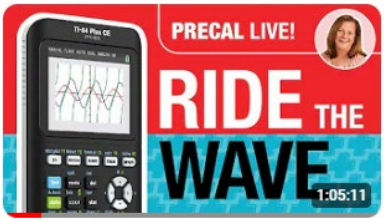
Thursday Night Precalculus Series
January 11, 2024

In this AP Precalculus Live session, we will explore periodic functions and their characteristics.



Introducing Periodic Functions for AP Precalculus Unit 3

About the Lesson

- This Teacher Notes guide is designed to be used in conjunction with the AP Precalculus Live session and Student Problems document that can be found on-demand:
  - https://www.youtube.com/live/hizywSt0N8g?si=qF1r5peeX3EQ2Aiu
  - *Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the "Live Session" but can be enriched by that resource.*
- This session involves exploring features of the graphs of periodic functions and their characteristics, such as:
  - Finding the period.
  - Determining intervals of increase and decrease.
  - Determining concavity.
  - Determining rates of change.
- Students should be able to use the TI-Nspire to verify these features of a periodic function.
- **Class Discussion**: Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

Materials:

TI-Nspire document
- Periodic Functions.tns

Student document
- PreCal_problems_01_11*Solutions*
- PRECAL_problems_solutions_01_11

YouTube
- https://www.youtube.com/live/hizywSt0N8g?si=qF1r5peeX3EQ2Aiu
- ***Documents and materials can be downloaded from this site.***

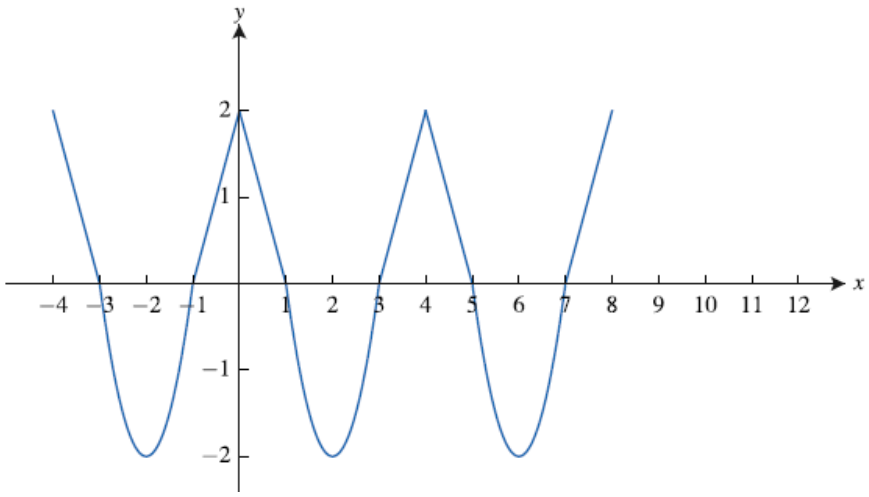
AP Precalculus Learning Objectives

- 3.1.A: Construct graphs of periodic relationships based on verbal representations
- 3.1.B: Describe key characteristics of a periodic function based on a verbal representation.

Source: AP Precalculus Course and Exam Description, The College Board

Problem 1. (a) – (c)

The graph of a periodic function f is shown.



(a) What is the period, p, of the function?

(b) Sketch the next period of the given graph.

(c) Determine whether each function is periodic. If it is, state the period. If it is not, explain why.

(i) $y = f\left(\frac{1}{2}(x-1)\right)$

(ii) $y = -f(x)$

(iii) $y = f(-x)$

(iv) $y = f(2x)$

(v) $y = f(x^2)$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Class Discussion:

What does it mean for a function to be periodic? How do we find the period?

Possible Answers: A function is periodic if there exists a value for p so that $f(x+p) = f(x)$. Visually there is a "chunk" of the graph that gets repeated every p units. When we look at a graph, we are trying to determine that part of the graph that gets repeated.

Class Discussion:

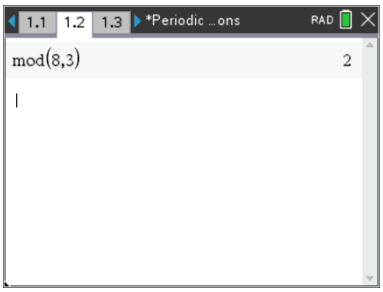
How can we use the transformations (additive and multiplicative) from Units 1 and 2 to explain whether or not each function in (c) is also periodic?

Possible Answers: Using (c) (i), the $(x-1)$ is a horizontal translation to the right 1 unit and the $\frac{1}{2}$ is a horizontal dilation by a factor of 2, so the resulting graph is periodic. A sketch of the new function is also very helpful.

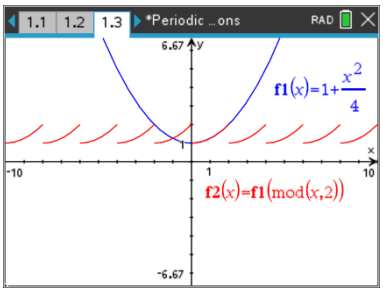
Teacher Note: Spend time on Topic 3.1 to emphasis vocabulary and the transformations from Unit 1 and 2 with "nontraditional" periodic functions.

Demonstration of the generation of a periodic function on the TI-Nspire.
The demonstration of generating a periodic function using the modulus function is beneficial to the teacher in writing or creating questions and documents to use in class. The demonstration of using the TI-84 follows the demonstration on the TI-Nspire.

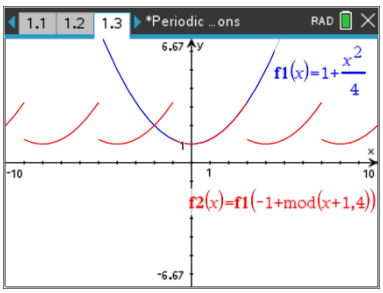
Technology Tip: The mod (modulus) function is an operator that gives the remainder. The operator mod(8, 3) yields 2 because 8 divided by 3 has a remainder of 2.



Graph $f1(x)=1+\frac{x^2}{4}$. We want to repeat the piece of the function from $x=0$ to $x=2$ to create a periodic function.
Graph $f2(x)=f1(\text{mod}(x,2))$.

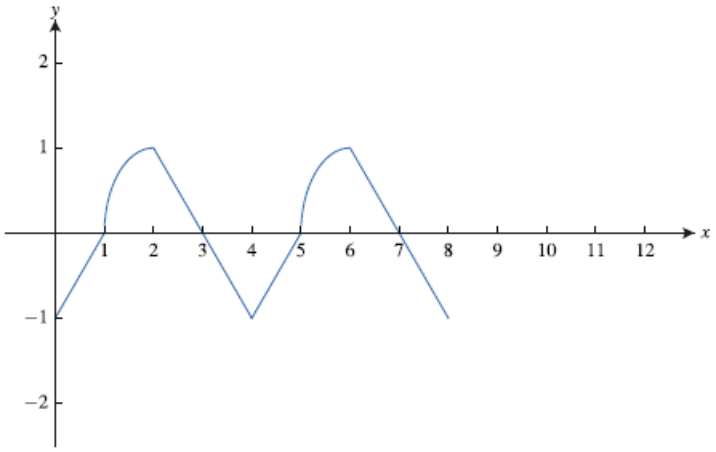


Now try $f2(x)=f1(-1+\text{mod}(x+1,4))$.



Problem 2.

The graph of a periodic function f is shown below.



(a) Sketch another cycle of the function on the interval $[8,12]$.

(b) Find $f(14)$ and $f(-1)$.

(c) Find the open intervals for $0 \le x \le 8$ on which the function is increasing and concave down.

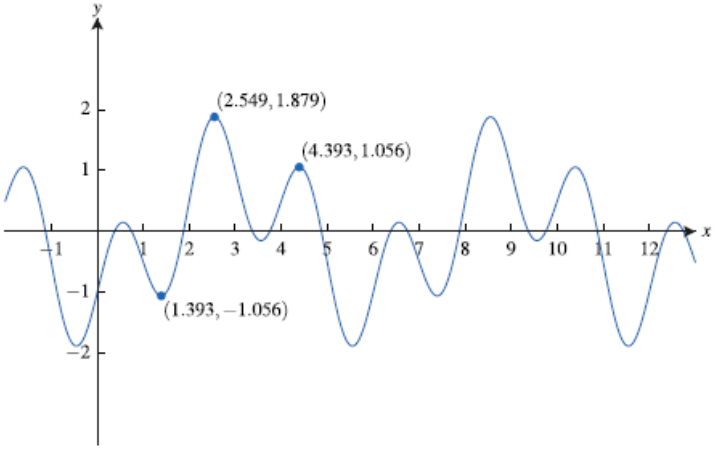
(d) Find the open intervals for $0 \le x \le 8$ on which the function is decreasing and concave up.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 3.

The graph of a periodic function f is shown below.



(a) Write an expression for a function g that is a horizontal translation of the graph of f which would be the exact same graph as that of f.

(b) Using the period of f, find the number of complete cycles of the graph of f in the xy - plane on the interval $0 \le x \le 350$.

Sample Solution:

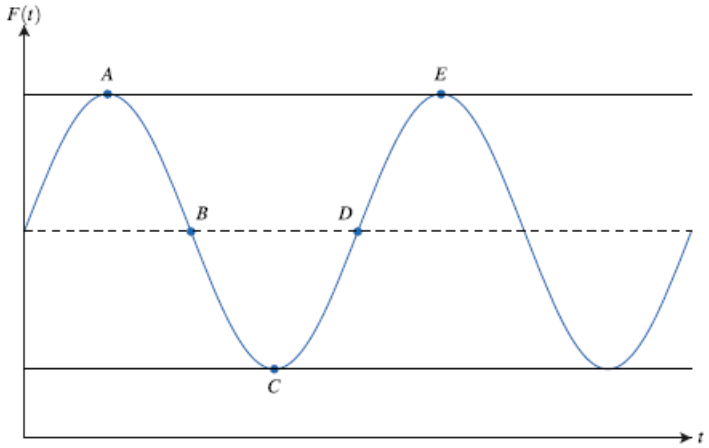
Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 4.

The blades of a large industrial fan rotate in a clockwise direction and rotate at a rate of 10 revolutions per second. Let point *A* be at the tip of the blade that is straight up at time *t* = 0. Point *A* is 75 inches from the floor. Each blade has a length of 14 inches from the center.

Let the periodic function *F* model the distance between point *A* and the floor, in inches, as a function of time *t* in seconds.

(a) Use the given information to find possible coordinates $\left(t, F\left(t\right)\right)$ of the points *A, B, C, D,* and *E* on the graph below.



(b) Use the graph of $y = F\left(t\right)$ and the intervals from A to B, B to C, etc. to find an interval on which the graph of F is increasing and concave down.

(c) Find an interval on which the graph of F is decreasing and concave down.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The graphing application can be used to explore periodic functions.
- The graphing application can be used to explore the behavior of a periodic function.

For more videos from the AP Precalculus Live series, visit our playlist
https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI

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