Finding the Shortest Path

Exploration 2

Most geometry utilities have reflection tools for finding the images of objects. To use this feature, you must first define a mirror line and the points to be reflected.

a.

1. Construct a mirror line segment, $\overline{AB}$.
2. Construct two points, *P* and *Q*, on the same side of $\overline{AB}$.
3. Reflect points *P* and *Q* in $\overline{AB}$ and label the images P' and Q', respectively.
4. Construct $\overline{PQ'}$ and $\overline{QP'}$.
5. Construct a point *C* at the intersection of $\overline{AB}$ and $\overline{PQ'}$.
6. Measure $\angle PCA$ and $\angle QCB$.
7. Construct $\overline{PP'}$ and $\overline{QQ'}$.
8. Label the intersection of $\overline{AB}$ and $\overline{PP'}$ as point *D* and the intersection of $\overline{AB}$ and $\overline{QQ'}$ as point *E*. Your construction should now resemble the one in Figure **11** below.

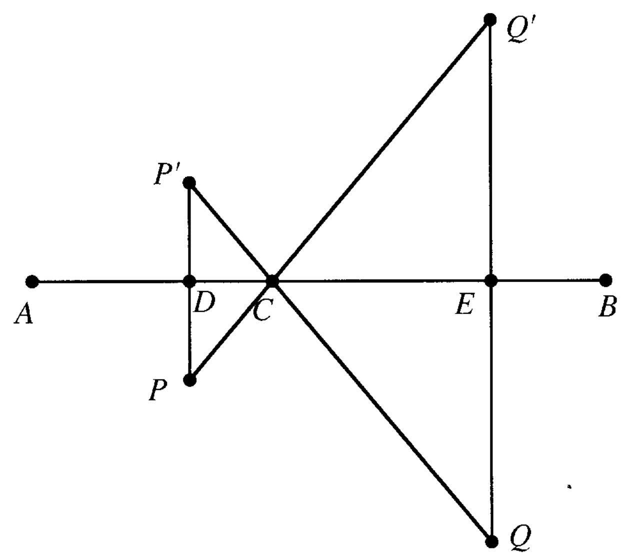

Figure 11: Reflection modeled on a geometry utility

9. Measure $\angle PDA$ and $\angle QEB$.
10. Measure the distances from $\overline{AB}$ to points *P*, P', *Q*, and Q'.
11. Measure $\overline{EQ}$ and $\overline{EQ'}$.

b. Use your measurements in Part **a** to make a conjecture about the relationship between $\overline{AB}$ and $\overline{QQ'}$.

c. Select and move various points on your construction. Note any relationships you observe among the measurements of angles and segments as you move the points.

d. Complete Steps **1–3** below using your construction from Part **a**.

1. Construct a point S anywhere on $\overline{AB}$.
2. Measure the total distance traveled when moving from Q to S and then from S to P.
3. By moving S along $\overline{AB}$, find the point where the total distance traveled in Step **2** is shortest.

Mathematics Note

The **perpendicular bisector** of a segment is the line that is perpendicular ($\perp$) to the segment and divides the segment into two congruent parts.

In Figure **12**, for example, line m is the perpendicular bisector of $\overline{CC'}$ because line m and $\overline{CC'}$ are perpendicular and $\overline{CM} \cong \overline{C'M}$.

A **reflection** in a line is a pairing of points in a plane so that the **line of reflection** (or **mirror line**) is the perpendicular bisector of every segment connecting a point in the preimage to its corresponding point in the image. Every point on the line of reflection is its own image.

In Figure **12**, parallelogram $C'D'E'F'$ is the image of parallelogram $CDEF$. Line m is the line of reflection since it is the perpendicular bisector of each segment joining a point in the preimage to its corresponding point in the image.

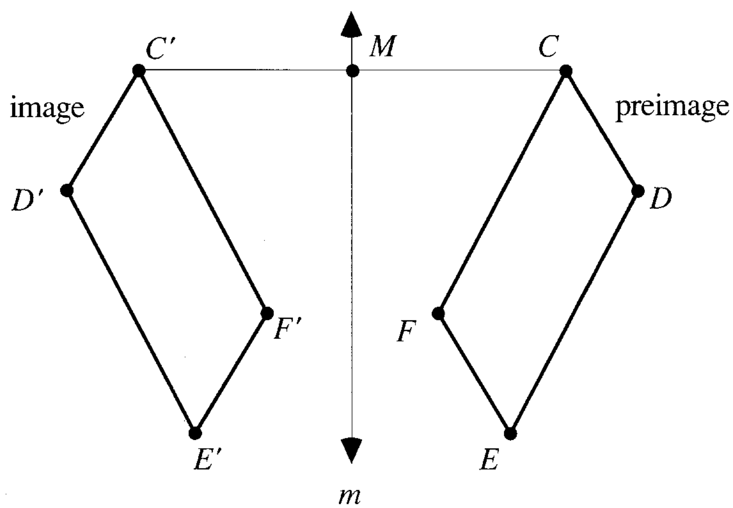

Figure 12: Reflection of parallelogram *CDEF* in line *m*