

That's the Way the Ball Bounces: Height and Time for a Bouncing Ball

Picture a bouncing ball. Between impacts with the floor, the ball rises and slows, then descends and speeds up. For any particular bounce, if the ball's height is plotted as a function of time, the resulting graph has a parabolic shape. In other words, the relationship between height and time for a single bounce of a ball is quadratic. This relationship is expressed mathematically as

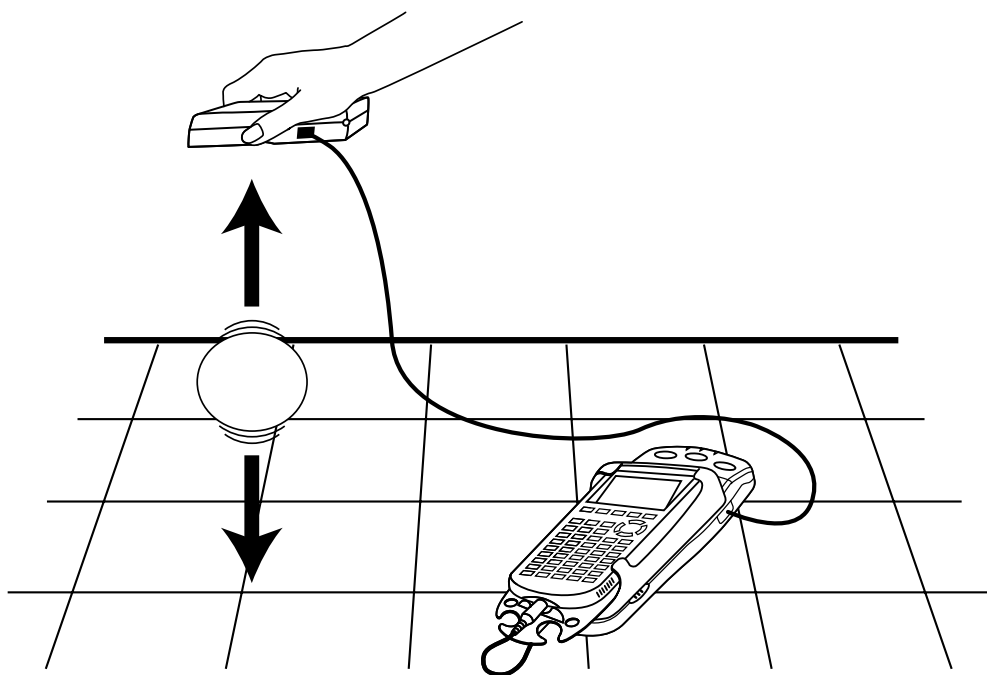
$$y = ax^2 + bx + c$$

where y represents the ball's height at any given time x . Another form of a quadratic equation is

$$y = a(x - h)^2 + k$$

where h is the x -coordinate of the vertex, k is the y -coordinate of the vertex, and a is a parameter. This way of writing a quadratic is called the *vertex form*.

In this activity, you will record the motion of a bouncing ball using a Motion Detector. You will then analyze the collected data and model the variations in the ball's height as a function of time during one bounce using both the general and vertex forms of the quadratic equation.



OBJECTIVES

- Record height versus time data for a bouncing ball.
- Model a single bounce using both the general and vertex forms of the parabola.

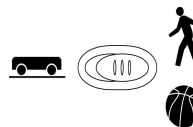
MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator
CBR 2 or Go! Motion and direct calculator cable
or Motion Detector and data-collection interface

EasyData application
ball (racquetball or basketball size)

PROCEDURE

1. Set up the Motion Detector and calculator.
 - a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
 - b. Turn on the calculator and connect it to the Motion Detector. (This may require the use of a data-collection interface.)
2. Position the Motion Detector about 1.5 m above the floor, so that the disc is pointing straight downward.
3. Set up EasyData for data collection.
 - a. Start the EasyData application, if it is not already running.
 - b. Select **[File]** from the Main screen, and then select **New** to reset the application.
 - c. Select **[Setup]** from the Main screen, and then select **Ball Bounce**.
4. Practice dropping the ball so that it bounces straight up and down beneath the Motion Detector. Minimize the ball's sideways travel. Dropping the ball from about waist high works well. The ball must never get closer than 0.5 m from the detector. Be sure to pull your hands away from the ball after you drop it so the Motion Detector does not detect your hands.
5. Select **[Start]** to begin data collection. The Ball Bounce instructions will appear on the calculator screen. Follow the instructions to collect data.
6. When data collection is complete, a graph of distance versus time will be displayed. Examine the graph; it should contain a series of parabolic regions. Check with your teacher if you are not sure whether you need to repeat the data collection.



If you are satisfied with the graph, proceed to the Analysis section. To repeat data collection, select **[Main]** to return to the Main screen and repeat Step 5.

ANALYSIS

1. Select the data corresponding to the ball's position between two bounces.
 - a. From the distance graph, select **[Anlyz]**, and then select **Select Region...** from the menu.
 - b. If a warning is displayed on the screen; select **[OK]** to begin the region selection process.
 - c. Use the **[D]** and **[C]** keys to move the cursor to the left edge of the parabolic region of your choice.
 - d. Select **[OK]** to mark the left bound.
 - e. Use the **[D]** and **[C]** keys to move the cursor to the right edge of the same parabolic region.
 - f. Select **[OK]** to select the region. After a moment you will see the selected portion of your graph filling the width of the screen.

2. Trace across the graph to determine the x - and y -coordinates of the vertex of the parabola (in this case, the maximum point on the curve). Round these values to the nearest hundredth, and record them in the first Data Table on the Data Collection and Analysis sheet.

⇒ Answer Question 1 on the *Data Collection and Analysis* sheet.

3. The model from physics for the distance versus time motion of a ball in free fall is quadratic. Fit a quadratic model $y = a(x - h)^2 + k$ to your data. In this vertex form, h is the x -coordinate of the vertex, k is the y -coordinate of the vertex, and a is a parameter.

Since you have values for the parameters h and k of your model, you can try plotting the model using a guess for the a parameter. First, enter your model equation for graphing.

- a. Press Y= .
 - b. Press CLEAR to remove any existing equation.
 - c. Enter the vertex form of the parabola using your values for the parameters h and k . For example, if your equation is $y = A(x - 4)^2 + 5$, enter $A*(x-4)^2 + 5$ in the Y_1 field.
 - d. Press 2nd [QUIT] to return to the home screen.
4. Next, set a value for the parameter a , and then look at the resulting graph. To obtain a good fit, try several values for a . Use the steps below to store different values to the parameter a . Start with $a = 1$. Experiment until you find one that provides a good fit for the data. *Test both positive and negative values for a .*
 - a. Enter a value for the parameter a . Press STO A ENTER to store the value in the variable A.
 - b. Press GRAPH to see your data with the model graph superimposed.
 - c. Press 2nd [QUIT] to return to the home screen.

Use your optimized value for a and the values of h and k you determined earlier to complete the vertex form of the equation.

⇒ Answer Question 2 on the *Data Collection and Analysis* sheet.

5. It is also possible to express any quadratic function in the standard form of $y = ax^2 + bx + c$, where the coefficient a is the same as the coefficient you just found for the vertex form, and b and c are other parameters related to the h and k you already know. To determine the coefficients b and c , expand the vertex form of your equation and collect like terms.

Round the corresponding values of a , b , and c to the nearest tenth and record them in the middle column of the second Data Table on the *Data Collection and Analysis* sheet.

6. Another way to determine the parameters is to use the calculator to perform a quadratic regression on the data to determine the parabola that fits the data best.
 - a. Press STAT and use the cursor keys to highlight CALC.
 - b. Press the number adjacent to QuadReg to copy the command to the home screen.
 - c. Press 2nd [L1] , 2nd [L6] , to enter the lists containing the data.
 - d. Press VAR and use the cursor keys to highlight Y-VARS.
 - e. Select Function by pressing ENTER .
 - f. Press ENTER to copy Y_1 to the expression.
 - g. On the home screen, you will now see the entry QuadReg L1, L6, Y1. This command will perform a quadratic regression with L1 as the x -values and L6 as the y -values. The resulting

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regression curve will be stored in equation variable Y_1 . Press **ENTER** to perform the regression.

Copy the parameters a , b and c that appear on your calculator screen to the third column of the second Data Table.

h. Press **GRAPH** to see your graph.

⇒ Answer Questions 3-5 on the *Data Collection and Analysis* sheet.

EXTENSION

How does the value of a vary from one parabolic section to the next? Collect another run of data, and determine a new value of the parameter a using any method you like. Explain why the values of a are in close agreement for both bounces. What does a measure?

CALCULUS EXTENSION

Take the second derivative of the modeling equation. What is the physical significance of this value?

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLE

Vertex	
x-coordinate	y-coordinate

Parameters	Values calculated from vertex form	Values from regression
<i>a</i>		
<i>b</i>		
<i>c</i>		

QUESTIONS

1. In this activity, the ball bounced straight up and down beneath the detector, yet the plot you see might seem to depict a ball that is moving sideways as it bounces up and down. Explain why the graph looks the way it does.

2. Write the vertex form of the parabola from Step 6.

3. Are the values of a , b , and c in the quadratic regression equation consistent with the values you determined earlier?

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4. Describe how the parameter a affects the graph of $y = a(x - h)^2 + k$. Specifically, how do the magnitude of a and the sign of a change the graph?

5. Suppose you had chosen the parabolic section for the bounce just to the right of the one you actually used in this activity. Describe how the parameters h and k would change, if at all, if this different parabolic section were to be fit with the equation $y = a(x - h)^2 + k$.