EXPLORATIONS ™

Activity 3

- Use the CellSheet™ application to organize data

- Find two positive integers equal to or greater than zero with a given sum and the maximum product

Finding the
Maximum Product

Introduction

With the CellSheet™ App, you can organize data into a spreadsheet and look for patterns in your data. These patterns can give you hints to the solution of a problem. In this activity, you will practice organizing data and looking for patterns.

Problem

Find the two positive integers whose sum is equal to 32 and whose product is the greatest.

One way to solve the problem is to use a logical and organized procedure to look at the product of all the pairs of positive integers whose sum is 32. You are going to list all the positive integers whose sum is 32. Then you are going to find the product of each pair and see which product is the greatest. This would take quite a while if all of the calculations were done manually. However, with a spreadsheet, the calculations can be done automatically and very quickly.

Exploration

1. Create a new spreadsheet with the CellSheet App, and name it **MAX**.

2. Enter **0** in cell A1 and **32** in cell B1.

3. a. Type the formula **=A1*B1** in cell C1 to find the product of the two numbers, 0 and 32.

MAX	A	B	C
1	0	32	
2			
3			
4			
5			
6			
C1: =A1*B1			

b. Type the formula **=A1+1** in cell A2.

c. Type the formula **=$B$1–A2** in cell B2.

Note: The $ symbol is typed with the keystrokes ⎡2nd⎤ ⎡RCL⎤.

d. Type the formula **=A2*B2** in cell C2.

MAX	A	B	C
1	0	32	0
2	1	31	31
3			
4			
5			
6			
A2:C2			Paste Menu

With the three formulas you entered in row 2, you have created patterns.

What pattern do you notice in column A? _____

What pattern do you notice in column B? _____

What pattern do you notice in column C? _____

If you copy these formulas to other cells, they will continue to perform the same calculations but on cells in relatively the same place. The exception is that the formula in cell B2 will always refer to cell B1 since that cell is anchored in the formula because of the dollar signs **(=$B$1–A3)**. **$B$1** is called an *absolute reference*. That is, as this formula is copied, that part of the formula preceded by a dollar sign will not change.

4. Copy the contents of cells A2, B2, and C2 down through row 33.

MAX	A	B	C
1	0	32	0
2	1	31	31
3			
4			
5			
6			
A2:C2			Paste Menu

a. Place the cursor on cell A2, press ⎡Y=⎤, and move the cursor to the right, highlighting cells A2, B2, and C2.

b. Press ⎡ZOOM⎤ to select **Copy**, then move the cursor to cell A3 and press ⎡TRACE⎤ to paste the formulas.

c. Move the cursor to cell A4, and again press ⎡TRACE⎤.

As you paste each row, watch the values in column C. What do you notice about the values?

Note: Copying formulas may take time, so be patient.

As you paste, you also want to look at the formulas that were copied. You will notice that they use cells in the same relative position as did the formulas in row 3.

d. Continue this process until you have pasted the formulas into row 33.

Note: There are other ways to copy, but this row-by-row method allows you to see each row as it is created.

MAX	A	B	C
28	27	5	135
29	28	4	112
30	29	3	87
31	30	2	60
32	31	1	31
33	32	0	0
A33: =A32+1			[Menu]

5. Press [2nd] [QUIT] to end the copying mode.

All the possible two-digit sums are exhausted by row 33. Take a look at the columns. What do you notice about the numbers?

What patterns can you find in column A? column B? column C?

What happens to the addends after row 16?

This property, called *symmetry,* is an important concept of mathematics.

MAX	A	B	C
15	14	18	14
16	15	17	15
17	16	16	16
18	17	15	17
19	18	14	18
20	19	13	19
A20: =A19+1			[Menu]

Look at column C. Is there a pattern in the rate of change from one product to the next?

6. Move to cell D2. Enter the formula **=C2–C1**. Copy that formula into the range from cell D3 to D33.

What pattern do you notice in column D?

MAX	B	C	D
28	5	135	
29	4	112	
30	3	87	
31	2	60	
32	1	31	
33	0	0	
D3:D33		[Paste]	[Menu]

7. To take a look at the data graphically, select **Menu** by pressing GRAPH. Select **Charts** > **Scatter**. Enter the **XRange** and **YRange** values as shown. The notation **A1:A33** means that the cells from A1 to A33 are included.

```
SCATTER CHART
XRange:A1:A33
YRange1:C1:C33
YRange2:
YRange3:
Title:
AxesOn AxesOff
DrawFit Draw
```

8. Press ENTER until the scatter plot appears. The graph depicts the product as a function of the first positive integer.

    Notice two things: there is a maximum, and the graph is symmetric.
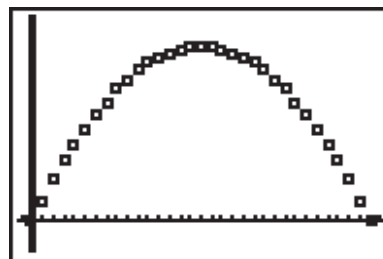
9. Press 2nd [QUIT] to return to the spreadsheet.

Student Worksheet

Name _____

Date _____

Navigating the CellSheet™ Application

1. If you copy a formula from one cell and paste it into another, the references in the formula will change relative to the cells. To keep a reference in a formula as an absolute value, you put a _____ before the letter and number of the cell by pressing _____.

Solving the Problem

2. Complete this table using the information found in the CellSheet App.

First Number	Second Number	Product
0	32	0
1	31	31
2	30	
3	29	
4	28	
5	27	
6	26	
7	25	
8	24	
9	23	
10	22	
11	21	
12	20	
13	19	
14	18	
15	17	
16	16	

3. The two integers equal to or greater than zero whose sum is 32 and whose product is the greatest are _____ and _____. The largest product possible with two integers equal to or greater than zero whose sum is 32 is ____.

4. The table in Question **2** shows all the possible combinations of positive integers whose sum is 32 but is only half the size of the table you constructed in the spreadsheet. The mathematical property that explains why the products of the first 15 pairs of numbers are the same as the last 15 pairs of numbers is

_____. (Notice that 16 + 16 is not repeated.)

Analyzing the Data

5. The table has several patterns. The values in column A increase by _____ each time as the values in column B decrease by _____. The values in column C, the products, increase each time up to row 16, but the rate of increase decreases by _____ for each new pair of numbers. This pattern is demonstrated in the table in Question **6**.

6. Complete the table.

First #	Second #	Product	Difference of Successive Products
0	32		
1	31	31	31
2	30	60	29
3	29	87	27
4	28		
5	27		
6	26		
7	25		
8	24		
9	23		
10	22		
11	21		
12	20		
13	19		
14	18		
15	17		
16	16		

7. Modify the existing spreadsheet to find the two integers equal to or greater than zero whose sum is 40 and whose product is the greatest. You will need to change the content of cell B1.

8. Find the two integers equal to or greater than zero whose sum is 29 and whose product is the greatest.

9. What generalization or rule can you make about finding two integers equal to or greater than zero whose sum is an even number and whose product is the greatest? What generalization or rule can you make about finding two integers equal to or greater than zero whose sum is an odd number and whose product is the greatest?

Extending the Activity

1. A farmer is building a fence to enclose a rectangular area adjacent to his barn. The fence will enclose three sides of the area, and a side of the barn will enclose the fourth side. What is the largest area that can be enclosed with 120 feet of fence? What is the minimum length required for the side of the barn to be able to enclose this area?

   _____

   Create a spreadsheet with the CellSheet™ App that allows you to investigate each possible combination of positive integers that represent the width and length of the enclosed area.

Teacher Notes

A c t i v i t y 3

Finding the Maximum Product

Preparation

The problem of finding the two positive integers whose sum is 32 and whose product is the greatest is not necessarily a difficult one. However, the strategy and organizational skills that this activity promotes are important. Students will see the benefits of using a spreadsheet to perform the tedious calculations that must be done to exhaust the possibilities of this search. The problem in this activity requires students to think about how the search is performed.

Elicit Questions

We want students to find two positive integers whose sum is 25 and whose product is the greatest. What two positive integers can fit both rules? What generalization can students make based on their answers? Check if the generalization will hold in all cases.

A	B	A times B
7	18	126
8	17	136
9	16	144
10	15	150
11	14	154
12	13	156

Management

Give students some time to look for patterns in the data presented. Have students offer as many patterns as they can find. Being able to identify patterns in numerical data is essential to mathematical fluency.

Answers to Exploration Questions

3. Answers will vary. At this point, students can formulate hypotheses about the patterns. Later in the Exploration, they will confirm their hypotheses. The values in column A increase by 1, the values in column B decrease by 1, and the values in column C increase from zero to 31.

5. Values in column C increase to row 16, then begin to decrease.

   Answers will vary. Students should note that values in column A increase by one while values in column B decrease by one. Values in column C increase to row 16, then begin to decrease.

   After row 16, the addends are inverted from the rows before 16 so that in row 17 the two addends are 17 and 15 and in row 15 they were 15 and 17.

6. The difference in values decreases by 2 at each row.

Answers to the Student Worksheet

Navigating the CellSheet™ Application

1. **$**; 2nd STO▶

Solving the Problem

2. Complete this table using the information found in the CellSheet App.

First Number	Second Number	Product
0	32	*0*
1	31	*31*
2	30	*60*
3	29	*87*
4	28	*112*
5	27	*135*
6	26	*156*
7	25	*175*
8	24	*192*
9	23	*207*
10	22	*220*
11	21	*231*
12	20	*240*
13	19	*247*
14	18	*252*
15	17	*255*
16	16	*256*

3. 16, 16, 256

4. Symmetry

5. One, one, two

6.

First #	Second #	Product	Difference of Successive Products
0	32	*0*	
1	31	*31*	31
2	30	*60*	29
3	29	*87*	27
4	28	*112*	25
5	27	*135*	23
6	26	*156*	21
7	25	*175*	19
8	24	*192*	17
9	23	*207*	15
10	22	*220*	13
11	21	*231*	11
12	20	*240*	9
13	19	*247*	7
14	18	*252*	5
15	17	*255*	3
16	16	*256*	1

7. The two positive integers whose sum is 40 that have the greatest product are 20 and 20. The product is 400.

8. The two positive integers whose sum is 29 that have the greatest product are 14 and 15. The product is 210.

9. If the number is even, the two integers are the same and will be half of the even number. If the number is odd, half of the number is not an integer. So, the two integers are the integers on either side of half of the number.

Extending the Activity

1. Enter **120** in cell A1. For simplicity, start with **40** in cell A2 as the length of the side opposite the barn. The formula **=($A$1−A2)/2** should be entered in cell B2. This formula calculates the other dimension of the area. The formula in cell C2, **=A2*B2**, calculates the area of the fenced-in region.

   The enclosed area increases as the side opposite the barn increases. This continues until that side reaches a length of 60 feet.

   From the spreadsheet, it is apparent that the maximum area is obtained when the side opposite the barn is 60 feet long and the other two sides are 30 feet long. The maximum area, under these conditions, is 1,800 square feet.