Angles of Depression and Elevation

Math Objectives

- Students will discuss the graphical and algebraic relationships between Angles of Depression and Elevation.
- Students will revisit and apply their right triangle trigonometry skills.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Angle of Elevation
- Angle of Depression
- Parallel
- Alternate Interior Angles
- Bearing

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content Topic 3 Geometry and Trigonometry:
  3.3a Applications of right and non-right-angled triangle Trigonometry, including Pythagoras.
  3.3b Angles of elevation and depression.
  3.3c Construction of labelled diagrams from written statements.
- As a result, students will:
  - Apply this information to real world situations

Teacher Preparation and Notes

- This activity is done with the use of the TI-84 family as an aid to the problems.

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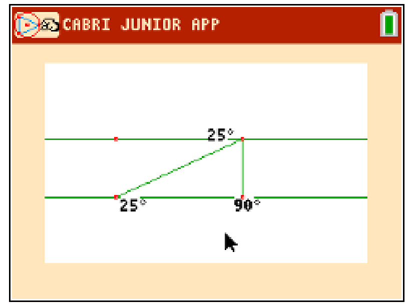
Activity Materials

- Compatible TI Technologies:
  TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

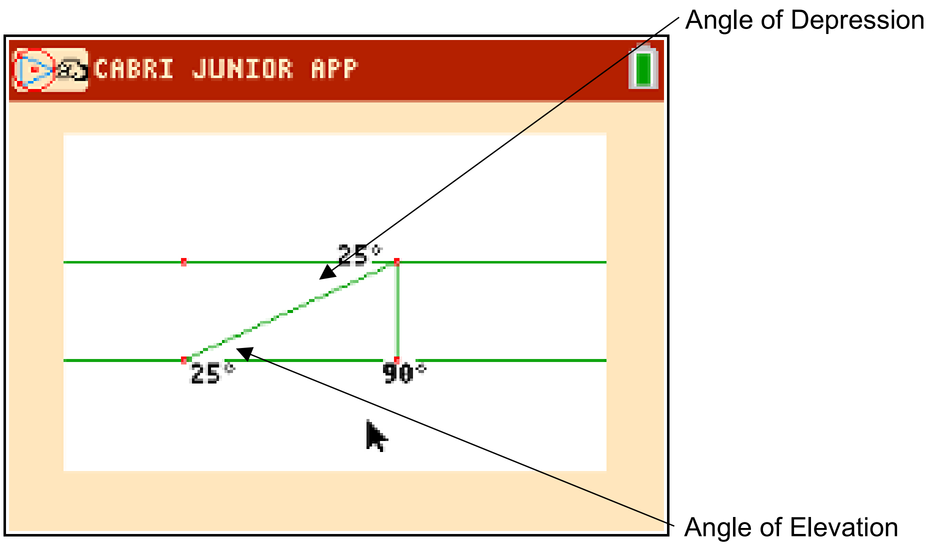
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
Angles_of_Depression_and_Elevation_Student-84.pdf
Angles_of_Depression_and_Elevation_Student-84.doc

The prior knowledge needed for this activity is understanding what the angles of depression and elevation are and what their relationship is with one another.

Looking at the picture below, discuss with a partner the relationships you see. Share your results with the class.

Using the same picture below, label both the angle of elevation and angle of depression. Remember, when looking up, the angle of elevation is the angle created with the person's horizontal line of sight and the upward tilt of their head to look at an object. Also, when looking down, the angle of depression is the angle created with the person's horizontal line of sight and the downward tilt of their head to look at an object.

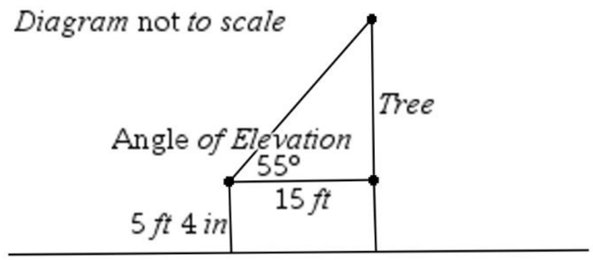


Possible discussion points: Students should be discussing the idea that the horizontal lines of sight are a pair of parallel lines and that the hypotenuse of the right triangle is a transversal. This situation would make the angle of elevation and the angle of depression alternate interior angles, therefore making them equal. Pay close attention to the placement of the angle of elevation outside the right triangle, not inside.

> **Teacher Tip:** Although the topics of right triangle trigonometry, angles of depression and elevation, and their algebraic and graphical relationships is learned in Geometry, Algebra 2, and Precalculus, many students forget about these relationships, how to label graphs with this information, and how to apply them to different problems. Make sure you are circling the classroom as they discuss each of these problems to ensure they are making the connections.

Problem 1

Mr. Jeffries took his Geometry class outside for class today. Their goal was to find the height of the tall tree behind the school. The students are to work in pairs. One pair of students used a tape measure to walk away from the tree and stop 15 ft. away. One of the pair stood at that spot 15 ft. away and looked straight at the tree. Her partner used a measuring app on her phone that allowed her to measure angles with her camera. She lined her phone's camera up with her partner's head and measured the angle of elevation to be 55°. Using the student's information, and the fact that the student who looked up at the tree was 5 ft. 4 in. tall, draw a diagram, label the angle of elevation and find the height of the tree.



$\tan 55° = \frac{tree\ height}{15}$

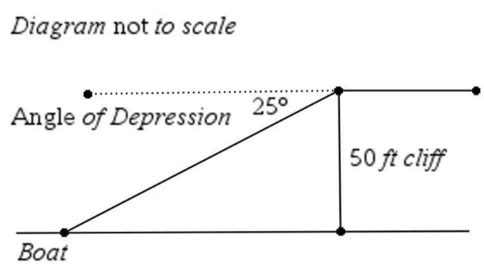
$tree\ height = 15 \tan 55°$

$tree\ height + student\ height = 21.4222 \ldots + 5.333 \ldots$

$tree\ height = 26.8\ ft$

Problem 2

Robert was standing on an Oceanside cliff enjoying the view, when all of a sudden he noticed a fire which seemed to be coming from a boat off shore. He decided he needed to contact the coast guard and give them as much information about the boat as he could. Because he visited this cliff often, he knew the cliff to be 50 ft. high. Robert estimated his angle of depression to be roughly 25°. Draw a diagram, label the angle of depression and find the distance the distressed boat is from the shoreline.



Since the angle of depression is 25°, this is equal to the interior angle by the boat (alternate interior angles). Therefore, the angle opposite the cliff is 25°.

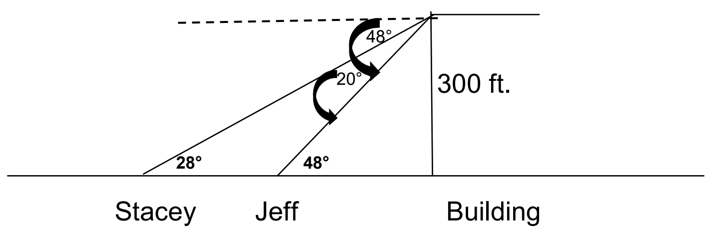
$\tan 25° = \frac{50}{distance\ to\ shore}$

$distance\ to\ shore = \frac{50}{\tan 25°}$

$distance\ to\ shore = 107\ ft$

Problem 3

Alex was bird watching with her binoculars at the top of a 300 ft. building. When she looked down toward the street at an angle of approximately 48°, she noticed her friend Jeff walking toward the building. She then lifted her binoculars another 20° and saw her friend Stacey walking toward the building as well. Using the diagram below, find the distance between Jeff and Stacey, labelling any angle of elevation or depression used.



Solution:

There are several methods students can use to find Stacey's angle of elevation, one possible method is:

$90° - 48° = 42°$

The sum of angles $42° + 20° = 62°$, which makes the angle of elevation at Stacey $90° - 62° = 28°$. You can now find the distance from Stacey to the building.

$\tan 28° = \frac{300}{Stacey\ Distance}$

$Stacey\ Distance = \frac{300}{\tan 28°}$

$Stacey\ Distance = 564.218\ ft$

Given the angle of 48°, this also gave us an angle of elevation from Jeff of 48°, so you can now find the distance from Jeff to the building.

$\tan 48° = \frac{300}{Jeff\ Distance}$

$Jeff\ Distance = \frac{300}{\tan 48°}$

$Jeff\ Distance = 270.121\ ft$

Now to find the distance between Stacey and Jeff we subtract:

$564.218 - 270.121 = 294\ ft$

Problem 4

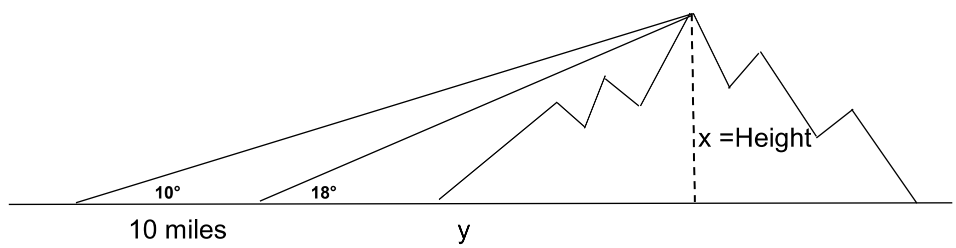
The Scott family was talking a road trip out west and were approaching a mountain range. The family was debating how tall the highest point was in front of them. When they first looked at the top of the mountains their angle of elevation was 10°. After driving another 10 miles, their new angle of elevation was 18°. Using the diagram below, find the approximate height of the highest point of the mountain range labelling the angles of elevation.

Solution:

Create two equations using the given information.

$\tan 18° = \frac{x}{y}$

$\tan 10° = \frac{x}{y+10}$



Now solve for y in the first and substitute it into the second.

$y = \frac{x}{\tan 18°}$

$\tan 10° = \frac{x}{\frac{x}{\tan 18°}+10}$

$\frac{x \tan 10°}{\tan 18°} + 10 \tan 10° = x$

$10 \tan 10° = x - \frac{x \tan 10°}{\tan 18°}$

$10 \tan 10° = x\left(1 - \frac{\tan 10°}{\tan 18°}\right)$

$x = \frac{10 \tan 10°}{1-\frac{\tan 10°}{\tan 18°}}$

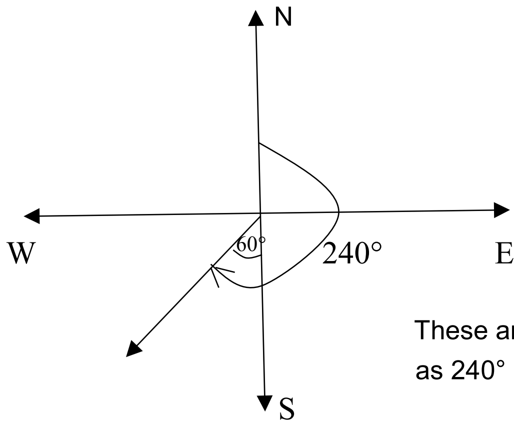
$x = 3.86\ miles$

Teacher Tip: This is a good place to discuss if there may be an easier way to find this height, maybe using non-right-triangle trigonometry.

Further Extension:
Bearings

A bearing is used to indicate the direction of an object from a given point. Three figure bearings are measured clockwise from North and can be written as three figures, such as 135°, 063°, or 275°.

Compass bearings are measured from either N or from S, and can be written such as N 40° E, N 75° W, S 36° E, or S 45° W. Below is an example diagram of a compass bearing.

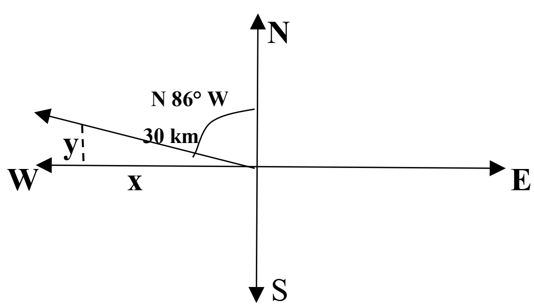


These are equivalent but can be written as 240° (from the north) or S 60° W.

Problem 5
A boat travels on a bearing of N 86° W for 30 km.

(a) How many miles north has the boat travelled?

Solution:



Now that we see what this looks like, we need to find the complement to 86°.
$90° - 86° = 4°$
To find the miles north travelled, we need to find y:
$\sin 4° = \frac{y}{30}$
$y = 2.09\ km$

(b) How many miles west has the boat travelled?

Solution:

To find the miles west travelled, we need to find x:
$\cos 4° = \frac{x}{30}$
$x = 29.9\ km$

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*