EXPLORATIONS

Activity 11

Curve Ball

When a ball bounces up and down, the height of the ball as a function of time can be modeled with a quadratic function. The resulting graph of this function would then be a parabola. This relationship is usually expressed in one of two ways. The form $y = ax^2+bx+c$, is commonly known as the standard form of a quadratic equation. It can also be written as $y = a(x-h)^2+k$, which is known as the vertex form of a quadratic function.

Introduction

In this experiment, you will record the motion of a bouncing ball using a CBL™/CBL 2™ and TI InterActive!™ software. You will then analyze the collected data and attempt to find a model for the height of the ball as a function of time.
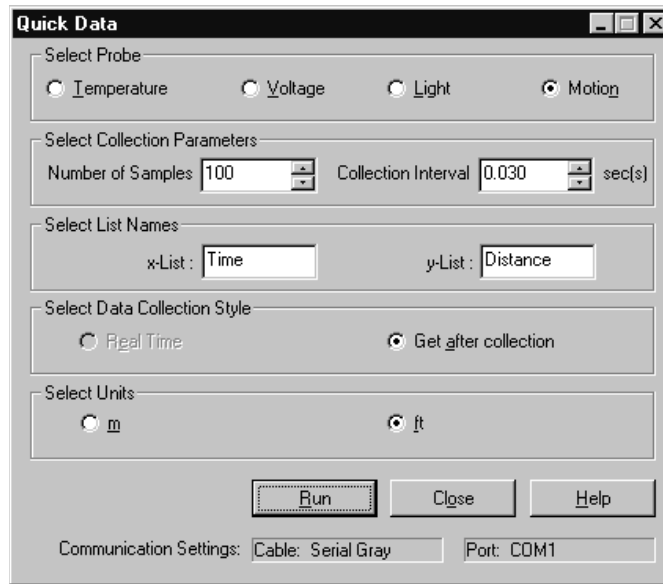
Equipment Required

♦ Computer

♦ TI InterActive! software

♦ CBL/CBL 2 (or a CBR™)

♦ TI-GRAPH LINK™ cable

♦ Motion detector

♦ Ball (racquetballs work well with this activity)
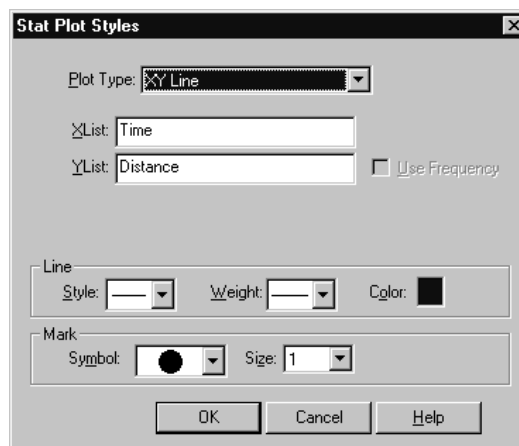
Instructions

1. Plug the TI-GRAPH LINK cable into your computer.

2. Plug the other end of the TI-GRAPH LINK cable into the CBL/CBL 2.

3. Plug the motion detector into the **CH1** port on the CBL/CBL 2.

4. Start TI InterActive! The software opens a new, blank document.

5. Title your new document *Curve Ball* and add your name and the date. Click the Save

   button ![save icon] to save and name your document.

6. Click the List button ![list icon] to open the Data Editor.

7. From the Data menu, choose Quick Data.

8.   Adjust the Quick Data dialog box settings so that they match the ones shown below.


Collecting the Data

1.   Be sure that the ball is bounced on a smooth, level surface. Do not allow anything to obstruct the path between the detector and the ball while the data is being collected.

2.   Hold the motion detector about five or six feet above the floor and parallel to it.

3.   To begin data collection, click Run on the Quick Data dialog box.

4.   Release the ball, taking care that the hands of the ball dropper are moved away from the path of the motion detector upon release of the ball.

5.   When data collection finishes, click the Zoom Statistics button [icon]. The viewing boundaries adjust automatically to show all the plotted data.

6.   If the scatter plot looks confusing, click the Stat Plots tab of the Functions dialog box, then click the box to the right of the check box. This will bring up the Stat Plot Styles shown above. Change the Plot Type to **XY line** and click **OK**.

7. Your plot should appear to be a number of parabolas opening upward and rising from

    left to right. If you are satisfied with your data, click on Save to Document ![icon] to save
    the graph in your TI InterActive!™ document. If you are not satisfied, close the Graph
    window and return to step **3** to collect a new data set.

Recording the Data

1. The plot you recorded is a distance versus time graph. Convert the recorded distances
    to heights by subtracting each value from the distance to the floor. The distance to the
    floor will be the maximum distance point that you collected.

2. Double-click on listname **L1**, type **time** in the Formula box, and click OK. Double-click
    on listname **L2**, type **max(distance)-distance** in the Formula box, and click OK. This will
    convert your distance readings to heights.

3. Click the Graph button ![icon], and then click the Stat Plots tab of the Functions dialog
    box. In the uppermost text box, type **L1** to specify it as the list containing the x
    coordinates. Press the Tab key and move to the second text box. Type **L2** to specify
    the list containing the y coordinates.

4. Press Enter, then click the Zoom Statistics button ![icon]. The viewing boundaries adjust
    automatically to show all the plotted data.

5. For this activity, you will analyze only one bounce of the ball. To choose a particular

    bounce, press ![Trace] and move to the first point on the left side of any bounce.
    Record the coordinates of this point in the table below. Continue to trace to the last
    point on the right side of the bounce and record its coordinates in the table below.

X	Y

6. Click the Save to Document button ![icon] to close the graph and bring up the Data
    Editor.

7. Scroll downward, observing the values in **L1** until you find the x-value of the first point
    you entered in the table above. Place the cursor in this cell. Click and drag down and
    over so that all the values in lists **L1** and **L2** are highlighted from the first value in the
    table above to the last value in the table above.

8. Press Ctrl+C to copy the selected values from **L1** and **L2**.

9. Move the cursor to the first cell in **L3** and press Ctrl+V to paste the selected values into
    L3 and **L4**.

10. Click the Graph button ![icon], and then click the Stat Plots tab. In the uppermost text
    box, type **L3** to specify it as the list containing the x coordinates. Press the Tab key and
    move to the second text box. Type **L4** to specify the list containing the y-coordinates.

11. Press Enter to turn the plot on. Click the Zoom Statistics button [image]. The viewing boundaries adjust automatically to show all the plotted data.

12. Click the Save to Document button [image] to save this graph in your TI InterActive!™ document.

13. Close the Data Editor.

Analysis and Questions

1. In this activity, the ball bounced straight up and down beneath the detector. The original plot, however, seemed to depict a ball moving sideways, rising, and upside down. Explain why this is so.

2. Click on the graph of the single parabola in your TI InterActive! document and press **Trace** to move along the plot of your parabola. Estimate the *x*- and *y*-coordinates of the vertex, round these values to the nearest hundredth, and record them in the table below.

x-coordinate	y-coordinate

3. The theoretical model for an object in free fall is quadratic. We will attempt to fit the data with the vertex form of a quadratic equation $y = a(x-h)^2+k$. In this model the coordinates (h, k) represent the coordinates of the vertex and a is a constant. Substitute the values for *h* and *k* from the table above in this equation and record it below.

$$y =$$

4. Close the Trace Value dialog box and click on the f(x) tab of the Functions dialog box. Enter the equation above, but substitute a -1 for *a*. Click Enter to graph the function. How well does it fit the data?

5. To obtain a good fit of the data you will need to adjust the value of *a*. Return to the Functions dialog box and try different values of a until you achieve a good model for the data. When you are satisfied with the fit, record your final values and equation below.

$a =$ $h =$ $k =$ $y =$

6.   Describe how changing the value of *a* effects the shape of the parabola.

7.   TI InterActive!™ can find its own model for the data by calculating the quadratic curve of best fit.

    a.   Click Save to Document to close the Graph window.

    b.   Click on Stat Calculations Tool .

    c.   Click the down arrow next to Calculation Type, scroll down the list and click on Quadratic Regression.

    d.   In the text box labeled X List, type **L3**; in the box labeled Y List, type **L4**.

    e.   Click Calculate to find the regression equation $y = ax^2 + bx + c$ and its variables. Record the regression equation values of *a*, *b*, and *c* below:

a	
b	
c	

8.   In the space below, record TI InterActive!'s regression model using the values found above.

$$y =$$

9.   Click Save Results to close the Statistical Regression Calculation dialog box and return to your TI InterActive! document.

Click the graph of your parabola saved in the TI InterActive! document and click on the f(x) tab on the Functions dialog box. Enter the model you recorded in step **8** in an available text box. Press Enter to see the graph of the model. How does it compare visually with the vertex form you had previously graphed?

10. Close the Graph window to return to your TI InterActive! document. Click on Math

Box . Type **expand(a*(x-h)^2+k,x)**, substituting the values you found for *a*, *h*, and *k* in step **5** above, and press Enter. Record the results in the space below.

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11. How does the standard form of the vertex form of the model compare with the standard form of the regression equation?

12. Save and print your TI InterActive!™ document.

Extensions

♦   Repeat this activity using different balls and different drop heights. What effect do these changes have on the value of *a* in either of your models?

♦   Choose another bounce to model from your original data set, other than the one you chose in this activity. Discuss the changes that result in the vertex form of your model. Can you come up with a quick way to find the equation of any bounce?

♦   Connect the TI-GRAPH LINK™ cable to your calculator and use TI InterActive! to send the data you collected from the computer to the calculator. Analyze the data on your calculator using the same methods applied in this activity.

Teacher Notes

Activity 11: Curve Ball

- ♦ CBL/CBL 2
- ♦ List Manipulation
- ♦ Quadratic Function

Activity Notes

- ♦ If there is not enough room near the computer to collect this data, it can be collected using the calculator with a CBL™, CBL 2™, or CBR™. The data can then transferred to a TI InterActive!™ document.

- ♦ Caution students to be sure that nothing blocks the path between the motion detector and the ball. Hands and cords are common obstructions.

Sample Data

Note: Due to the large number of data points collected, the sample data contains only the points of the selected bounce.

Time (sec)	Height(ft)	Time(sec)	Height(ft)	Time (sec)	Height(ft)
.66	0.12	.93	2.28	1.20	2.19
.69	0.38	.96	2.39	1.23	2.04
.72	0.72	.99	2.47	1.26	1.86
.75	1.03	1.02	2.52	1.29	1.65
.78	1.32	1.05	2.53	1.32	1.42
.81	1.57	1.08	2.52	1.35	1.15
.84	1.79	1.11	2.49	1.38	0.85
.87	1.98	1.14	2.41	1.41	0.52
.90	2.15	1.17	2.32	1.44	0.15

Analysis and Questions - Key

1. The data represents the distance from the motion detector to the ball with respect to time. The plot moves to the right because time is increasing. The data appears to be rising because the distance to the ball is increasing as the maximum heights of the ball decrease.

2. The vertex lies at approximately (1.05, 2.53).

3. $y = a(x - 1.05)^2 + 2.53$

4. The vertex is correct, but the graph is upside down and very wide.

5. $a = -16$, $h = 1.055$, $k = 2.53$, $y = -16(x - 1.055)^2 + 2.53$

6. Positive values for a cause the parabola to open upward, while negative values make it open downward. As the absolute value of a increases, the parabola appears to become thinner as a result of the vertical stretch taking place. As the absolute values of a decrease, the parabola appears to become wider.

7. a = -15.9167, b = 33.5786, c = -15.1804

8. $y = -15.9167x^2 + 33.5786x - 15.1804$

9. The models are virtually identical.

10. $y = -16x^2 + 33.76x - 15.2784$

11. The models are very close.