Spring Constant – ID: 9901

By Irina Lyublinskaya

Time required
45 minutes

Topic: Force and Motion

- *Predict and describe the effect of balanced forces on an object.*
- *Measure or calculate the net force on an object.*
- *Determine a spring constant.*

Activity Overview

In this activity, students explore the relationship between displacement and restoring force for an elastic spring. Students use simulations to compare springs with various spring constants in spring-mass systems and determine the relationship between spring constant and restoring force for the same displacement, and between spring constant and displacement for the same restoring force. Based on these explorations, students then solve problems involving calculating spring constants.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *Vernier Dual-Range Force Sensor*
- *copy of the student worksheet*
- *pen or pencil*
- *mass hanger with masses*
- *EasyLink™ or Go!™Link interface*
- *spring*
- *safety goggles*
- *ruler*
- *ring stand and clamp*

TI-Nspire Applications

Graphs & Geometry, Data & Statistics, Notes, Calculator

Teacher Preparation

Before carrying out this activity, you should review with students the concept of static equilibrium.

- *The screenshots on pages 2–10 demonstrate expected student results. Refer to the screenshots on pages 11 and 12 for a preview of the student TI-Nspire document (.tns file). Pages 13–15 show the student worksheet.*
- ***To download the .tns file and student worksheet, go to education.ti.com/exchange and enter "9901" in the search box.***

Classroom Management

- *This activity is designed to be **student-centered**, with the teacher acting as a facilitator while students work cooperatively. The student worksheet guides students through the main steps of the activity and includes questions to guide their exploration. Students should record their answers to the questions on blank paper.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

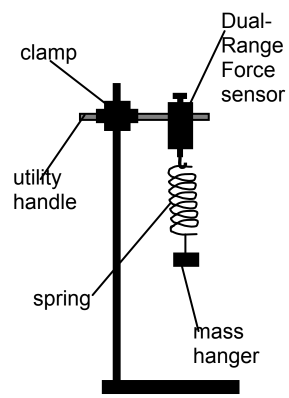
The following questions will guide student exploration during this activity:

- What is the relationship between the restoring force of a stretched elastic spring and the displacement of the spring?
- What is the meaning of the spring constant?

The purpose of this activity is to provide students with an opportunity to explore the relationship between the restoring force of an elastic spring and its displacement and to derive Hooke's law. The activity also gives students a chance to define the spring constant and analyze several situations illustrating the behavior of springs with various spring constants.

This activity consists of three problems. In the first problem, students explore the relationship between the restoring force of an elastic spring and its displacement, derive Hooke's law, and define the spring constant. In the second problem, students compare springs with different spring constants in simulated experiments. In the third problem, students apply these ideas to problem solving.

Problem 1 – Hooke's law



Step 1: Students should open the file **PhyAct_9901_Spring_Constant.tns**, read the first two pages, and then move to page 1.3, which contains an empty *Data & Statistics* application. Then, students should set up the spring and force sensor to collect data. They should use the utility handle to connect the force sensor to the ring stand with the clamp. The hook on the force sensor should be pointing downward. They should hang the spring off the hook on the force sensor, and hang the mass hanger off the bottom of the spring, as shown in the diagram to the right. (Make sure the switch on the force sensor is set to the correct range. The range used will depend on the amount of mass students apply to the spring. Make sure the springs used are stretchy enough to extend measurably when moderate amounts of mass are added.) When students have set up their experiments, they should answer question 1.

Q1. What do you think is the relationship between the length of the spring (that is, its extension from its equilibrium position) and the force the spring is exerting?

A. *Students' answers will vary.*

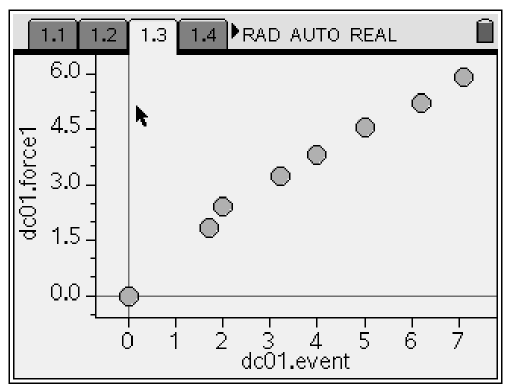
Step 2: Next, students should move to page 1.3, insert a new data collection box (ctrl D), and connect the EasyLink or Go!Link connector to their handheld or computer. A force measurement should appear in the data collection box. Students should wait until the measurement has stabilized, and then zero the sensor. They set up the data collection to **Events with Entry** mode, and begin the experiment.

Step 3: Next, students collect a data point representing the equilibrium position of the spring (i.e., the length of the spring when no mass is attached.)

Step 4: Students add a small amount of mass to the hanger, wait for the reading to stabilize, measure the length of the spring, and collect another data point.

Step 5: Students repeat Step 4 several more times, until they have collected at least six data points.

Step 6: When students have collected all of their data points, they stop the experiment, close the data collection box, and disconnect the force sensor.



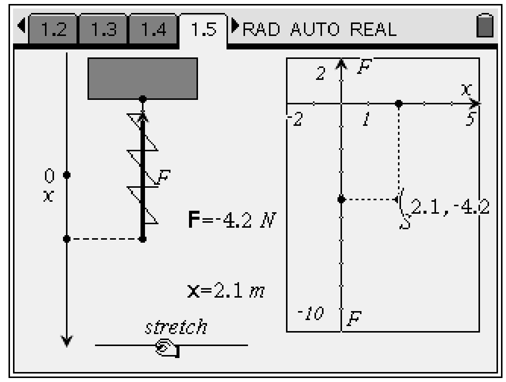
Step 7: Students examine a plot of stretch vs. restoring force (**dc01.force1** vs. **dc01.event**). If the graph does not automatically appear on page 1.3, students can create the graph by selecting **dc01.event** for the *x*-axis and **dc01.force1** for the *y*-axis. After students examine the graph, they should answer questions 2 and 3.

Q2. Describe the shape of the graph.

A. *The length of the spring seems to be directly proportional to the force exerted by the spring.*

Q3. Does the graph match the prediction you made in question 1? If not, explain why you made the prediction you did. What assumptions did you make that were incorrect?

A. *Students' answers will vary. Encourage metacognitive thinking to help students identify any errors in their reasoning.*



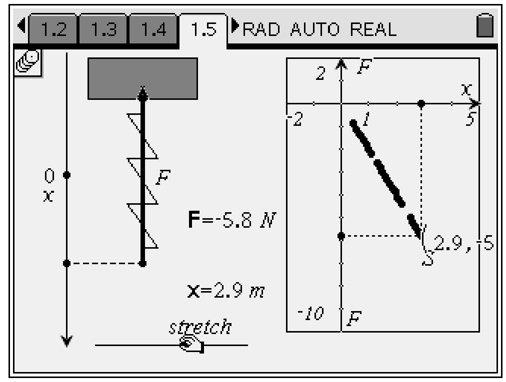
Step 8: Next, students should read the text on page 1.4 and then move to page 1.5, which shows a simulation of a spring attached to a fixed object; it is similar to the setup students just used to collect data. In this simulation, the natural (rest) length of the spring is 2.5 m. The force is calculated using the equation $F = -kx$, where $k = 2$ N/m. The displacement (x) is shown on the x-axis of the graph on the page, and the force is shown on the y-axis. Thus, point S has coordinates (x, F). (The x-axis is also shown on the simulation diagram for reference.) Students should use the *stretch* slider to change the stretch of the spring and observe the effect of the stretch on the restoring force. Students can observe the physical stretching of the spring, the change in the magnitude of the restoring force vector, and the movement of point S on the graph as they vary the displacement (stretch) of the spring. The stretch values range from 0 m to 5 m. The range of values for force is from 0 N to 10 N. Students should vary the displacement of the spring and then answer questions 4 and 5.

Q4. Describe the relationships between the stretch (displacement) of the spring and the magnitude and direction of the restoring force.

A. *The restoring force is always opposite in direction to the spring's displacement, and the magnitude of the force is directly proportional to the magnitude of the displacement.*

Q5. Does this simulation show the same relationship between stretch and restoring force that you observed in your data collection?

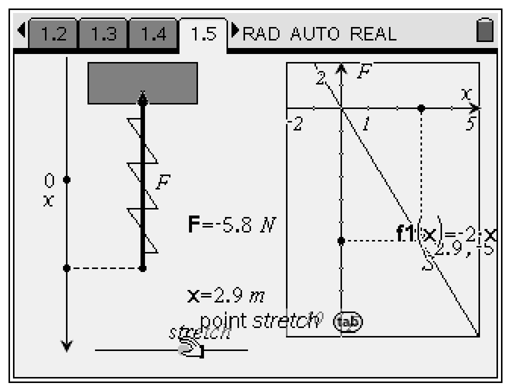
A. *Students' answers will vary, but if they were reasonably precise in collecting the data, the relationships should be similar.*



Step 9: Next, students should use the **Geometry Trace** tool (**Menu > Trace > Geometry Trace**) to put a trace on point *S* in the graph on page 1.5. Note: Make sure students do not press (esc) after selecting point *S* with the **Geometry Trace** tool. After students have selected point *S,* they should use (ctrl)(click) to grab point *stretch* and drag it along the slider and observe the shape of the trace. Then, they should answer question 2.

Q6. What mathematical relationship does there appear to be between *x* and *F*? That is, what equation of the form *F*(*x*) appears to describe the locations of point *S* as the string is stretched?

A. *Based on the geometry trace and observed values of* F *and* x, *students should be able to determine that the equation F(x) = –2x fits the data on the location of point* S*.*



Step 10: Next, students erase the geometry trace (**Menu > Trace > Erase Geometry Trace**). Then, they graph the function *F*(*x*) that they determined in question 6. To do this, they should first press (ctrl)(G) to display the function entry line. They should then enter the equation for *F*(*x*) that they derived in question 6 and press (enter). The graph of the line will appear. They can press (ctrl)(G) again to hide the entry line.

Step 11: Next, students should again vary the stretch of the spring using the *stretch* slider. They should observe whether point *S* follows the function they graphed. They should read the text on pages 1.6 and 1.7, and then answer questions 7–11.

Q7. Was the equation you predicted in question 6 correct? If not, explain any errors in your reasoning. If your prediction was incorrect, find the correct relationship before proceeding with the rest of the questions.

A. *Student answers will vary. Encourage student discussion of any incorrect answers, and make sure students obtain the correct equation before moving on to the rest of the activity.*

Q8. What is the meaning of the slope in this equation?

A. *The slope of the line indicates how much force the string produces for each unit increase in displacement. Encourage student discussion of this point, as some students may not immediately see the relationship.*

Q9. Which would be easier to stretch, a spring with a large spring constant or one with a small spring constant? Explain your answer.

A. *The larger* k *is, the harder it is to stretch the spring. The spring constant represents the stiffness or strength of the spring.*

Q10. What is the spring constant for the spring on page 1.5?

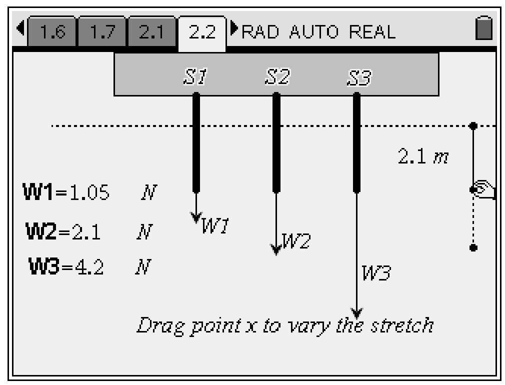
A. *The equation relating force and displacement for this spring is* $F(x) = -2x$*, so the spring constant is* $k = 2$ *N/m.*

Q11. What is the spring constant for the spring you used during the data collection in Steps 1–7? (Hint: Use the **Regression** or **Movable Line** tools to find the slope of the best-fit line through your data points.)

A. *Student answers will vary. Encourage students to compare their results with the listed values of the constants for the springs, and to discuss the possible causes of any differences. Note that, because students recorded the length of the spring (instead of recording the displacement of the spring), the best-fit equation for their data will have the form F = kx instead of F = –kx.*

Problem 2 – Comparing springs with different spring constants

Step 1: Next, students should read the text on page 2.1 and then move to page 2.2. Page 2.2 shows a diagram of three springs that are hanging vertically and have the same natural (rest) length but different spring constants. Three different weights are attached to the free ends of the springs, and all three springs stretch the same distance. Students should vary the stretch by moving slider *x* and observe what weights are required to maintain this relationship. After exploring the simulation, students should answer questions 7 and 8.



Q12. Which spring has the smallest spring constant? Which has the largest spring constant? Answer without doing any calculations. Explain your answers.

A. *According to Hooke's law, the stronger the spring, the more force is required to stretch it the same distance. The largest weight (force) is required to stretch spring 3, so spring 3 is the stiffest, and it has the largest spring constant. Spring 1 requires the least weight (force) to stretch it a given distance, so spring 1 is the least stiff and has the smallest spring constant.*

Q13. Use the *Calculator* application on page 2.3 to calculate the spring constant for each spring.

A. *Students should use the variables defined on page 2.2 in order to complete the calculations. If **x** = 1.9 m, **W1** = 0.95 N, **W2** = 1.9 N, and **W3** = 3.8 N, then the following calculations apply:*

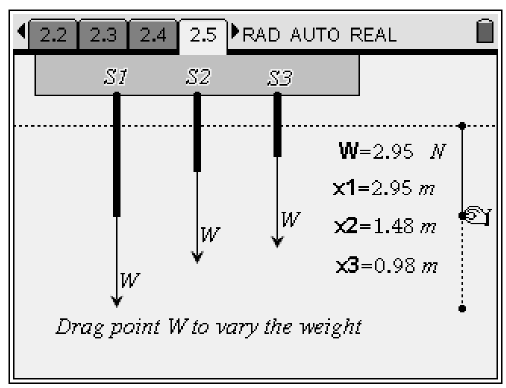
$$k_1 = \frac{\mathbf{W1}}{\mathbf{x}} = \frac{0.95\ N}{1.9\ m} = 0.50\ N/m$$

$$k_2 = \frac{\mathbf{W2}}{\mathbf{x}} = \frac{1.9\ N}{1.9\ m} = 1.0\ N/m$$

$$k_3 = \frac{\mathbf{W3}}{\mathbf{x}} = \frac{3.8\ N}{1.9\ m} = 2.0\ N/m$$

If you wish and time allows, you may discuss with students how they could have set the displacement to allow them to calculate the value of k *for each spring without needing to do any calculations. They should realize that setting* x *at 1 m would allow them to "calculate"* k *for each spring by inspection (i.e., the value of* k *would be equal to the value of* W *for that spring).*

Step 2: Next, students should read the text on page 2.4 and then move to page 2.5. Page 2.5 shows three springs suspended vertically. The same amount of weight is attached to each of the three springs. Students should move slider *W* to vary the weight on the springs and observe the resulting displacements of the springs. Then, students should answer questions 14 and 15.



Q14. Which spring has the smallest spring constant? Which has the largest spring constant? Answer without doing any calculations. Explain your answers.

A. *In this situation, the same weight stretches all three springs, so the spring that stretches the most is the weakest spring and has the smallest spring constant. Similarly, the spring that stretches the least is the strongest spring and has the largest spring constant. Therefore, spring 1 has the smallest spring constant, and spring 3 has the largest spring constant.*

Q15. Use the *Calculator* application on page 2.6 to calculate the spring constant for each spring on page 2.5.

A. *Again, students should use the displayed variables to find the spring constants. If* ***W*** *= 1.85 N,* ***x1*** *= 1.85 m,* ***x2*** *= 0.92 m, and* ***x3*** *= 0.62 m, then the following calculations apply:*

$$k_1 = \frac{\mathbf{W}}{\mathbf{x1}} = \frac{1.85\ N}{1.85\ m} = 1.0\ N/m$$

$$k_2 = \frac{\mathbf{W}}{\mathbf{x2}} = \frac{1.85\ N}{0.92\ m} = 2.0\ N/m$$

$$k_3 = \frac{\mathbf{W}}{\mathbf{x3}} = \frac{1.85\ N}{0.62\ m} = 3.0\ N/m$$

Q16. A spring hangs vertically next to a ruler. The end of the spring is next to the 15 cm mark on the ruler. When a 2.5 kg mass is attached to the end of the spring, the end of the spring lines up with the 46 cm mark. What is the spring constant of this spring? Assume that the mass of the spring is negligible.

A. *Students should use the* Calculator *application on page 2.6 to solve this problem. The rest position of the spring is 15 cm. Therefore, the displacement of the spring is 46 cm – 15 cm = 31 cm. Because the stretched spring is not moving, the forces acting on it must be balanced. This means that the restoring force produced by the spring is equal to the downward force on the spring. The downward force is the weight of the mass, which is given by the equation W = mg. Rearranging the equation F = –kx to solve for* k *yields the equation* $k = -\frac{F}{x}$. *Substituting the given values yields the following:*

$$k = -\frac{F}{x} = -\frac{-(2.5\ kg)(9.8\ m/s^2)}{0.031\ m} = 790\ N/m$$

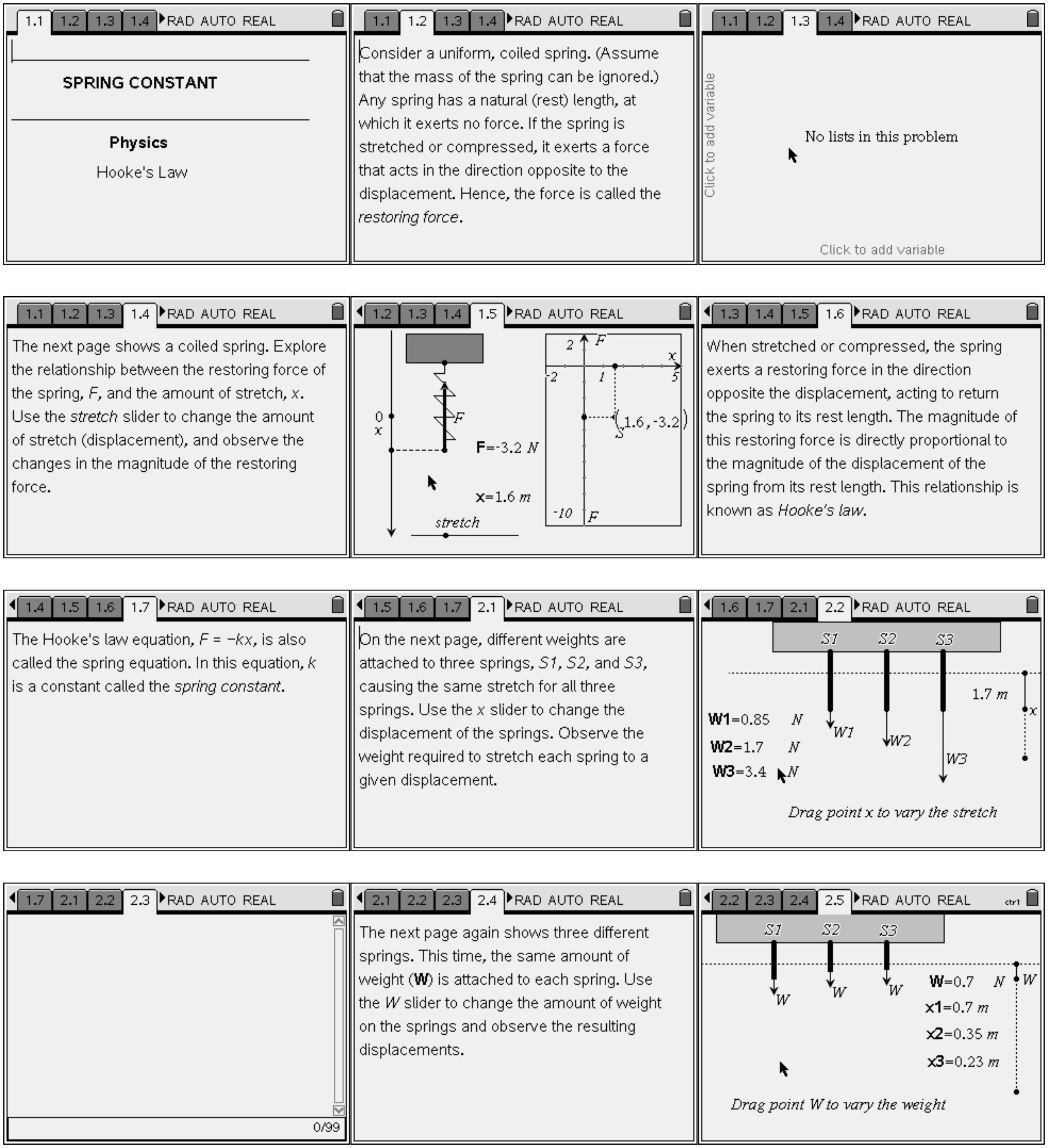
*Note that the restoring force (*F*) is given a negative magnitude because it acts in a direction opposite that of the displacement.*

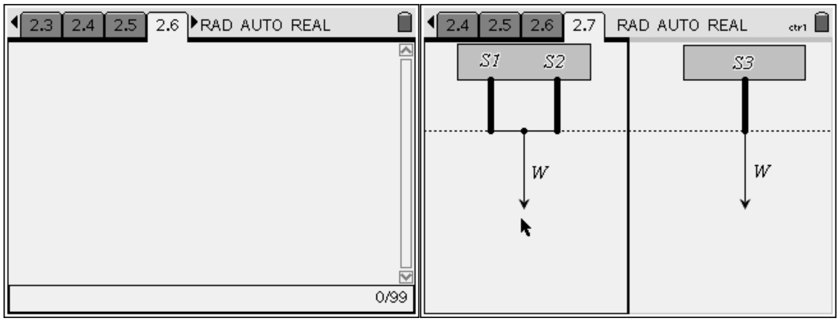
Q17. Two identical springs with spring constants of 3 N/m, *S1* and *S2,* support a weight of 30 N, as shown in the left-hand diagram on page 2.7. Each spring stretches by 10 cm. *S1* and S*2* are then replaced by a single spring, *S3,* that stretches the same distance under the same weight. What is the spring constant of *S3?*

A. *Students should be able to reason that the single spring in the second situation has to be as strong as the two springs in the first situation, so the single spring's spring constant should be twice as large as those of the two initial springs (that is, 6 N/m). To verify this reasoning, students should apply the ideas of static equilibrium to both situations, as shown below:*
two springs: $W = -2k_1x$
one spring: $W = -k_2x$
$-2k_1x = -k_2x$
$k_2 = 2k_1$

Spring Constant – ID: 9901

(Student)TI-Nspire File: *PhyAct_9901_Spring_Constant.tns*




2.3 2.4 2.5 2.6 RAD AUTO REAL
0/99
2.4 2.5 2.6 2.7 RAD AUTO REAL
S1 S2
S3
W
W

Spring Constant

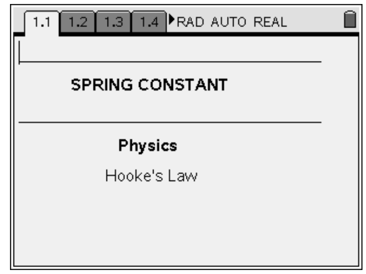
ID: 9901

Name ______________________

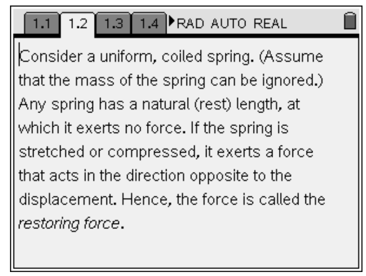
Class ______________________

In this activity, you will explore the following:

- *the relationship between the displacement of a spring and the restoring force exerted by the spring*
- *the effects of the spring constant on the characteristics of a spring*



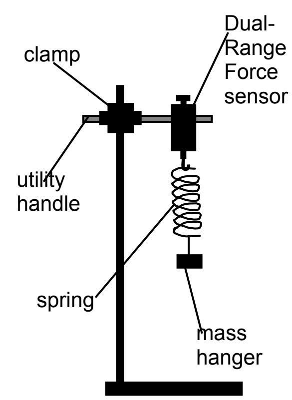
Open the file **PhyAct_9901_Spring_Constant.tns** on your handheld or computer and follow along with your teacher for the first two pages. Move to page 1.2 and wait for further instructions from your teacher.



In this activity, you will collect data on the length of a spring with different amounts of mass attached to it. You will use your data to determine the relationship between length and restoring force. You will then use your data, together with several simulations, to learn about spring constants and their effects on the characteristics of springs.

Problem 1 – Hooke's law

Step 1: In the first part of this activity, you will collect data on restoring force and the length of a spring. First, you must set up the spring and force sensor to collect data. Use the utility handle to connect the force sensor to the ring stand with the clamp. The hook on the force sensor should be pointing downward. Hang the spring off the hook on the force sensor, and hang the mass hanger off the bottom of the spring. Your setup should resemble the diagram at right. Once your experiment is set up, answer question 1.



Q1. What do you think is the relationship between the length of the spring (that is, its extension from its equilibrium position) and the force the spring is exerting?

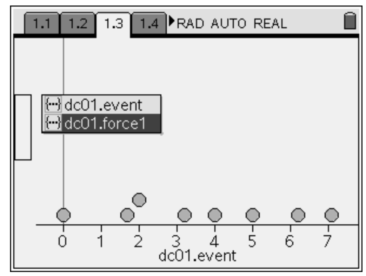
Step 2: Move to page 1.3, insert a new data collection box (ctrl D), and connect the EasyLink or Go!Link connector to your handheld or computer. A force measurement should appear in the data collection box. Wait until the measurement has stabilized, and then zero the sensor (**Menu > Sensors >Zero**). Set up the data collection to **Events with Entry** mode (**Menu > Experiment > Set Up Collection > Events with Entry**), and begin the experiment by pressing the "play" button (►).

Step 3: Use the ruler to measure the length of the spring. This is the equilibrium length. You will be recording the displacement of the spring from this position as you add mass to the mass hanger. Press the button in the lower left corner of the data collection box. A dialog box should open. Type 0 for the event (because the displacement of the spring from its equilibrium position is 0 cm) and click OK.

Step 4: Add some mass to the hanger. The amount of mass you add should be enough to stretch the spring a small amount. Wait until the force reading has stabilized and the spring is no longer moving. Measure the length of the spring. Press the lower left button in the data collection box to record this data point. The event value you should enter should be the difference between the length of the spring and the equilibrium length. For example, if the equilibrium length was 5.5 cm, and the new length is 6.2 cm, then the value you should enter is 0.7.

Step 5: Repeat Step 4 at least five more times, so that you have at least six data points. Add a small amount of mass to the mass hanger at each step.

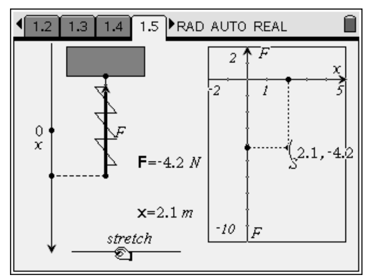
Step 6: When you have collected all of your data points, end the experiment by clicking on the upper left-hand box in the data collection box. Close the data collection box and disconnect the sensor from the TI-Nspire.



Step 7: The data you collected should be displayed in the *Data & Statistics* application on page 1.3. If they are not, use the application to display the data. Use **dc01.event** as your *x*-values and **dc01.force1** as your *y*-values. Then, answer questions 2 and 3.

Q2. Describe the shape of the graph.

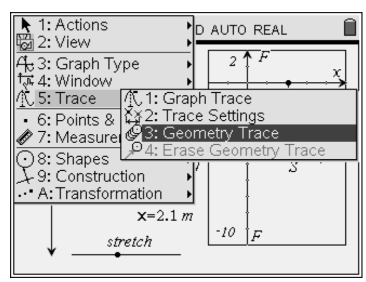
Q3. Does the graph match the prediction you made in question 1? If not, explain why you made the prediction you did. What assumptions did you make that were incorrect?



Step 8: Read the text on page 1.4, and then move to page 1.5. Page 1.5 shows a simulation of a spring attached to a fixed object; it is similar to the setup you just used to collect data. Vary the displacement of the spring using the *stretch* slider, and then answer questions 4 and 5.

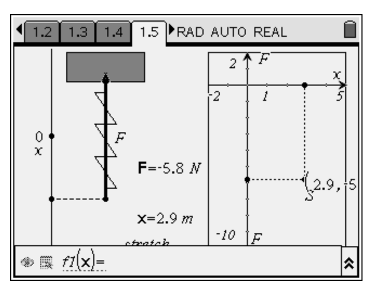
Q4. Describe the relationships between the stretch (displacement) of the spring and the magnitude and direction of the restoring force.

Q5. Does this simulation show the same relationship between stretch and restoring force that you observed in your data collection?



Step 9: Next, use the **Geometry Trace** tool (**Menu > Trace > Geometry Trace**) to put a trace on point *S* on page 1.5. Then, drag point *stretch* and slide it along the line segment. Observe the resulting graph of point *S*. Then, answer question 6.

Q6. What mathematical relationship does there appear to be between *x* and *F*? That is, what equation of the form *F*(*x*) appears to describe the locations of point *S* as the string is stretched?



Step 10: Next, erase the **Geometry Trace** (**Menu > Trace > Erase Geometry Trace**). Press ctrl G to show the function line. Enter the equation you identified in question 6.

Step 11: Use the *stretch* slider to change the extension of the spring. Observe whether point *S* appears to follow the function you graphed. Read the text on pages 1.6 and 1.7, and then answer questions 7–11.

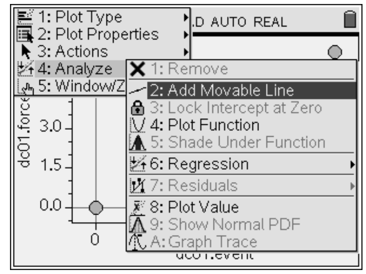
Q7. Was the equation you predicted in question 6 correct? If not, explain any errors in your reasoning. If your prediction was incorrect, find the correct relationship before proceeding with the rest of the questions.

Q8. What is the meaning of the slope in this equation?

Q9. Which would be easier to stretch, a spring with a large spring constant or one with a small spring constant? Explain your answer.

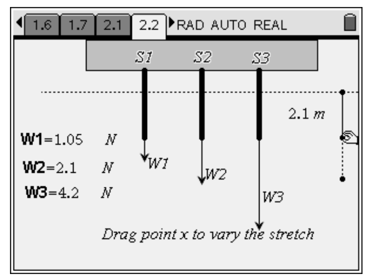
Q10. What is the spring constant for the spring on page 1.5?

Q11. What is the spring constant for the spring you used during the data collection in Steps 1–7? (Hint: Use the **Regression** or **Movable Line** tools to find the slope of the best-fit line through your data points.)



Problem 2 – Comparing springs with different spring constants

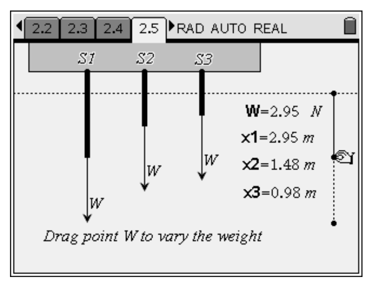
Step 1: Read the text on page 2.1 and then move to page 2.2. Page 2.2 shows a diagram of three springs that are hanging vertically and have the same natural (rest) length but different spring constants. Three different weights are attached to the free ends of the springs, and all three springs stretch the same distance. Vary the stretch by moving slider *x* and observe what weights are required to maintain this relationship. After exploring the simulation, answer questions 12 and 13.



Q12. Which spring has the smallest spring constant? Which has the largest spring constant? Answer without doing any calculations. Explain your answers.

Q13. Use the *Calculator* application on page 2.3 to calculate the spring constant for each spring.

Step 2: Next, read the text on page 2.4 and then move to page 2.5. Page 2.5 shows three springs suspended vertically. The same amount of weight is attached to each of the three springs. Move slider *W* to vary the weight on the springs and observe the resulting displacements of the springs. Then, answer questions 14–17.



Q14. Which spring has the smallest spring constant? Which has the largest spring constant? Answer without doing any calculations. Explain your answers.

Q15. Use the *Calculator* application on page 2.6 to calculate the spring constant for each spring on page 2.5.

Q16. A spring hangs vertically next to a ruler. The end of the spring is next to the 15 cm mark on the ruler. When a 2.5 kg mass is attached to the end of the spring, the end of the spring lines up with the 46 cm mark. What is the spring constant of this spring? Assume that the mass of the spring is negligible.

Q17. Two identical springs with spring constants of 3 N/m, *S1* and *S2,* support a weight of 30 N, as shown in the left-hand diagram on page 2.7. Each spring stretches by 10 cm. *S1* and S2 are then replaced by a single spring, *S3,* that stretches the same distance under the same weight. What is the spring constant of *S3?* Suppose the angle of your ramp were larger than it is. How would the acceleration coefficient in these equations change? Explain your answer.

