

## Area of the Missing Square

ID: 11751

Time required  
40 minutes

## Activity Overview

*In this activity, students will be introduced to an area model for representing a quadratic equation. Students will explore the relationship between the value of  $b$  and  $c$ , in  $y = x^2 + bx + c$ , form of the quadratic equation. The relationship will be examined with integer and non-integer values in order to help students recognize a pattern. Students will then apply their knowledge by answering several questions using the relationship.*

## Topic: Quadratic Equations

- *Completing the Square*
- *Factoring*
- *Perfect Square Trinomials*

## Teacher Preparation and Notes

- *The teacher should be prepared to introduce the first problem of the document so students understand the “algebra tile” area representation of a quadratic equation. Teachers could use algebra tiles to introduce the activity if available.*
- *This activity explores completing the square when the coefficient of  $x^2$  is one. The teacher should use the pattern recognition by the students to extend the activity to coefficient values other than one after the activity.*
- *Students may record responses on the worksheet or in the .tns file.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “11751” in the quick search box.***

## Associated Materials

- *Alg2Week15\_Square\_worksheet.doc*
- *Alg2Week15\_Square.tns*
- *Alg2Week15\_Square\_Soln.tns*

## Suggested Related Activities

*To download any activity listed, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter the number in the quick search box.*

- *Bridge on the River Quad (TI-Nspire technology) — 9531*
- *Bridge on the River Quad (TI-84 Plus family) — 9530*
- *Completing the Square (TI-Nspire technology) — 11532*
- *Parabola Graphing: Completing the Square (TI-Nspire technology) — 9493*

**Problem 1 – Introduction**

In this problem, the teacher will introduce the area model of a quadratic equation. The use of squares for  $x^2$  and integers, and rectangles for  $x$  terms should be explained. The teacher should discuss the length of each side of the figures and the area of each figure. The teacher should also note that when the square is completed it can be factored into a perfect square.

Area =  $x^2 + x + x + c$   
 Area =  $x^2 + 2x + c$

What is the area of the missing square that completes the larger square?

Discussion Questions:

- What is the area of each figure?
- How do I find the area of the missing square?
- How are the side lengths of each figure related?
- How does the factored form relate to the length and width of the completed square?
- How does the quadratic relate to the area of the square?

A square of length and width 1 completes the square. The area of the larger square is now:  
 Area =  $x^2 + 2x + 1$   
 Area =  $(x+1)^2$

**Problem 2 – Integer Lengths**

In this problem, students will use a slider to increase the length of the square in increments of one. They will find the area of the square needed to complete the square. Students should observe the relationship between the coefficient of  $x$  and  $c$ .

Discussion Questions:

- How does the coefficient of  $x$  in the area equation change each time  $l$  increases by one?
- How is the area of the grey or missing square related to the length of  $l$ ?

Area =  $x^2 + b \cdot x + c$   
 Area =  $x^2 + 4 \cdot x + c$

$l = 2$

Write the Area equation using the values of  $b$  and  $c$  for each value of  $l$ .

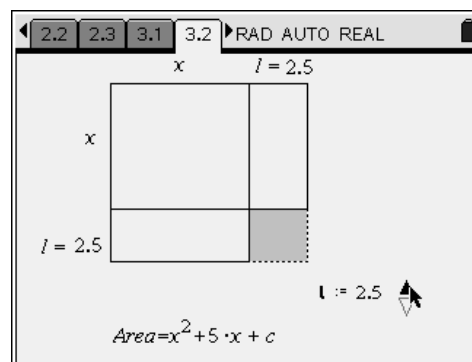
Value of $l$	Area Equation
2	$A = x^2 + 4x + 4$
3	$A = x^2 + 6x + 9$
4	$A = x^2 + 8x + 16$
5	$A = x^2 + 10x + 25$

**Problem 3 – Non-integer Lengths**

In this problem, students will use a slider to increase the length of the square in increments of 0.5. They will find the area of the square needed to complete the square. Students should observe the relationship between the coefficient of  $x$  and  $c$ . Students will answer questions about the pattern they found.

Discussion Questions:

- How does the coefficient of  $x$  in the Area equation change each time  $l$  increases by one-half?
- How is the area of the grey or missing square related to the length of  $l$ ?
- How is the coefficient of  $x$  related to the length of the grey square?
- How is the coefficient of  $x$  related to the value of  $c$ ?
- What is a formula or method for finding the value of  $c$  without using the calculator?



Write the Area equation using the values of  $b$  and  $c$  for each value of  $l$ .

Value of $l$	Area Equation
1.5	$A = x^2 + 3x + 2.25$
2.5	$A = x^2 + 5x + 6.25$
3.5	$A = x^2 + 7x + 12.25$
4.5	$A = x^2 + 9x + 20.25$

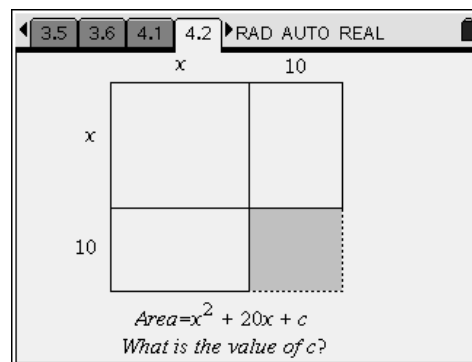
**Problem 4 – Applying Your Knowledge**

In this problem, students will answer questions applying their knowledge for completing the square.

The discussion after reviewing the solutions to the application problems should prepare to extend the activity, at a future time, to completing the square for quadratics with coefficients of  $x^2$  not equal to one.

Discussion Questions:

- How would you complete the square if the coefficient of  $x^2$  is not equal to one?



**Solutions – student worksheet**

1. The coefficient is double the length of the grey square.
2. If you take half of the coefficient and square it you get  $c$ .

3.  $c = \left(\frac{b}{2}\right)^2$

Page 4.2: 100

Page 4.3: 49

Page 4.4: 1.44

Page 4.5:  $\frac{25}{4}$

Page 4.6:  $x^2 + 4\sqrt{2}x + c$

Page 4.7: 24

Page 4.8: Add three. Half of 4 squared is 4 and we already have 1 so add three to get 4.

Page 4.9:  $\left(\frac{b}{2}\right)^2$