***Inverse Functions***

by

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**Textbook Correlation: Key Topic**

- Pre-Requisites: Functions and Equations

**NCTM Principles and Standards:**

- Process Standard
  - Representation
  - Connections

Recall that

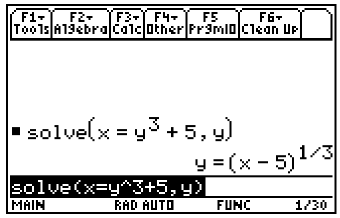
1. In order for the inverse of $y = f(x)$ to be a function, $f$ must be a one-to-one function.
2. If the point (a,b) is a point on the graph of $y = f(x)$, then the point (b,a) must be a point on the graph of the inverse of $f$.
3. Geometrically, two functions are inverses of each other if their graphs are reflections of each other with respect to the line $y = x$.
4. If $f$ and $g$ are inverses, then $g(f(x))=f( g(x)) = x$.

**Exercises**:

1. Given the function $f(x) = x^3 + 5$, solve for the inverse function if it exists. Verify your results.

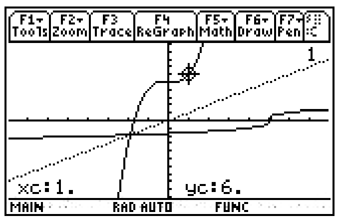
**Solution**:

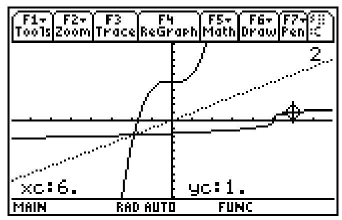
a. Let $y = x^3 + 5$.
b. Interchange $x$ and $y$. The inverse relation is $x = y^3 + 5$.
c. Solve for $y$. Use the **solve** command on the Home screen.



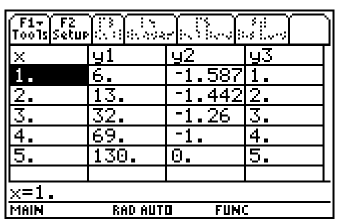

$y = (x\text{-}5)^{1/3}$ is the inverse relation. Is it a function?

To validate the result graphically, enter **y1(x) =** $\mathbf{x^3}$ **+ 5**, **y2(x) = (x-5)**$^{\mathbf{1/3}}$, and **y3(x) = x** in the **Y=** editor. Check to see if y1 and y2 are reflections of each other with respect to the dotted line y3(x) = x. Use **F3** (Trace) to check reflection points. The screens below illustrate (1,6) on y1 and (6,1) on y2. Recall that after pressing F3 you can type in the the *x* coordinate and press enter.



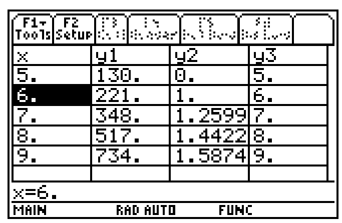




Check the **Table** to validate numerically.



| x | y1 | y2 | y3 |
|---|---|---|---|
| 1. | 6. | -1.587 | 1. |
| 2. | 13. | -1.442 | 2. |
| 3. | 32. | -1.26 | 3. |
| 4. | 69. | -1. | 4. |
| 5. | 130. | 0. | 5. |

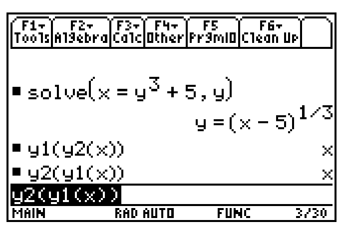
x=1.



| x | y1 | y2 | y3 |
|---|---|---|---|
| 5. | 130. | 0. | 5. |
| 6. | 221. | 1. | 6. |
| 7. | 348. | 1.2599 | 7. |
| 8. | 517. | 1.4422 | 8. |
| 9. | 734. | 1.5874 | 9. |

x=6.

To verify the result analytically calculate y1(y2(x)) and y2(y1(x)) on the **HOME** screen. If both yield x as a result, then the functions are inverses as illustrated below.



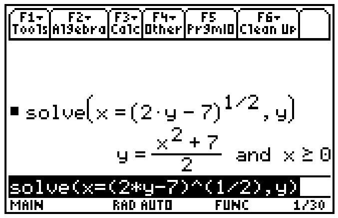

2. Given the function f($x$) = $(2x\ \text{-}7)^{1/2}$, solve for the inverse function if it exists. Verify.

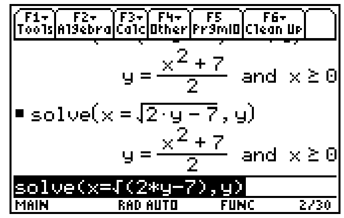
**Solution**:

a. Let $y = (2x\ \text{-}7)^{1/2}$ or $y = \sqrt{(2x\ \text{-}7)}$ .

b. Interchange $x$ and $y$. The inverse relation is $x = (2y\ \text{-}7)^{1/2}$.

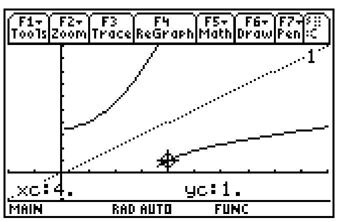
c. Solve for *y*. You can use the **solve** command on the Home screen.

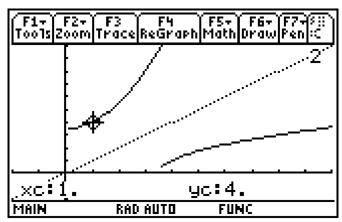


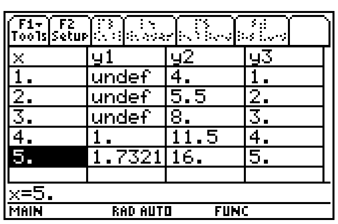

**Answer**: $y = (x^2 + 7)/2$, $x \geq 0$ is the inverse relation.
Is it a function?
Why is it necessary to have a restricted domain?
Is there another correct solution?

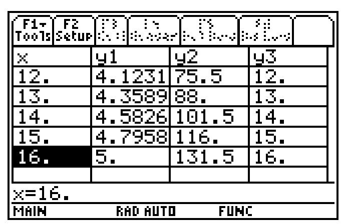
To validate the result graphically, enter $\mathbf{y1(x) = (2x - 7)^{1/2}}$, $\mathbf{y2(x) = (x^2 + 7)/2 \mid x \geq 0}$, and **y3(x) = x** in the **Y=** editor. Check to see if y1 and y2 are reflections of each other with respect to the dotted line y3(x) = x. Use **F3** (Trace) to check reflection points. The screens below illustrate (1,6) on y1 and (6,1) on y2. Recall that after pressing F3 you can type in the *x* coordinate and press enter.



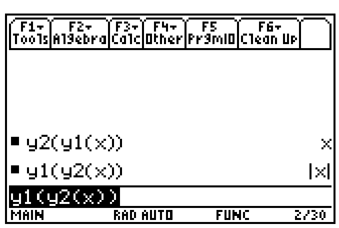




Check the **Table** to validate numerically.




To validate the result analytically calculate y1(y2(x)) and y2(y1(x)) on the **HOME** screen. If both yield x as a result, then the functions are inverses as illustrated below.



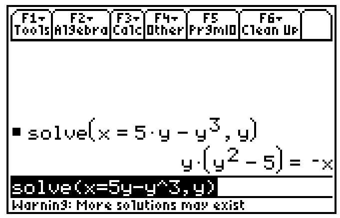

3. Consider the function $f(x) = 5x - x^3$. What is the inverse relation? Is it a function? Verify your results.

**Solution**:

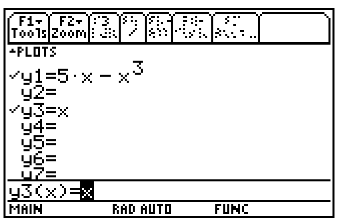
a. Let $y = 5x - x^3$.

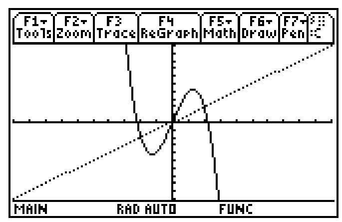
b. Interchange $x$ and $y$. The inverse relation is $x = 5y - y^3$.

c. Solve for $y$. You can use the **solve** command on the Home screen.

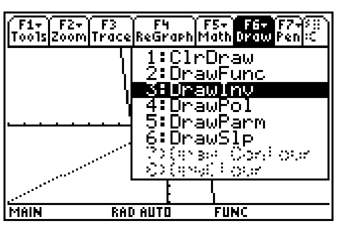


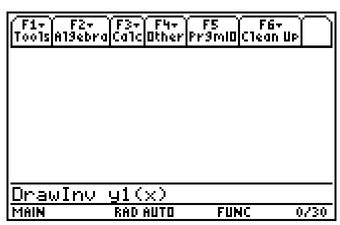

In this case, $y$ cannot be expressed as a function of $x$. Investigate graphically. Enter **y1(x) = 5x - x³** and **y3(x) = x** in the **Y=** Editor and **GRAPH** in a standard window.

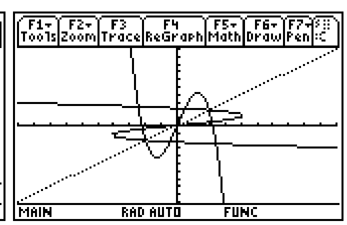



Clearly, y1 is not a one-to-one function. Its inverse relation is not a function. To graph the inverse, press **2nd**, **F1** for **F6** (Draw). Select **3:DrawInv**. The DrawInv command will appear on the entry line of the Home Screen. Type **y1(x)** and press **ENTER**. The inverse relation will appear on the graph screen as shown below.




4. Find ways to restrict the domain of f($x$) so that the inverse will be a function.

**Additional Exercise**:

Use the procedures described above to explore the inverse of $f(x) = e^x$