EXPLORATIONS

Activity 10

That's a Stretch

Objectives

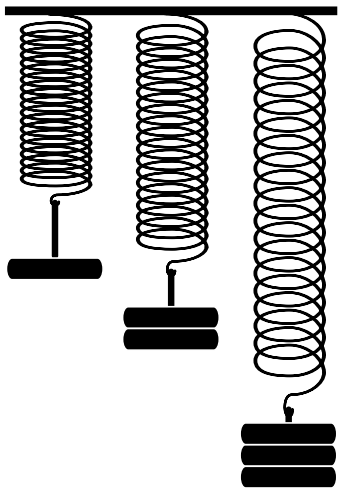
- To determine the relationship between the stretch of a spring and the number of weights in a cup suspended from the spring
- To find the *y* value of a function, given the *x* value
- To use interpolation to find unknown values
- To use technology to find a best fit line
- To use technology to plot a set of ordered pairs

Materials

- TI-83 Plus
- Calculator Based Ranger™ (CBR™) — optional
- Slinky® cut in half, one per group
- Small bathroom paper cups or film containers, one per group
- Marbles, pennies, or other small objects such as cubes, at least 40 per group
- Large paper clips, one per group
- Meter stick, one per group

Introduction

When you bounce a basketball, the shape of the ball temporarily changes. When you pluck a string on a guitar, the shape of the string changes. Once you release the string, it springs back to its original form. When a weight is suspended from a spring, the spring stretches. If additional weights are added, the spring stretches even more. Once the weights are removed, the spring returns to its original shape.



The basketball, the string on the guitar, and the spring are said to be *elastic*. If an external force is applied to an object, it creates stress within the object that causes it to become deformed. *Elasticity* is the property of a body that causes it to return to its initial size and shape after being compressed or stretched. Not all materials return to their initial state after a force is applied. These materials are said to be *inelastic*. Some examples of objects that are inelastic are clay, lead, and dough.

For many materials, the amount of stretch or compression is directly proportional to the applied force. This relationship was first expressed by British physicist Robert Hooke and is known as *Hooke's Law*.

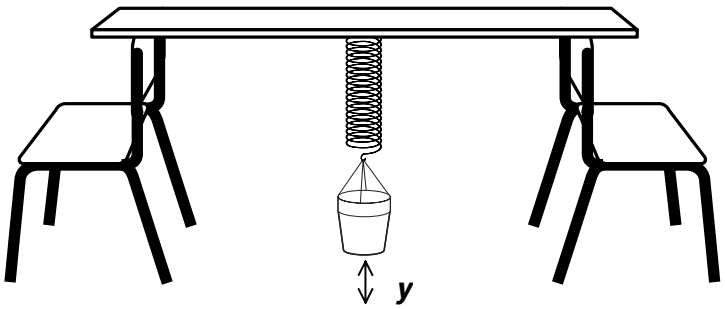
Problem

Steel is an *elastic* material. Many springs are constructed of steel. What would happen if you suspended objects from a steel spring? Would the spring stretch at a constant rate or an exponential rate?

Collecting the data — Part I

Each group of students should obtain one meter stick, a cup, a Slinky®, and 40 marbles or other small objects from your teacher.

Using a paper clip, create a handle on the cup (see diagram below). Hang the cup on the Slinky®. Place the Slinky on the center of the meter stick. Place the meter stick across two chairs as shown in the diagram below.



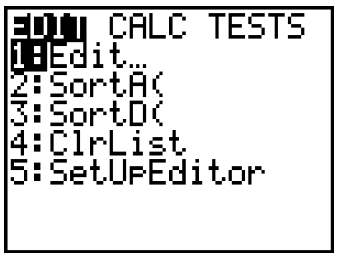
1. Measure the distance in centimeters from the floor to the bottom of the cup. Record the distance in the table on the **Data Collection and Analysis** page.
2. Place five of the objects that you are using in the cup. When the Slinky® is stable, measure the distance from the floor to the bottom of the cup. Record the distance in the table on the **Data Collection and Analysis** page.
3. Place five additional objects in the cup. When the Slinky® is stable, measure the distance from the floor to the bottom of the cup. Record the distance in the table on the **Data Collection and Analysis** page.
4. Continue placing additional objects in the cup in increments of five and measure the distance from the floor to the bottom of the cup. Record the distances in the table on the **Data Collection and Analysis** page.

Setting up the TI-83 Plus

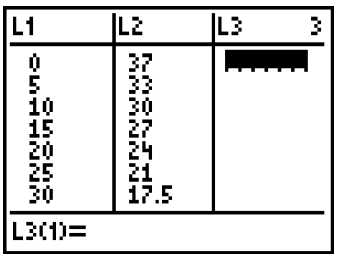
Before starting your data collection, make sure that the TI-83 Plus has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

Entering the data in the TI-83 Plus

1. Press [STAT] and select **1:Edit** by pressing [ENTER].

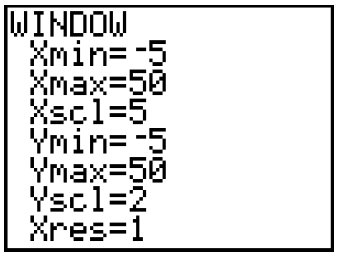


2. Enter the number of objects in **L1**.
3. Enter the distance from the floor to the bottom of the cup in **L2**.



Setting up the window

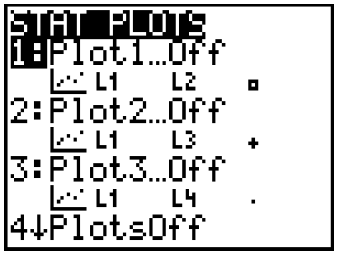
1. Press [WINDOW] to set up the proper scale for the axes.
2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.



3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. Set the **Xscl** to **5**.
4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.
5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **2**.

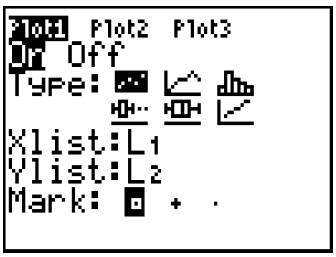
Graphing the data: Setting up a scatter plot

1. Press [2nd] [STAT PLOT] and select **1:Plot1** by pressing [ENTER].

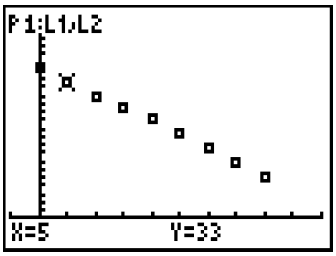


2. Set up the plot as shown by pressing ENTER ▼ ENTER ▼ 2nd **1** ENTER 2nd **2** ENTER ENTER.

Note: *Press ▼ ▼ if* **L1** *and* **L2** *are already displayed.*



3. Press GRAPH TRACE to see the plot.



Analyzing the data

Finding a trend line

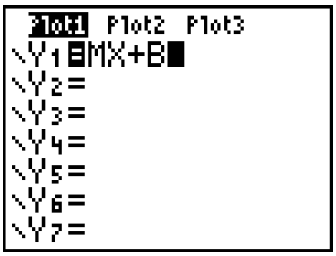
The data that you collected appears to be linear; therefore, you will find a linear equation for the line.

1. The *y*-intercept of a line is the point at which the line crosses the *y*-axis. The y-intercept of the trend line is the first value in **L2**. Record the *y*-intercept of the line on the **Data Collection and Analysis** page.

2. Store the *y*-intercept to **B** in the TI-83 Plus. Press STO▶ ALPHA [B] ENTER to store this value to **B**. (**B** is located above 1 on the TI-83 Plus.)

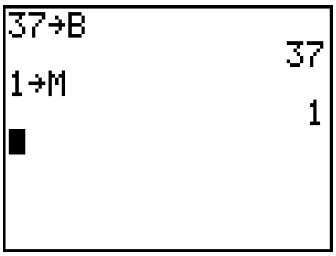
3. Enter the slope-intercept form of a linear equation in **Y1**.

4. Press Y= ALPHA [M] X,T,Θ,*n* + ALPHA [B] to place the equation *Y* = *MX* + *B* in the Y= menu.



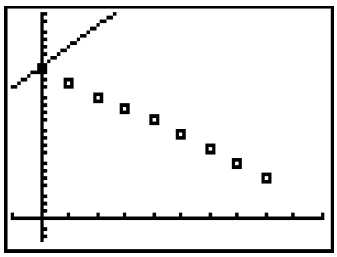
Use a guess and check method to find the *slope* of the line.

5. Store **1** to **M** in the TI-83 Plus. Press 2nd [QUIT] to return to the Home screen. Press **1** STO▶ ALPHA [M] ENTER.



6. Press GRAPH to see the graph of the trend line.

How should you adjust the value of the slope?

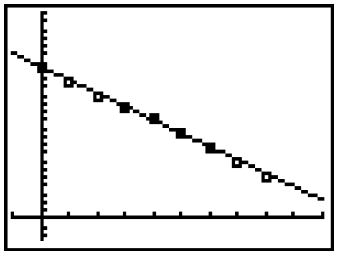


7. Continue to guess and check different values for the slope until you find a value that appears to fit the data like the example shown.

8. Change the value of the slope and store it to **M**.

9. Press 2nd [QUIT] ◀ ◀ ◀ DEL 2nd DEL and type the new value for the slope.

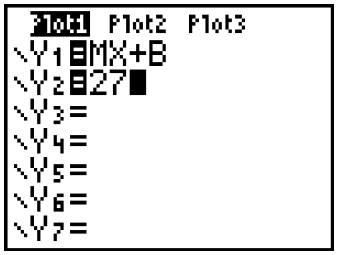
10. Press GRAPH to see the graph of the trend line. Record the slope and the equation of the line on the **Data Collection and Analysis** page.



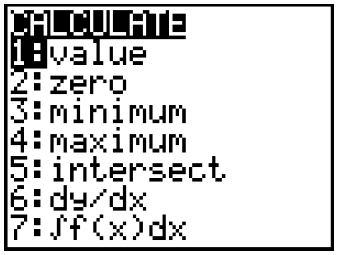
Finding the intersection

Determine the point of intersection using the data that you collected.

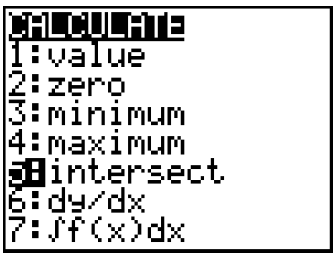
1. Press Y= and ▼ until you are in the first position for **Y2**. Type **2 7**.



2. Press 2nd [CALC].

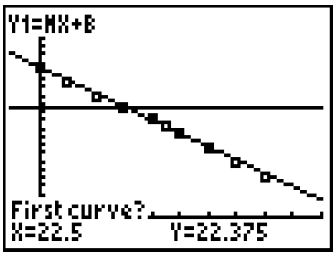


3. Select **5:intersect** and press ENTER.



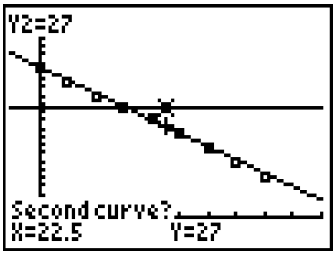
4. The calculator will prompt you for the *First curve*. Make sure the cursor is flashing on the regression line and then press ENTER.

Note: *Use ▲ and ▼ to select the function. Once you have selected the regression line press ENTER.*

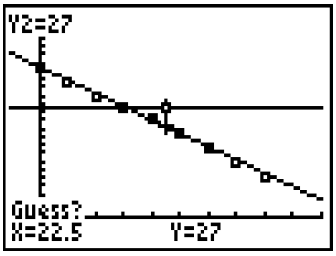


5. The calculator will prompt you for the *Second curve*. Make sure the cursor is flashing on the second line and then press ENTER.

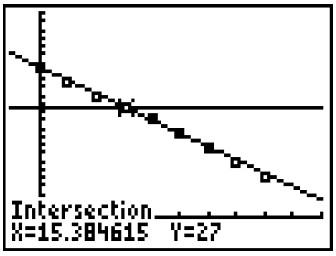
Note: *Use ▲ and ▼ to select the function. Once you have selected the regression line press ENTER.*



6. The calculator will prompt you to *Guess*. Try to estimate the coordinates of the point of intersection.

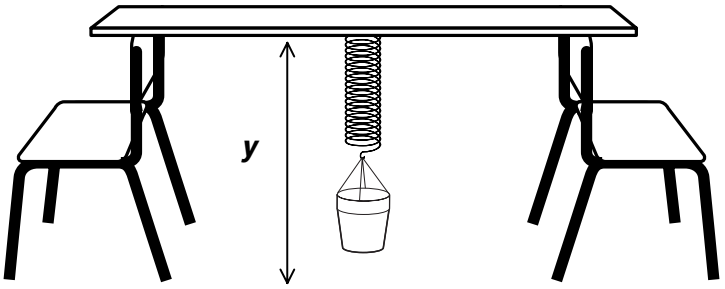


7. Press ENTER to find the exact point of intersection. The *x* value is the desired distance.



Answer Part I questions 1 through 7 on the **Data Collection and Analysis** page.

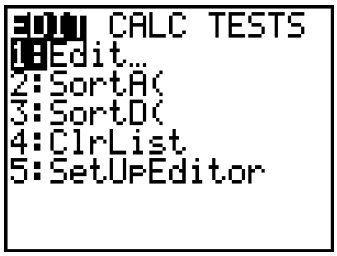
Collecting the Data — Part II



1. Remove all objects from the cup. Measure the length of the Slinky® and the cup. Place five of the objects that you are using in the cup. When the Slinky is stable, measure the length of the Slinky and the cup. Record the length in the table on the **Data Collection and Analysis** page.
2. Place five additional objects in the cup. When the Slinky® is stable, measure the length of the Slinky and the cup. Record the length in the table on the **Data Collection and Analysis** page.
3. Continue placing additional objects in the cup in increments of five and measure the length of the Slinky® and the cup. Record the lengths in the table on the **Data Collection and Analysis** page.
4. After you have collected all of the data, set up **Plot2** using the number of objects versus the length of the Slinky® and the cup. Use the procedures from Part I's **Analyzing the data: Finding a trend line** section to find an equation for the number of objects versus the length of the Slinky and the cup. Record the values on the **Data Collection and Analysis** page.

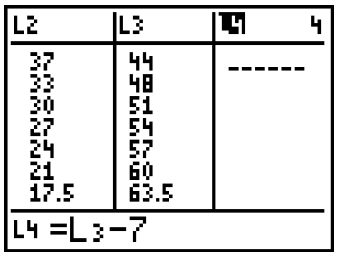
Use the following steps to find the length of the Slinky® for each of the data items listed.

5. Measure the length of the cup with the paper clip. Record the length on the **Data Collection and Analysis** page.
6. Press [STAT] and select **1:Edit** by pressing [ENTER].



7. Press [▲] and [▶] enough times to move the cursor to the top of **L4**.
8. Press [2nd] [L3] [−] **7**.

Note: *The number 7 was used in this example because the cup and paper clip measured 7 centimeters. Use the length of the cup and the paper clip.*



9. Press ENTER.

L2	L3	L4 4
37	44	37
33	48	41
30	51	44
27	54	47
24	57	50
21	60	53
17.5	63.5	56.5

L4(1)=37

10. Use **Plot3** to set up a plot of the number of objects versus the length of the Slinky®. Use the procedures from Part I's **Analyzing the data: Finding a trend line** section to find an equation for the number of objects versus the length of the Slinky. Record the values on the **Data Collection and Analysis** page.

11. Enter equations (1), (2), and (3) in **Y1**, **Y2**, and **Y3** respectively.

Use equations (1), (2), and (3) to answer Part II questions 1 through 6 on the **Data Collection and Analysis** page.

Extensions

Collect the distance by using a Calculator Based Ranger™ (CBR™). Place the CBR on the floor under the cup. Glue or tape a paper plate on the bottom of the cup. Press APPS and select **2:DataLogger**. Collect the distance data and store the data in **L2**. Follow the procedures for the activity above.

Data Collection and Analysis

Name _____

Date _____

Activity 10: That's a Stretch

Collecting the data — Part I

Record your data from Part I in the table below.

Number of objects in cup	Distance from floor to bottom of cup (cm)
0	
5	
10	
15	
20	
25	
30	
35	
40	

Analyzing the data — Part I

The *y*-intercept is: _____.

Slope = _____ Equation of Line (1): _____

Use your equation from number 7 in the **Analyzing the data: Finding a trend line** section to answer questions 1 through 7.

1. What is the *independent variable* for this activity? _____

2. What is the *dependent variable* for this activity? _____

3. Explain what the *y*-intercept represents.

_____

_____

4. Explain what the *slope* represents.

_____

_____

5. Use your equation to find the distance from the floor to the bottom of the cup if 13 objects were placed in the cup. _____

(Press 2nd [CALC] **1:value** ENTER. Type **1 3** ENTER. The *y* value is the desired distance.)

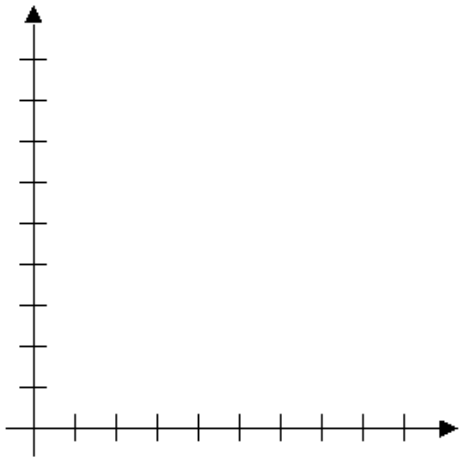
6. Jennifer did this activity with 40 pennies and Mustafa did this activity with 40 small candies (M&M's® or Skittles®). Draw a sketch of the lines produced by Jennifer and Mustafa on the same set of axes. Label the axes. Identify which line represents Jennifer's data and which line represents Mustafa's data.

a. Which person had a line with the smaller *slope*?

_____

b. Which person had a line with the greater *y*-intercept?

_____



7. Using the data that you collected, determine how many objects were used if the distance measured 27 centimeters. _____

Collecting the data — Part II

Record your data from Part II in the table below.

Number of objects in cup	Length of Slinky® and cup (cm)	Length of Slinky® (cm)
0		
5		
10		
15		
20		
25		
30		
35		
40		

Analyzing the data — Part II

The *y*-intercept is: _____.

Slope = _____ Equation of Line (2): _____

Length of Cup and Paper Clip = _____

The *y*-intercept is: _____.

Slope = _____ Equation of Line (3): _____

1. How do the *slopes* of the lines in equations (1), (2), and (3) compare?

_____

_____

2. What is the meaning of the *slope* in equations (2) and (3)?

Equation (2): _____

_____

Equation (3): _____

_____

3. Explain the meaning of the *y*-intercept in equations (2) and (3).

Equation (2): _____

_____

Equation (3): _____

_____

4. How far would the Slinky® stretch if 13 objects were placed in the cup?

_____

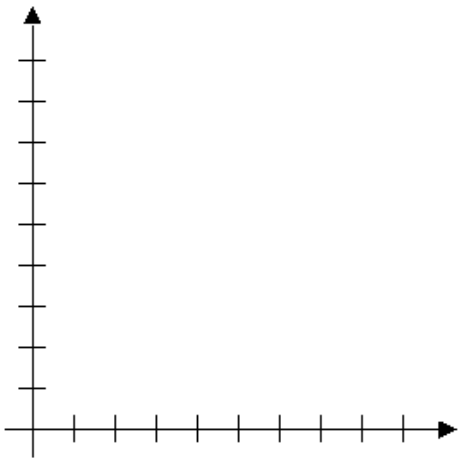
5. How many objects would it take to stretch the Slinky® a distance of 75 centimeters?

_____

6. Repeat question number 6 in Part I, but this time sketch a graph for the length of the Slinky®. Answer the question in **a** and **b** below.

a. _____

b. _____



Teacher Notes

Activity 10

That's a Stretch

Objectives

- To determine the relationship between the stretch of a spring and the number of weights in a cup suspended from the spring
- To find the *y* value of a function, given the *x* value
- To use interpolation to find unknown values
- To use technology to find a best fit line
- To use technology to plot a set of ordered pairs

Materials

- TI-83 Plus
- Calculator Based Ranger™ (CBR™) — optional
- Slinky® cut in half, one per group
- Small bathroom paper cups or film containers, one per group
- Marbles, pennies, or other small objects such as cubes, at least 40 per group
- Large paper clips, one per group
- Meter stick, one per group

Preparation

- You can suspend the meter stick across two desks or two chairs.
- You can use marbles, pennies, small cubes, or candy for objects to place in the cup.
- This activity explores both positive and negative slopes. Part II of the activity allows students to examine the *y*-intercept of a line.

Answers to Data Collection and Analysis

Collecting the data

- Sample data, Part I.

Number of objects in cup	Distance from floor to bottom of cup (cm)
0	37
5	33
10	30
15	27
20	24
25	21
30	17.5
35	14
40	10.5

- Sample data, Part II.

Number of objects in cup	Length of Slinky® and cup (cm)	Length of Slinky® (cm)
0	44	37
5	48	41
10	51	44
15	54	47
20	57	50
25	60	53
30	63.5	56.5
35	67	60
40	70.5	63.5

Analyzing the data — Part I

Use your equation from number 7 in the **Analyzing the data: Finding a trend line** section to answer questions 1 through 7.

1. What is the *independent variable* for this activity?

The independent variable for this activity is number of objects.

2. What is the *dependent variable* for this activity?

The dependent variable for this activity is the distance from the floor to the bottom of the cup measured in centimeters.

3. Explain what the *y*-intercept represents.

The y-intercept is the distance, in centimeters, of the cup from the floor with zero objects in the cup.

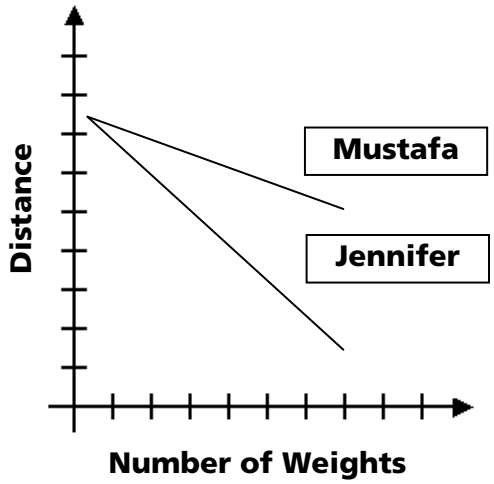
4. Explain what the *slope* represents.

The slope represents the number of centimeters that the distance decreases from the bottom of the cup to the floor each time an object is added to the cup.

5. Use your equation to find the distance from the floor to the bottom of the cup if 13 objects were placed in the cup.

For the sample data, the distance is 28.55 cm.

6. Jennifer did the activity with 40 pennies and Mustafa did the activity with 40 small candies (M&M's® or Skittles®). Draw a sketch of the lines produced by Jennifer and Mustafa on the same set of axes. Label the axes. Identify which line represents Jennifer's data and which line represents Mustafa's data.



a. Which person had a line with the smaller slope?

Mustafa

b. Which person had a line with the greater *y*-intercept?

They have the same y-intercept.

7. Using the data you collected, determine how many objects were used if the distance measured 27 centimeters?

The TI-83 Plus returns a value of 15.384615; however, the answer must be an integer. Therefore, the value is at least 16 objects. Check and discuss students' answers.

Analyzing the data — Part II

1. How do the *slopes* of the lines in equations (1), (2), and (3) compare?

The slopes of all of the lines are equal in absolute value. However, slopes of equations (2) and (3) are positive while the slope of equation (1) is negative.

2. What is the meaning of the *slope* in equations (2) and (3)?

Equation (2): The number of centimeters that the length of the spring with the cup increases each time an object is added to the cup.

Equation (3): The number of centimeters that the length of the spring without the cup increases each time an object is added to the cup.

3. Explain the meaning of the *y*-intercept in equations (2) and (3).

Equation (2): The initial length, in centimeters, of the spring with the cup.

Equation (3): The initial length, in centimeters, of the spring without the cup.

4. How far would the Slinky® stretch if 13 objects were placed in the cup?

45.5 cm

5. How many objects would it take to stretch the Slinky® a distance of 75 centimeters?

The TI-83 Plus returns a value of 58.461538; however, the answer must be an integer. Therefore, the value is at least 59 objects. Check and discuss students' answers.

6. Repeat question number 6 in Part I, but this time sketch a graph for the length of Slinky®. Answer the question in **a** and **b** below.

a. *Mustafa had the graph with the smaller slope.*

b. *They have the same y-intercept.*