



MATH AND SCIENCE @ WORK

AP* CALCULUS Student Edition



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SPACE SHUTTLE ASCENT

Background

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries, and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

Since its first flight in 1981, the space shuttle has been used to extend research, repair satellites, and help with building the International Space Station, or ISS. However, by 2010 NASA plans to retire the space shuttle in favor of a new Crew Exploration Vehicle, or CEV. Until then, space exploration depends on the continued success of space shuttle missions. Critical to any space shuttle mission is the ascent into space.



Figure 1: Space Shuttle Discovery at lift-off during STS 121.



Figure 2: Onlookers view the launch of STS 121.

The ascent phase begins at liftoff and ends at insertion into a circular or elliptical orbit around the Earth. To reach the minimum altitude required to orbit the Earth, the space shuttle must accelerate from zero to 8,000 meters per second (almost 18,000 miles per hour) in eight and a half minutes. It takes a very unique vehicle to accomplish this.

There are three main components of the space shuttle that enable the launch into orbit. The orbiter is the main component. It not only serves as the crew's home in space and is equipped to dock with the International Space Station, but contains maneuvering engines for finalizing orbit. The external tank, the largest component of the space shuttle, supplies the propellant to the orbiter's three main maneuvering engines. The two solid rocket boosters, the third component, provide the main thrust at launch and are attached to the sides of the external tank (Figure 1). The components of the space



shuttle experience changes in position, velocity and acceleration during the ascent into space. These changes can be seen when one takes a closer look at the entire ascent process (Figure 3).

The ascent process begins with the liftoff from the launch pad. Propellant is being burned from the Solid Rocket Boosters, or SRB, and the external tank, or ET, causing the space shuttle to accelerate very quickly. This high-rate of acceleration as the space shuttle launches through the Earth's atmosphere causes a rapid increase in dynamic pressure, known as Q in aeronautics. The structure of the space shuttle can only withstand a certain level of dynamic pressure (critical Q) before it suffers damage. Before this critical level is reached, the engines of the space shuttle are throttled down to about 70%. At about one minute after launch the dynamic pressure reaches its maximum level (max Q). The air density then drops rapidly due to the thinning atmosphere and the space shuttle can be throttled to full power without fear of structural damage.

At about 2 minutes after launch, the atmosphere is so thin that the dynamic pressure drops down to zero. The SRB, having used their propellant, are commanded by the space shuttle's onboard computer to separate from the external tank. The jettison of the booster rockets marks the end of the first ascent stage and the beginning of the second.

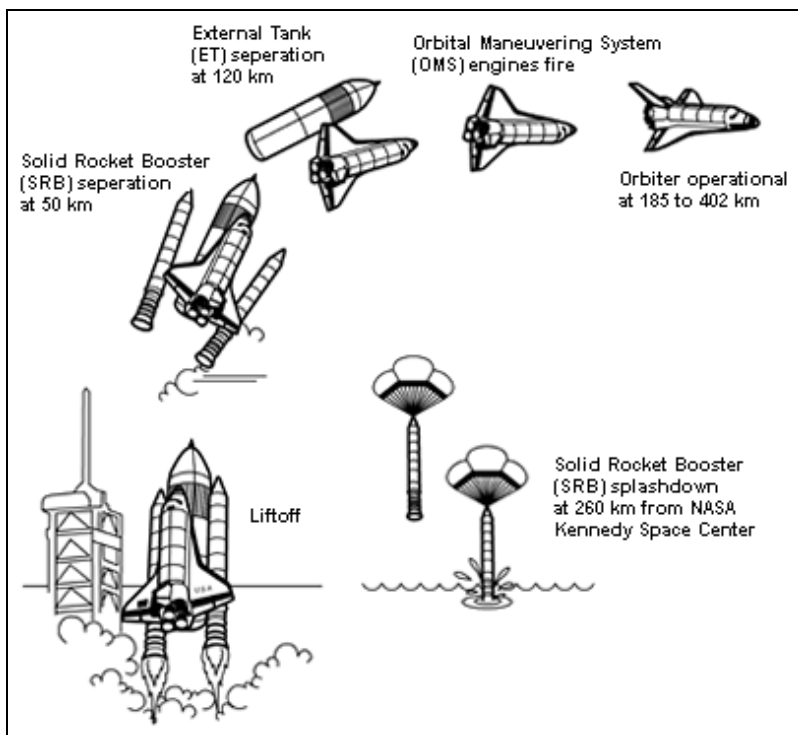


Figure 3: Space Shuttle Ascent Process

The second stage of ascent lasts about six and a half minutes. The space shuttle gains more altitude above Earth and the speed increases to the nearly $7,850 \text{ m/s}$ ($17,500 \text{ mph}$) required to achieve orbit. The main engines are commanded by the onboard computer to reduce power, ensuring that acceleration of the space shuttle does not exceed 29.4 m/s^2 (3 g). Within thirty seconds the space shuttle reaches Main Engine Cut-Off, or MECO. For the next eleven seconds, the space shuttle coasts through space. At nine minutes, the command to jettison the nearly empty external tank is given by the onboard computer leaving the space shuttle's maneuvering engines to control any movement needed to achieve final orbit.

As we look to the future of space exploration, the ascent stage will remain a critical part of any successful mission.



Problem

Open the TI-Nspire document (ShuttleAscent.tns), read through the problem set-up, and complete the questions in the document. Table 1 is also on page 1.7 of the TI-Nspire document. It is provided here for you to refer to as you work through the problem.

Table 1: STS-121 Ascent Data

Time (s)	Altitude (m)	Velocity (m/s)	Acceleration (m/s ²)
0	-8	0	2.45
20	1244	139	18.62
40	5377	298	16.37
60	11,617	433	19.40
80	19,872	685	24.50
100	31,412	1026	24.01
120	44,726	1279	8.72
140	57,396	1373	9.70
160	67,893	1490	10.19
180	77,485	1634	10.68
200	85,662	1800	11.17
220	92,481	1986	11.86
240	98,004	2191	12.45
260	102,301	2417	13.23
280	105,321	2651	13.92
300	107,449	2915	14.90
320	108,619	3203	15.97
340	108,942	3516	17.15
360	108,543	3860	18.62
380	107,690	4216	20.29
400	106,539	4630	22.34
420	105,142	5092	24.89
440	103,775	5612	28.03
460	102,807	6184	29.01
480	102,552	6760	29.30
500	103,297	7327	29.01
520	105,069	7581	0.10

Note: Notice from the table that the altitude is negative at liftoff. Zero altitude can be described as a specific distance from the center of the Earth. Since the Earth is not perfectly spherical the location of the launch just happens to be below this specified point. Also, because this is a calculated number, some degree of error may be present.



- A. Answer the following based on Table 1 (TI-Nspire page 1.7).
- I. Create a scatterplot of the space shuttle's altitude in relation to time. Remember to adjust your viewing window.
 - II. Use the regression functions on your TI-Nspire handheld to determine which type of function best fits the data (i.e. linear, quadratic, cubic, quartic, exponential, power, etc.). Store the equation in $f1(x)$.
- B. Write the function describing the space shuttle's altitude. Let h stand for altitude and t stand for time. In writing the equation, you will need to decide what decimal place to round the coefficients to. An alternative to this is to write the coefficients using significant figures. In that case, you will need to decide how many significant figures to use. Explain your reasoning for your decision. (In your handheld, keep the original equation in $f1(x)$ to be used for following questions.)
- C. Set $f2(x)$, the velocity function, to be the first derivative and apply the first derivative test to $f2(x)$ to determine the relative extrema on the interval $[0, 520]$. Remember to adjust your viewing window. Explain what the extrema represent about the altitude of the space shuttle. How does this compare with the data in Table 1 (TI-Nspire page 1.7)?
- D. Determine the points of inflection of the altitude function. Keep your function needed to find the points of inflection in $f3(x)$. Describe the concavity of the altitude function.
- E. Analyze the altitude function along the interval $[0, 160]$. What is happening to the space shuttle when the concavity changes?



- F. Answer the following based on Table 1 (TI-Nspire page 1.7).
- I. Make a scatterplot of the velocity versus time.
 - II. Explain what is happening to the velocity of the space shuttle over the interval $[0, 520]$.
 - III. Determine the regression equation for velocity. Write the velocity function letting v stand for velocity and t stand for time. Store this equation in $f_4(x)$.
- G. Answer the following based on Table 1 (TI-Nspire page 1.7).
- I. Create a scatterplot of acceleration vs. time.
 - II. Identify where the following points occur on the graph.
 - Max Q
 - Solid Rocket Booster Separation
 - III. Why does the space shuttle's acceleration increase in a quadratic fashion along the interval $[120, 460]$?
 - IV. What is happening to the acceleration on the interval $[480, 500]$ and why?
 - V. Why does the acceleration drop drastically along the interval $[500, 520]$?



H. Using the scatterplot from question G, part I, answer the following:

- I. Is there one function that best fits the acceleration data? Why or why not?

- II. What is the acceleration function that describes the interval $[120, 460]$? Let a stand for acceleration and t stand for time. Store this equation in $f5(x)$ on your handheld. The data for time and acceleration during this interval are stored in ***maintime*** and ***mainacceleration*** in columns E and F of the Lists and Spreadsheets page.

- I. Compare the graphs in $f2(x)$ (the first derivative of the altitude function) and in $f4(x)$, (the velocity function found by doing the regression of the data). Do this by hiding all other equations and plots. Do not clear the other equations. Remember to adjust your viewing window. How do these graphs compare? Explain the differences in what they are representing.

- J. Now compare the graphs in $f3(x)$ (the second derivative of the altitude function) and in $f5(x)$ (the acceleration function found by doing the regression of the data). How do these graphs compare? Explain the differences in what they are representing.