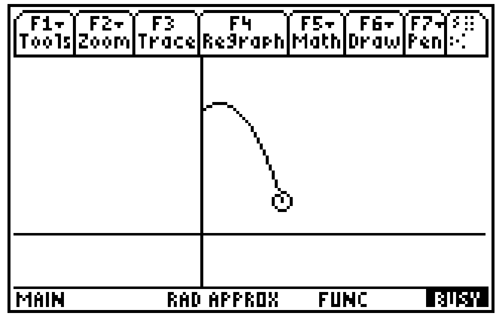
- *CalcWeek16_XtremeCalcPt2_Worksheet_TI89.doc*
- *tanimat2.89p*

Suggested Related Activities

- *Time Derivatives (TI-Nspire CAS technology) — 9537*
- *Extrema (TI-Nspire CAS technology) — 9414*
- *Graphical Derivatives (TI-Nspire CAS technology) — 8499*
- *Parachuting (TI-89 Titanium) — 3021*
- *Move My Way (TI-84 Plus) — 4324*

Part 1 – Extreme Cyclist

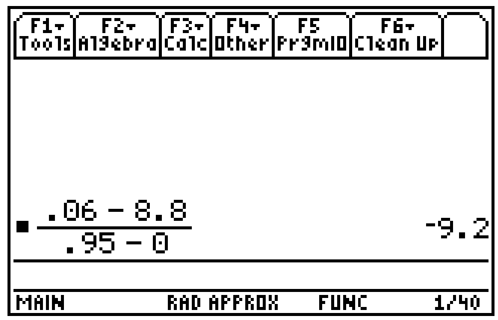
Students are to enter a position-time graph that represents an extreme bicyclist as he is jumping off a ledge and landing safely on the ground. Students should enter the given equation and settings and watch as the graph of the function is drawn. Students should think about the slope of the tangent line to the position graph.



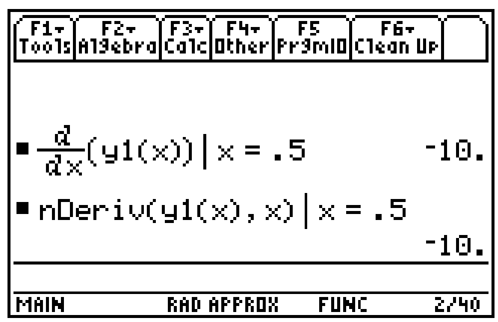
Discussion question

- *How does average velocity differ from instantaneous velocity?*

Students are asked to find the average velocity of the given function between *t* = 0 s and 0.95 s. They should calculate the slope between these two points. Students can use the home screen to find the answer, but should show their work on their worksheets. Point out to students that the slope is obviously negative.

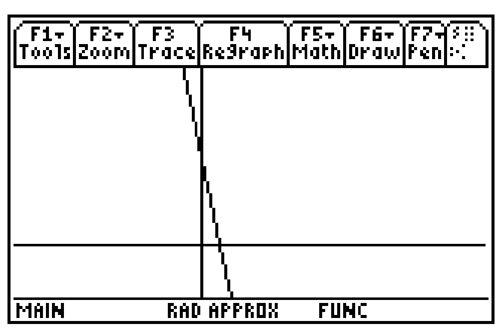


Explain that the velocity function is the derivative of the position function. Students will calculate the velocity at *t* = 0.5 algebraically on the HOME screen. Students can use the **Derivative** command or **nDeriv** command to aid them with this task.



Students can graphically verify their answer using the program *tanimat2*. On the HOME screen, students should enter **tanimat2()** and once the program begins, select **Interactive** in the Main menu, enter 1 for number of points (must press [alpha] + 1), and select **TANGENTS ONLY**. To find the slope at *t* = 0.5, students should type 0.5 and press [ENTER] twice. The slope will appear on the screen. Students should then press [ENTER] to continue and select **6:QUIT** from the Main Menu. This slope should be *v*(0.5) = –10.

Students are to find the velocity-time function and graph it in **y1**. The slope of this graph is –32. Even though the velocity is zero at the peak, students should notice that a parabolic position-time graph indicates a constant acceleration.

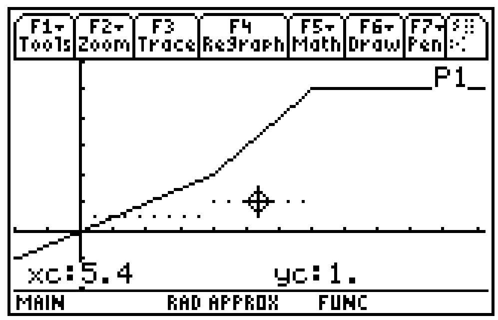


Student Solutions

1. The cyclist has an initial positive velocity. The slope of the tangent of the position-time graph is positive. The height *y* is increasing. Up is the positive *y* direction.
2. (0.06 – 8.8)/(0.95 – 0)= –9.2 ft/s
3. $s'(0.5) = v(0.5) = -10$ ft/s
4. $v(t) = 6 - 32t$
5. Acceleration is constant –32 ft/s^2.
6. Velocity was always decreasing because the acceleration was negative.
7. Sometimes. Zero is not a positive or negative number. Speed is always ≥ 0.
8. When the velocity and acceleration are both in the same direction, the speed is increasing.

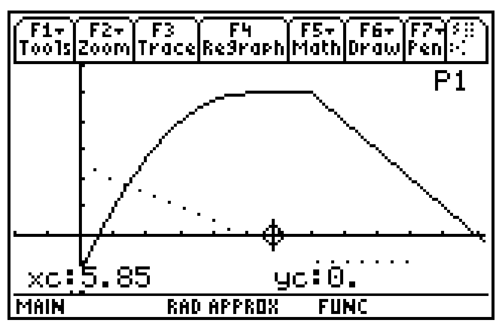
Part 2 – Predict the Graph

Students will examine a distance-time graph and consider what the corresponding velocity-time graph looks like. It is recommended for students to sketch their prediction of the velocity-time graph with the same time scale as the distance time graph.



To view the velocity-time graph, students will run the *tanimat2* program again. However, this time, students should select **2:Animated** from the Main Menu, then **LOW** for sampling rate, and display **TANGENT&PTS**. Students should find the velocity-time graph when $0 \le t \le 10$. The velocity-time graph will be drawn on the screen with the position-time graph.

As students analyze the motion of the object, a common mistake is "graph as picture errors." This is when a student considers a graph to be like a photograph of a situation rather than an abstract mathematical representation. Warn students to not use this thought when answering Question 10.



Students will repeat this process for another position-time graph on page 5.2.

Student Solutions

9. Student answers will vary. Students should have sketched their prediction of the velocity-time graph.

10. The solution is that the graph depicts an object that starts at the origin moving at a constant velocity; it suddenly moves at a higher constant speed and then abruptly stops. These jumps in the velocity time graph make this *extreme motion*.

11. When $t = 5$ seconds, the acceleration exists because the left limit of the rate of change of velocity equals the right limit as t approaches 5.

12. When $t = 7$, the acceleration does not exist because the limit from the left does not equal the right limit of the slope.