### Implicit Differentiation

by

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# **Textbook Correlation: Key Topic**

• Derivatives

# **NCTM Principles and Standards:**

- Process Standard
  - Representation
  - Connections

#### **Implicit Differentiation**

#### **Exercises:**

1. Given  $x^2 - y^2 = 9$ , compute  $\frac{dy}{dx}$  using implicit differentiation.

### Solution:

Taking the derivative of  $x^2 - y^2 = 9$  with respect to x on the calculator treats y as a constant. Instead, take the derivative of  $x^2 - y(x)^2 = 9$  with respect to x. Why?

Solve for  $\frac{dy}{dx}$  by replacing the derivative notation with the variable name "dydx." Evaluate the variable "dydx" using the solve command. Reproduce the screens illustrated below.

F1+ F2+ F3+ F4+ F5 F6+ ToolsAl9ebraCalcOtherPr9mIOClean UP	F1+ F2+ F3+ F4+ F5 F6+ ToolsA19ebraCalcOtherPr9mlOClean Up	F1+ F2+ F3+ F4+ F5 ToolsAl9ebraCalcOtherPr9mIOClean Up
	$= \frac{1}{d \times (x - (g(x)) - g)}$	$= \frac{d}{dx} \left( \times^2 - (y(x))^2 = 9 \right) \left  \frac{d}{dx} (y) \right $
$=\frac{d}{d}\left[x^{2}-(y(x))^{2}=9\right]$	$2 \cdot x - 2 \cdot y(x) \cdot \frac{w}{dx}(y(x)) = 0$	$2 \cdot x - 2 \cdot dydx \cdot y(x) = 0$
$\alpha \times (1)$ $(2,1,1)$ $(2,1$	$= \frac{d}{dx} \left[ \times^2 - (y(x))^2 = 9 \right] \left  \frac{d}{dx} (y(x)) \right ^2$	solve(2·x - 2·dydx·y(x) = 6) study = X
$2 \times - 2 \cdot g(x) \cdot \frac{1}{dx}(g(x)) = 0$	$2 \cdot x = 2 \cdot dy dx \cdot y(x) = 0$	$\frac{dydx = \frac{y(x)}{y(x)}}{(2xx - 2xdydx + y(x) - 0, dydx)}$
MAIN RAD AUTO FUNC 1/30	MAIN RAD AUTO FUNC 3/30	MAIN RAD AUTO FUNC 3/30

2. Evaluate  $\frac{dy}{dx}$  at the point (5,4). **Answer**: 5/4.

3. Use the TI-89 (TI-92 Plus) to graph the hyperbola  $x^2 - y^2 = 9$  and draw the tangent line at the point (5,4). Did you get the same results?



4. Use implicit differentiation to find an equation of the tangent line to the asteroid  $x^{2/3} + y^{2/3} = 4$  at the point  $(-3(3)^{1/2}, 1)$ . Use the TI-89 (TI-92 Plus) to graph the astroid and draw the tangent line as illustrated in Exercise 3. Did you get the same results?



4. Given  $12x^2 + 3xy^2 - 7y = 0$ , compute  $\frac{dy}{dx}$  using implicit differentiation.